

# MIMO MAP Equalization and Turbo Decoding in Interleaved Space–Time Coded Systems

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**Abstract**—Decoding of space–time codes in frequency-selective fading channels is considered. The approach is based on iterative soft-in soft-out equalization and decoding. It is applicable to space–time coded systems that deploy symbol/bit interleavers. We focus on the equalization stage by extending the Ungerboeck equalizer formulation to a multiple-input multiple-output time-variant channel. The resulting structure comprises a bank of matched filters, followed by an *a posteriori* probabilities calculator that runs the Bahl–Cocke–Jelinek–Raviv/maximum *a posteriori* algorithm with an appropriate metric. Simulation results are reported for space–time bit-interleaved codes designed over the enhanced data rates for GSM evolution (EDGE) air interface.

**Index Terms**—Equalizers, iterative decoding, multiple transmit antennas, space–time codes, space–time interleaved codes, turbo processing.

## I. INTRODUCTION

SPACE–TIME (ST) coding has become a fertile research area, since it was shown that spectral efficiency and reliability over fading channels can be achieved by designing in a unified fashion coding and modulation with multiple transmit–receive antennas [1]. Several ST coding approaches have been proposed. Among these are ST trellis codes that combine coding and modulation in a single entity [2], orthogonal ST block codes [3], and ST bit-interleaved codes where coding and modulation are kept separated by a bit interleaver [4]. Although these codes have been designed for flat fading channels, they can be deployed in frequency-selective fading environments provided that equalization is used. The optimal decoding scheme calls for maximum-likelihood joint equalization and decoding using a hypertrellis that is constructed by taking into account the code as well the intersymbol interference (ISI) channel structures. When interleaving is deployed, a practical scheme is suggested by the turbo equalization concept [5], [6]: disjoint soft-in soft-out equalization and decoding, feedback from the decoder, and iterative processing. The optimum maximum *a posteriori* (MAP) ST equalizer computes soft information by joint equalization of the simultaneously transmitted signals [7]. We refer to it as a multiple-input multiple-output (MIMO) MAP equalizer.

For single-input single-output ISI channels, two well-known formulations of the maximum-likelihood sequence estimator

(MLSE)/MAP receiver have been developed. In Forney’s formulation [8], a sequence estimation algorithm that uses the Euclidean distance metric follows a whitening matched filter. In Ungerboeck’s formulation [9], the sequence estimation algorithm operates directly on the matched-filter output using a modified metric. The equivalence of the two receivers has been shown in [10]. In this letter, we follow Ungerboeck’s approach to derive a ST equalizer that is capable of delivering the *a posteriori* probabilities of the coded bits/symbols that are transmitted on overlapping time-variant ISI channels. The resulting structure comprises a bank of time-variant matched filters followed by a MAP/Bahl–Cocke–Jelinek–Raviv (BCJR) [11] processor that uses a modified metric. The modified metric requires an *a priori* transition probability that can be estimated by the outer decoder.

Recently, turbo ST processing has been proposed also in [12] and [13] for decoding of ST block/trellis codes with interleavers. Both [12] and [13] follow the approach of deploying a spatial–temporal whitening MIMO prefilter [14]. Further, the layered ST architecture in [13] requires multiple receive antennas. Instead, in our approach, the spatial–temporal whitening operation, as well as multiple receive antennas, are not required. It should also be noted that the ST decoding problem is intrinsically a multiuser detection problem that has been extensively investigated in the code-division multiple access (CDMA) literature [15].

In this letter, the proposed iterative equalization/decoding approach is described for application to a ST bit-interleaved coded architecture, Sections II–III. It can be applied for equalization and decoding of ST block/trellis codes with symbol interleavers [12], by concatenating the proposed ST equalizer with a soft-in soft-out decoder, e.g., MAP decoder, that exploits the structure of the ST code. In Section IV, we consider, as an example, the deployment of ST bit-interleaved codes over the enhanced data rates for GSM evolution (EDGE) air interface [16].

