

RESEARCH ARTICLE

Energy Efficient Resource Allocation for Quantity of Information Delivery in Parallel Channels[†]

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ABSTRACT

This paper deals with the problem of minimizing the energy required to transmit a certain amount of information bits. If there is no constraint, in a Gaussian channel, the solution leads us to the use of an infinite transmission time or an infinite bandwidth. In particular, the minimum required signal-to-noise ratio to transmit one bit is equal to -1.59 dB. To allow the development of new green communication services, a finite bandwidth and a finite time have to be used in practice. Thus, we focus on the transmission of a given number of bits over a set of parallel Gaussian channels when there is an energy constraint and we have the goal of minimizing either the highest transmission time (among the channels) or the average channel occupancy time. This is a resource allocation problem that is formulated by targeting a certain energy consumption factor, herein defined as the ratio between the energy required in the asymptotic regime and the energy required to transmit a certain quantity of information with limited (time, bandwidth) resources. The Pareto front for the joint optimization of the maximum transmission time and the average channel occupancy time is also derived. Several explicative numerical results of the derived resource allocation algorithms for energy efficient communications are also reported. Copyright © 0000 John Wiley & Sons, Ltd.

1. INTRODUCTION

Energy consumption of data communication systems is becoming an important issue due to the large use of devices. The evaluation of energy consumption can be done from various perspectives which include implementation and hardware issues as circuit consumption and non ideal performing signal processing algorithms for data recovery [1, 2].

In fact, it is well-known that the minimum signal energy per information bit that is required for reliable

communications in a Gaussian channel can be obtained from the minimum signal-to-noise ratio (SNR) that is equal to -1.59 dB. This result was firstly derived in [3] in the asymptotic regime assuming that the transmission of the information requires an infinite amount of time. More recently it has been extended to a general class of channels in [4] and it has been shown that it can be achieved as the bandwidth goes to infinity. In the green communications context, this lower limit gives the minimal transmit energy per bit required for reliable communication. Consequently, the efficiency of green communication systems can be measured using the bit-per-Joule (bit/J) metric [1, 5]. This performance metric has been studied taking into account various aspects both with pragmatic and information

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theoretic approaches [6]. In this paper we only focus on the energy efficiency in relation to capacity.

With delay tolerant applications and networks, the constraint on the time of transmission can be relaxed [7, 8] and the solution to the problem of minimizing the transmit energy leads to the use of an infinite time of transmission. However, from an information theoretic point of view, it becomes interesting to study and develop resource allocation schemes that ensure reliable communications with a given energy constraint near to the asymptotic energy limit but assuming a finite bandwidth and time of transmission. The objective is then to find the feasible and bounded set of parameters that provide a given energy efficiency. From an operational point of view, focusing on energy efficiency, instead of spectrum efficiency, can lead to the development of new and green communication services that exploit the underloaded usage periods of the networks and the mechanisms of load-shedding [9]. More than two orders of energy gain can then be expected [10, 11].

In this paper, to focus on the system energy efficiency and formulate the objective, we start by defining the energy efficiency factor β as the ratio between the asymptotic energy limit and the energy required to transmit an amount of bits with limited (time, bandwidth) resources. Then, the optimization problem becomes a resource allocation problem under the constraint given by the defined energy efficiency factor and under the constraint given by the quantity of information to be transmitted. The problem is solved for parallel independent Gaussian channels. The focus is given on particular solutions that minimize the time of transmission defined as the maximum among the transmission periods in the set of channels (referred to as transmission time), or on the solutions that minimize the average of the transmission periods in the set of channels (referred to as average channel occupancy time).

The remainder of the paper is organized as follows. Section 2 introduces the model of the communication system. Section 3 recalls some results on energy minimization. In this section, we also derive the energy lower bound for parallel channels and we analyze the new energy efficiency metric. The general problem is formulated in Section 4 and two simple cases are analyzed. Particular solutions are analysed in Section 5. These solutions address the problem of minimizing the transmission time and the problem of minimizing the

average channel occupancy time, two problems that are stated and studied in this Section 5. The joint optimization of these two time parameters is studied in Section 6. Section 7 concludes the paper.

2. SYSTEM MODEL

We consider n parallel independent channels. On the i th channel, the input-output relationship is

$$Y_i = h_i X_i + W_i, \quad i = 1, \dots, n \quad (1)$$

where X_i denotes the transmitted data signal with power P_i , Y_i is the received signal, and h_i is the complex scalar channel gain. The additive noise sample W_i is assumed to be a complex Gaussian random variable with zero-mean. The noise is assumed independent across the channels and white in the transmission band B_i , with power spectral density N_i .

With a Gaussian input signal, the maximum input-output mutual information in channel i , is the well-known channel capacity measured in bit-per-second-per-Hertz

$$C_i = \log_2 \left(1 + |h_i|^2 \frac{P_i}{B_i N_i} \right). \quad (2)$$

It should be noted that (2) can be used to derive a tight approximation of the maximal number of bits that can be transmitted when the number of bits is large. Other bounds can be obtained with finite and low number of bits to be transmitted [12].