## II. ST BIT-INTERLEAVED CODED SYSTEM MODEL

We consider a ST bit-interleaved coded system [4], [7] as depicted in Fig. 1. A convolutional encoder encodes a sequence of bits  $\{b_i\}$  into a coded bit sequence  $\{c_i\}$ . The coded bit sequence is appropriately interleaved and parsed into  $N_T$  branch sequences  $\{d_i^a\}$ ,  $a = 1, \dots, N_T$ . Each branch bit sequence is mapped into a complex symbol sequence  $\{x_k^a\}$  with a memoryless  $M$ -phase-shift keying (PSK) or  $M$ -quadrature amplitude modulation (QAM) mapper. After pulse shaping and radio frequency (RF) modulation, the signals are simultaneously transmitted by the transmit antennas. Let  $g_T(t)$  be the pulse shaping filter, then, the complex signal transmitted by antenna  $a$  is expressed as  $x^a(t) = \sum_{k=-\infty}^{+\infty} x_k^a g_T(t - kT)$ .

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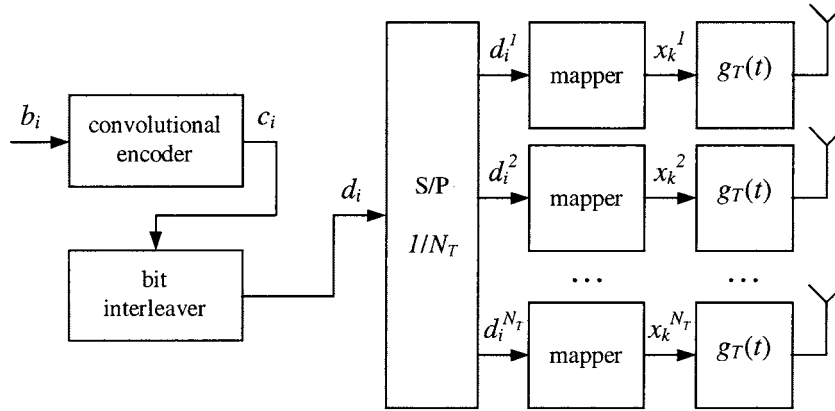


Fig. 1. ST bit-interleaved coded system.

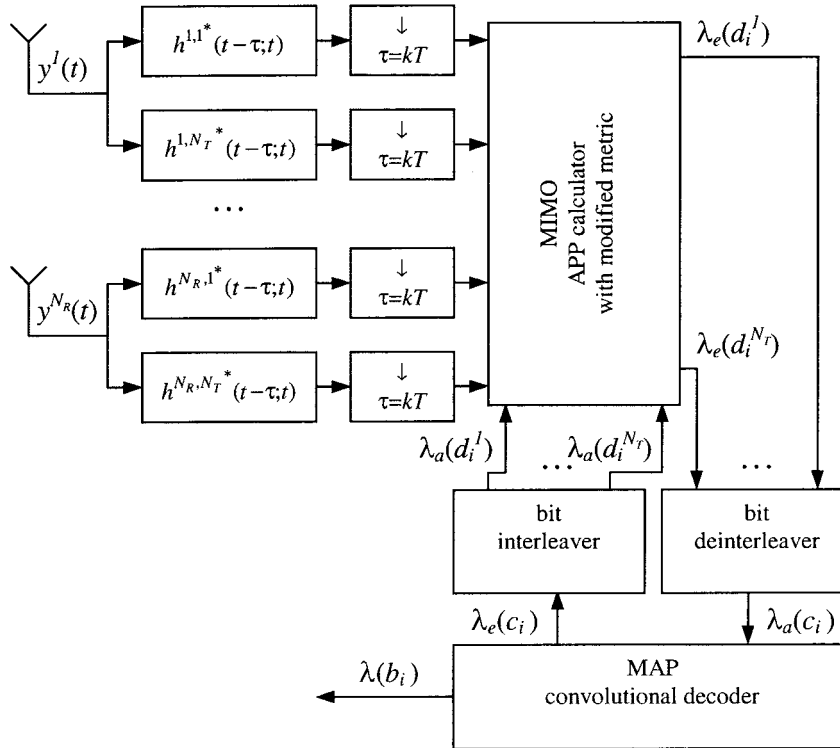


Fig. 2. Iterative (turbo) equalization and decoding receiver structure for ST bit-interleaved coded modulation.

The bit-interleaver is deployed to remove the correlation in the sequence of convolutionally encoded bits, which is an essential condition for turbo decoding. Further, it can be used to decorrelate the fading channel and maximize the diversity order of the system. When dealing with channels with memory, the overall system can be treated as a serially concatenated and interleaved coded system [5], [18]. Guidelines for the design of ST bit-interleaved codes that achieve full diversity are described in [4] and [17].

We assume deploying an array of  $N_R$  receive antennas (Fig. 2) and the channel to be time-variant with lowpass impulse response  $g_{ch}^{b,a}(\tau;t)$  (between receive antenna  $b$  and transmit antenna  $a$ ). Thus, the baseband complex received

signal at the output of the RF carrier demodulator [9] of receive antenna  $b$  is

$$y^b(t) = \sum_{k=-\infty}^{+\infty} \sum_{a=1}^{N_T} x_k^a h^{b,a}(\tau - kT; t) + \eta^b(t). \quad (1)$$

In (1),  $h^{b,a}(\tau;t) = \int_{\mathcal{R}} g_T(\tau - \lambda) g_{ch}^{b,a}(\lambda;t) d\lambda$  is the convolution of the pulse shaping filter with the lowpass channel impulse response at time  $t$ . The fading channel is, in general, considered frequency selective, such that ISI may arise. The complex noise processes  $\eta^b(t)$  are assumed to be zero-mean stationary white Gaussian and to be independent across the receive antenna branches. Further, we assume that the noise processes in

the transmission medium have double-sided power spectral density  $2N_0$ .

### III. ITERATIVE ST EQUALIZATION AND DECODING

Concatenating (Fig. 2) in an iterative fashion a ST soft-in soft-out equalizer with a soft-in soft-out convolutional decoder performs decoding [5], [7]. The equalizer delivers to the decoder the *a posteriori* probabilities of the coded bits<sup>1</sup> by observing the received signals over a finite time interval, as described in the next section. The decoder can be implemented with the MAP algorithm [11], [19] and provides new and improved *a posteriori* probabilities of the coded bits by exploiting the redundancy of the code. According to the turbo principle, extrinsic information  $(\lambda_e(d_i^a), \lambda_e(c_i))$  in Fig. 2) has to be exchanged between the equalizer and decoder in order to minimize the correlation with previously computed information. The extrinsic information on a given bit is defined as the *a posteriori* probability of that bit by taking into account all transmitted bits except the bit under consideration. If we operate in the logarithm domain, then the extrinsic information is obtained by subtracting the input log-likelihoods from the output log-likelihoods. In the final iteration, the decoder provides the decoded information bit sequence.

#### A. ST MIMO MAP Equalizer

The equalizer computes the *a posteriori* probabilities  $\lambda(d_i^m = \pm 1)$  of each coded and interleaved bit  $d_i^m$ ,  $m = 1, \dots, N_T$ , observing the received signals  $y^b(t)$ ,  $b = 1, \dots, N_R$ , over a finite time interval  $I$ , i.e.,  $\lambda(d_i^m = \pm 1) = P[d_i^m = \pm 1 | \underline{y}(t), t \in I]$ , where  $\underline{y}(t) = \{y^1(t), \dots, y^{N_R}(t)\}$ .

Now, consider the *a posteriori* probability of a given coded bit sequence  $\{\hat{d}_i\} = \{d_i^a, a = 1, \dots, N_T\}$ , i.e.,  $P[\{\hat{d}_i\} | \underline{y}(t)]$ . Since there is a one-to-one correspondence between the sequence of coded bits  $\{\hat{d}_i\}$  transmitted over the interval  $I$  and the sequence of symbols  $\{\hat{x}_k\} = \{\hat{x}_k^1, \dots, \hat{x}_k^{N_T}, kT \in I\}$ , we have  $P[\{\hat{d}_i\} | \underline{y}(t)] = P[\{\hat{x}_k\} | \underline{y}(t)]$ . From Bayes' rule and under the assumption of the bits to be independent due to interleaving, we obtain

$$P[\{\hat{x}_k\} | \underline{y}(t)] = K P[\underline{y}(t) | \{\hat{x}_k\}] \prod_{kT \in I} \prod_{a=1}^{N_T} \prod_{l=1}^N P[\hat{d}_{Nk+l}^a] \quad (2)$$

where  $K = 1/P[\underline{y}(t)]$ ,  $N = \log_2 M$  is the number of bits per antenna symbol, and  $P[\hat{d}_{Nk+l}^a]$  is the probability that bit  $d_{Nk+l}^a$  equals  $\hat{d}_{Nk+l}^a$ .

To proceed, let us define the state at time  $(k-1)T$  for some finite  $N_s$  as  $\hat{S}_{k-1} = (\hat{x}_{k-N_s+1}^1, \dots, \hat{x}_{k-N_s+1}^{N_T})$ ,

<sup>1</sup>It is equivalent to operating by exchanging between the equalizer and the decoder *a posteriori* probabilities  $\lambda(d_i = \pm 1) = P[d_i = \pm 1]$ , *a posteriori* log-likelihoods  $\Lambda(d_i = \pm 1) = \ln \lambda[d_i = \pm 1]$ , or *a posteriori* log-likelihood ratios  $L(d_i) = \Lambda(d_i = 1) - \Lambda(d_i = -1)$  [19].

$\dots, \hat{x}_{k-1}^1, \dots, \hat{x}_{k-1}^{N_T}$ ). Following Ungerboeck's equalizer formulation [9] (see the Appendix), the *a posteriori* probability of a coded bit sequence can be factored, apart from constant multiplicative terms, as

$$P[\{\hat{x}_k\} | \underline{y}(t)] \sim \prod_{kT \in I} \underbrace{\gamma_{ch}(\hat{S}_{k-1}, \hat{S}_k)}_{\gamma_k(\hat{S}_{k-1}, \hat{S}_k)} P(\hat{S}_k | \hat{S}_{k-1}) \quad (3)$$

where the *a priori* transition probability is defined as

$$P(\hat{S}_k | \hat{S}_{k-1}) = \prod_{a=1}^{N_T} \prod_{l=1}^N P[\hat{d}_{Nk+l}^a] \quad (4)$$

while the channel transition probability is defined as shown in (5) at the bottom of the page ( $\Re$  denotes real part).

Assuming knowledge of the equivalent channel impulse responses, the  $z$  parameters and  $s$  parameters in (5) are computed as follows:

$$z^{b,a}(kT) = \int_I h^{b,a*}(t - kT; t) y^b(t) dt \quad (6)$$

$$s^{b,a,c}(kT; nT) = \int_I h^{b,a*}(t - kT; t) h^{b,c}(t - nT; t) dt. \quad (7)$$

Given the factorization in (3), the *a posteriori* probabilities of the coded bits can be computed by the application of the MAP/BCJR algorithm [11]. From [19], the *a posteriori* probabilities of coded bit  $d_i^m$ , transmitted during time period  $kT$  by antenna  $m$ , can be obtained as

$$\lambda(d_i^m = \pm 1) \sim \sum_{(S_{k-1}, S_k) \in D(\pm 1)} \alpha_{k-1}(S_{k-1}) \gamma_k(S_{k-1}, S_k) \beta_k(S_k) \quad (8)$$

where the sum in (8) is computed over all state transitions corresponding to  $d_i^m = \pm 1$  with  $i = Nk + l$ , for a given  $l = 1, \dots, N$ . Further,  $\alpha_k(S_k)$  and  $\beta_{k-1}(S_{k-1})$  are recursively computed as

$$\alpha_k(S_k) = \sum_{S_{k-1} \in \Sigma} \alpha_{k-1}(S_{k-1}) \gamma_k(S_{k-1}, S_k) \quad (9)$$

$$\beta_{k-1}(S_{k-1}) = \sum_{S_k \in \Sigma} \beta_k(S_k) \gamma_k(S_{k-1}, S_k) \quad (10)$$

with  $\Sigma$  being the set of all possible states. The cardinality of such a set is  $|\Sigma| = M^{N_T(N_s-1)}$  with  $M$  modulation order and  $N_s$  such that  $s^{b,a,c}(kT; (k-m)T) = 0$  for  $|m| \geq N_s$ .  $N_s$  is finite assuming time-limited pulse shaping filters and channel impulse responses.  $\alpha_0(S_0)$  and  $\beta_L(S_L)$  are initialized with the known starting and ending states associated to the block of length  $L$  that we process.

#### B. Remarks

The  $z$  parameters are obtained by convolving each received signal with a bank of time-variant filters. The filters are matched to the equivalent impulse response that comprises the transmit

$$\gamma_{ch}(\hat{S}_{k-1}, \hat{S}_k) = \exp \left\{ \frac{1}{4N_0} \sum_{b=1}^{N_R} \sum_{a=1}^{N_T} \Re \left\{ \hat{x}_k^{a*} \left[ 2z^{b,a}(kT) - \sum_{c=1}^{N_T} \hat{x}_k^c s^{b,a,c}(kT; kT) - 2 \sum_{c=1}^{N_T} \sum_{m>0} \hat{x}_{k-m}^c s^{b,a,c}(kT; (k-m)T) \right] \right\} \right\}. \quad (5)$$

pulse and the time-variant channel. The outputs are then sampled at rate  $1/T$ . The  $s$  parameters are obtained by crosscorrelating the impulse responses of the transmit–receive antenna links. Each received signal is used in a maximal ratio combining fashion into the channel transition probability. For static channels, i.e., with no variation over the processing time window, the  $s$  parameters are time invariant and can be computed offline once, which simplifies complexity. In fact

$$\begin{aligned} s^{b,a,c}(kT; nT) &= s^{b,a,c}(kT - nT) \\ &= \int_{t+kT \in I} h^{b,a^*}(t) h^{b,c}(t + kT - nT) dt. \end{aligned} \quad (11)$$

If we assume a tapped delay line channel model  $g_{ch}^{b,a}(\tau; t) = \sum_{p=0}^{N_p} \alpha_p^{b,a}(t) \delta(\tau - pT)$  that can be considered time invariant over the duration of the transmit pulse, the  $z$ – $s$  parameters can be written, respectively, as

$$z^{b,a}(kT) = \sum_{p=0}^{N_p} \alpha_p^{b,a^*}(kT + pT) \hat{z}^b(kT + pT) \quad (12)$$

with  $\hat{z}^b(kT) = \int_I g_T^*(t - kT) y^b(t) dt$  and

$$\begin{aligned} s^{b,a,c}(kT; nT) &= \sum_{p=0}^{N_p} \sum_{p'=0}^{N_p} \left\{ \alpha_p^{b,a^*}(kT + pT) \right. \\ &\quad \left. \cdot \alpha_{p'}^{b,c}(kT + pT) \kappa_T(kT + pT - nT - p'T) \right\} \end{aligned} \quad (13)$$

with  $\kappa_T(\tau) = \int_I g_T^*(t) g_T(\tau + t) dt$ . If the autocorrelation of the transmit pulse is Nyquist, then  $\kappa_T(iT) = \delta(i)$ . It follows that by substituting such expressions into (5) (or equivalently, into (A6) and (A7) of the Appendix), the channel transition probability becomes

$$\begin{aligned} \gamma_{ch}(\hat{S}_{k-1}, \hat{S}_k) &\sim \exp \left\{ -\frac{1}{4N_0} \sum_{b=1}^{N_R} \left| \hat{z}^b(kT) \right. \right. \\ &\quad \left. \left. - \sum_{a=1}^{N_T} \sum_{p=0}^{N_p} \hat{x}_{k-p}^a \alpha_p^{b,a}(kT) \right|^2 \right\}. \end{aligned} \quad (14)$$

That is, the channel metric is based on the well-known Euclidean distance metric [7]. Further, the sequence of samples  $\hat{z}^b(kT)$  is obtained by filtering the received signal with a filter matched to the transmit pulse only.

The evaluation of the transition probability  $\gamma_k(S_{k-1}, S_k)$  requires knowledge of the corresponding *a priori* probability of the coded bits. It is, in practice, approximated with the interleaved extrinsic information provided by the outer decoder in the previous decoding iteration, i.e.,  $P[\hat{d}_{Nk+l}^a] = \lambda_a(\hat{d}_{Nk+l}^a)$ . Equally likely bits are assumed at the first equalization stage. The extrinsic *a posteriori* probabilities at the equalizer output are computed as  $\lambda_c(d_i^m = \pm 1) = \lambda(d_i^m = \pm 1) / \lambda_a(d_i^m = \pm 1)$ . This corresponds to evaluate (8) by taking into account, in the transition probability, the extrinsic information of all transmitted bits except the one under consideration.

We can compute the *a posteriori* probabilities of the coded symbols  $P[x_k^m | \underline{y}(t)]$  (instead of the coded bits) by simply com-

TABLE I  
CODE RATES AND MODULATION ORDERS FOR ST-BIC EDGE  
( $N_B$ : length of information bits block.  $R$ : convolutional code rate.)

Transmission Rate	$N_B$	1 TX	2 TX
1 Bit/s/Hz	400	$R=1/3$ , 8-PSK	$R=1/4$ , 4-PSK
2 Bit/s/Hz	800	$R=2/3$ , 8-PSK	$R=1/2$ , 4-PSK
3 Bit/s/Hz	1200	$R=1/1$ , 8-PSK	$R=1/2$ , 8-PSK

puting the sum in (8) over the set of state transitions corresponding to the desired symbol. For instance, this is the goal in an ST-coded system that deploys block/trellis codes with symbol interleavers [12]. In this case, the outer decoder has to provide the extrinsic state transition probabilities  $P(\underline{S}_k | \underline{S}_{k-1})$ , since the assumption of independence is valid at symbol level, and the bit-wise factorization in (4) does not hold.

The complexity of the equalizer algorithm is determined by the number of states, equal to  $|\Sigma| = M^{N_T(N_S-1)}$  and the number of transitions to/from each state, equal to  $M^{N_T}$ . Complexity reduction in the equalization stage can be achieved by operating in the logarithm domain and by introducing the MAX-LOG-MAP implementation of the BCJR algorithm [20]. Further, state reduction techniques can also be applied such as the  $M$ -algorithm [21] or set partitioning with decision feedback techniques [22].

#### IV. EXAMPLE OF APPLICATION TO THE EDGE SYSTEM

As an example, we consider the application of ST bit-interleaved coded modulation (STBICM) to the EDGE air interface [16]. According to [16], transmission rates of one, two, and three bits/s/Hz are achieved by deploying bit-interleaved tail terminated convolutional codes, with 8-PSK modulation. The transmission symbol period is  $T = 3.69 \mu s$ . A Gaussian filter (with normalized bandwidth equal to 0.3) is used for partial response pulse shaping. Blocks of coded bits are interleaved across four bursts. The bursts are frequency hopped. For low-mobility applications, we can assume that such bursts experience independent Rayleigh fading, with no channel variation over a single burst.

We seek performance enhancements through the deployment of ST bit-interleaved codes with two transmit antennas. This would allow minimal standard changes. We follow the code-construction criteria in [4] and [17], such that the bits at the output of the convolutional encoder are parsed into two streams. Each antenna bit stream is randomly interleaved and transmitted over four bursts. Code rates up to 1/4 are obtained by puncturing (from right to left) of a mother code with polynomials in octal notation (133, 171, 145, 145) for both the single and double transmit antenna system. In Table I, we summarize code rates, modulation orders and sizes of the information bit blocks. The mapping from bits to symbols is in accordance to the Gray rule.

Now, in Fig. 3, we report block-error rate simulation results versus average signal energy-to-noise ratio (normalized over the number of transmit antennas) for the coding schemes in Table I. Perfect knowledge of the channel state information is assumed. The COST typical urban (TU) channel model [16] is assumed

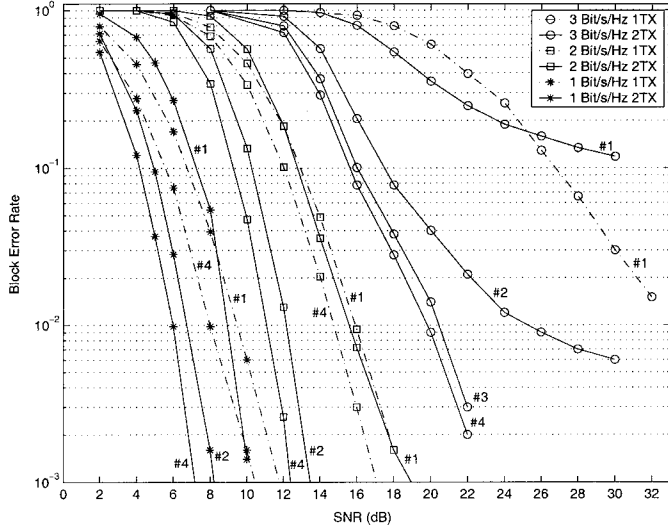


Fig. 3. Block error rate performance of STBICM over the EDGE system air interface with two transmit and one receive antennas and turbo MAP equalization (solid lines). Performance with one transmit antenna is also shown (dashed lines). Labels indicate the decoding iteration number. Convolutional coding rates and modulations specified in Table I. COST TU channel model. Transmission over four bursts with ideal frequency hopping and static fading on each burst. Uncorrelated fading across distinct antenna links.

with independent Rayleigh faded rays. The rays' relative power and delay are

$$P_{\text{dB}} = [-4, -3, 0, -2.6, -3, -5, -7, -5, -6.5, -8.6, -11, -10]$$

$\tau_{\mu s} = [0, 0.1, 0.3, 0.5, 0.8, 1.1, 1.3, 1.7, 2.3, 3.1, 3.2, 5]$ . The transmit-receive branches are assumed uncorrelated. With such a channel model, the ISI spans about three symbols requiring an equalizer with up to  $|\Sigma| = 4096$  states with 8-PSK and two transmit antennas. Complexity reduction is obtained by using the MAX-LOG-MAP approximation in both the equalizer and the decoder. Further, the number of states in the equalizer is reduced through decision feedback [22]. As we move on the trellis we include in the channel metric computation past decisions for the most recent symbols, i.e., in the transition  $(S_{k-1}, S_k)$  we use hard decisions for the symbols  $x_{k-2}^1, \dots, x_{k-2}^{N_T}, x_{k-3}^1, \dots, x_{k-3}^{N_T}, \dots$  that at time  $(k-1)T$  are associated to all taps with the exception of the main one. The hard decisions are obtained as feedback from the path history associated to the state  $S_{k-1}$ . Therefore, the number of trellis states reduces to  $|\Sigma_{\text{red}}| = M^{N_T}$ , in particular, with a single transmit antenna and 8-PSK, it reduces to  $|\Sigma_{\text{red}}| = 8$ , while with two transmit antennas it becomes  $|\Sigma_{\text{red}}| = 16$  with 4-PSK and  $|\Sigma_{\text{red}}| = 64$  with 8-PSK.

Curves with up to four iterations are shown for both the single and double transmit antenna system. At block-error rate =  $10^{-2}$  the double transmit antenna system gains 2, 4, and 13 dB over the single transmit antenna system, respectively, for data rates of 1, 2, and 3 bits/s/Hz. An error floor appears at the first decoding iteration in the 2 and 3 bits/s/Hz curves. This is due to the deployment of decision feedback; however, it is removed through iterative processing, as Fig. 3 shows.

## V. CONCLUSION

We have considered iterative (turbo) equalization and decoding of interleaved ST codes. From Ungerboeck's equalizer formulation, a MAP equalizer has been derived for general time-variant MIMO ISI channels. This ST equalizer is optimum in the *a posteriori* sense, and does not necessarily require multiple receive antennas. The performance of the proposed decoding approach has been evaluated over the EDGE air interface when deploying ST bit-interleaved convolutional codes. Other, performance results for various ST bit-interleaved coding schemes and channel scenarios are reported in [4], [7], [17], and [23]. The results show that this coding/decoding approach allows full exploitation of the temporal, spatial, and frequency diversities available in the system.

## APPENDIX

### CHANNEL PROBABILITY DENSITY FUNCTION (PDF) IN TIME-VARIANT MIMO CHANNEL

Let us assume a time-variant MIMO channel with equivalent impulse responses  $h^{b,a}(\tau; t)$  (between transmit antenna  $a$  and receive antenna  $b$ ), and let us assume zero-mean stationary complex additive white Gaussian noise processes that are independent across the receive branches. Then, the conditional channel PDF can be expressed as follows:

$$p\left[\underline{y}(t), t \in I \mid \{\hat{\underline{x}}_k\}\right] \sim \exp\left\{-\frac{1}{4N_0} \sum_{b=1}^{N_R} \Omega^b(\{\hat{\underline{x}}_k\})\right\} \quad (\text{A1})$$

$$\begin{aligned} \Omega^b(\{\hat{\underline{x}}_k\}) &= \int_I |y^b(t)|^2 \\ &\quad - \sum_{a=1}^{N_T} \sum_{kT \in I} \hat{x}_k^a h^{b,a}(t - kT; t) \Big|^2 dt \\ &= A + B + C. \end{aligned} \quad (\text{A2})$$

Following [9], the terms  $A$ ,  $B$ , and  $C$  are defined as follows:

$$A = \int_I |y^b(t)|^2 dt \quad (\text{A3})$$

$$B = - \int_I 2\Re \left\{ \sum_{a=1}^{N_T} \sum_{kT \in I} \hat{x}_k^{a*} h^{b,a*}(t - kT; t) y^b(t) \right\} dt \quad (\text{A4})$$

$$\begin{aligned} C &= \int_I \left\{ \sum_{a=1}^{N_T} \sum_{kT \in I} \hat{x}_k^{a*} h^{b,a*}(t - kT; t) \right. \\ &\quad \cdot \left. \sum_{c=1}^{N_T} \sum_{nT \in I} \hat{x}_n^c h^{b,c}(t - nT; t) \right\} dt. \end{aligned} \quad (\text{A5})$$

If we define the  $z$  parameters and  $s$  parameters as in (6) and (7), the terms  $B$  and  $C$  can be rewritten as

$$B = -2\Re \left\{ \sum_{a=1}^{N_T} \sum_{kT \in I} \hat{x}_k^{a*} z^{b,a}(kT) \right\} \quad (\text{A6})$$

$$C = \sum_{a=1}^{N_T} \sum_{kT \in I} \sum_{c=1}^{N_T} \sum_{nT \in I} \hat{x}_k^{a*} \hat{x}_n^c s^{b,a,c}(kT; nT). \quad (\text{A7})$$

Further

$$\begin{aligned}
C = & \sum_{a=1}^{N_T} \sum_{c=1}^{N_T} \sum_{kT \in I} \hat{x}_k^{a*} \hat{x}_k^c s^{b,a,c}(kT; kT) \\
& + \sum_{a=1}^{N_T} \sum_{c=1}^{N_T} \sum_{kT \in I} \sum_{n < k} \left\{ \hat{x}_k^{a*} \hat{x}_n^c s^{b,a,c}(kT; nT) \right. \\
& \left. + \hat{x}_n^{a*} \hat{x}_k^c s^{b,a,c}(nT; kT) \right\}. \quad (\text{A8})
\end{aligned}$$

Since  $s^{b,a,c}(kT; nT) = s^{b,c,a^*}(nT; kT)$ , both the main additive terms in (A8) are real. Further, if we exchange the indexes  $a$  with  $c$  in the second term and we set  $m = k - n$ , the term  $C$  can be written as follows:

$$\begin{aligned}
C = & \sum_{kT \in I} \Re \left\{ \sum_{a=1}^{N_T} \sum_{c=1}^{N_T} \hat{x}_k^{a*} \hat{x}_k^c s^{b,a,c}(kT; kT) \right. \\
& + 2 \sum_{a=1}^{N_T} \sum_{c=1}^{N_T} \sum_{m > 0} \hat{x}_k^{a*} \hat{x}_{k-m}^c \\
& \left. \cdot s^{b,a,c}(kT; (k-m)T) \right\}. \quad (\text{A9})
\end{aligned}$$

Thus, (5) is easily derived, summing  $A$ ,  $B$ , and  $C$ , i.e., (A1) can be written as the product of the channel transition probabilities in (5).

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