The maximum quantity of information expressed in bit that can be transmitted in T_i seconds in the channel with bandwidth B_i is $Q_i = C_i \times B_i \times T_i$. The corresponding consumed energy, which is the transmit energy during the transmission period T_i , is

$$J_i = P_i \times T_i = \left(2^{\frac{Q_i}{B_i}} - 1 \right) \frac{B_i T_i}{\gamma_i}, \quad (3)$$

where $\gamma_i = \frac{|h_i|^2}{N_i}$ is the normalized SNR in channel i obtained with unit transmit power and bandwidth. This energy is required to transmit the information bits Q_i during the transmission period T_i in channel i when the channel state information is known at both the transmitter and receiver sides.

Now, with n parallel channels, the total energy required to transmit $Q = \sum Q_i$ bits becomes

$$J = \sum_{i=1}^n J_i = \sum_{i=1}^n (2^{\frac{Q_i}{B_i T_i}} - 1) \frac{B_i T_i}{\gamma_i}. \quad (4)$$

In this paper, we provide results in the asymptotic regime assuming that Q_i and T_i are sufficiently large so that (2) and (3) give valid and sufficiently tight results. In the non-asymptotic regime, bounds tighter than the one given by the capacity formula (2) can be used [13].

3. ENERGY CONSIDERATIONS AND PRELIMINARIES

This section recalls some known results on energy consumption adapted to our system model. An energy efficiency metric, called energy efficiency factor, is also introduced and compared to the conventional energy efficiency metric.

3.1. Energy consumption minimization

We first formulate the allocation problem where the objective is the quantity of information to be transmitted and the resource is the energy to be minimized. This is obtained starting from the power allocation problem to maximize the transmission rate in Gaussian parallel channels.

Proposition 1. Under a certain transmission period allocation $\{T_i\}_{i=1}^n$, the information bit allocation $\{Q_i\}_{i=1}^n$ which minimizes the energy needed to transmit Q bits of information is

$$\begin{aligned} Q_i^* &= 0, \quad \lambda \gamma_i \leq 1, \\ Q_i^* &= B_i T_i \log_2(\lambda \gamma_i), \quad \lambda \gamma_i > 1, \end{aligned}$$

with λ such that $\sum_{i=1}^n Q_i^* = Q$.

Proof

See Appendix A. \square

This result is a water-filling solution applied to the energy minimization. If all $B_i = B$ for all $i \in [1, n]$, this solution is also the solution of the power minimization

problem under a bit-rate constraint $\sum_i C_i^* = C$ with $C_i^* = \frac{Q_i^*}{B_i T_i}$, and where Q_i^* and T_i go to infinity.

3.2. Lower bound of energy

It is well known that the minimal needed energy to transmit a certain amount of bits can be obtained in the ultra wide band regime [4] with particular conditions. The same result can also be obtained in the ultra wide time regime. This concept of ultra wide time is similar to the ultra wide band case when the bandwidth is replaced by the time of transmission. In both ultra wide cases, the product $B_i \times T_i$ goes to infinity. We recall here the following proposition [4].

Proposition 2. The asymptotic limit of energy needed to transmit Q_i bits over one channel of normalized SNR γ_i is

$$J_{i,0} = \lim_{B_i T_i \rightarrow \infty} J_i = \frac{Q_i \log_e 2}{\gamma_i}.$$

This proposition is another formulation of the minimum received signal energy per information bit required for reliable communications. It should be noted that the channel gain (or attenuation) is taken into account through the coefficient γ_i .

The use of ultra wide band or ultra wide time is not a sufficient condition to ensure the asymptotic regime. The correct condition to verify Proposition 2 is such that the product $B_i \times T_i$ must go to infinity, i.e., if only one parameter goes to infinity, the other shall not go to 0. In fact, if all the information bits Q_i are transmitted in only one symbol, the time T_i and the bandwidth B_i can be such that the product $B_i T_i$ is a certain constant. In this case $B_i T_i = \alpha$ and the needed energy

$$J_i = \left(2^{\frac{Q_i}{\alpha}} - 1 \right) \frac{\alpha}{\gamma_i} \quad (5)$$

is higher than $J_{i,0}$ for all finite α even if either B_i or T_i goes to infinity.

The result of Proposition 2 is now extended to the case of parallel independent channels.

Proposition 3. The asymptotic limit of energy needed to transmit Q bits through n independent parallel channels with normalized SNR $\{\gamma_i\}_{i=1}^n$ is obtained when all the information bits are transmitted over the best channel, so

that [11]

$$J_0 = \frac{Q \log_e 2}{\max_i \gamma_i}.$$

Proof

See Appendix B. \square

This solution is not the water-filling one or, in other words, it can be considered as a limiting case of water-filling: all the information bits are transmitted over the channel with the highest normalized SNR, called hereafter the best channel. This result is not surprising. The best way to minimize the needed energy is to use the channel which needs the least energy.

The minimal energy limit can not be reached under finite time or bandwidth constraints. If we try to minimize the needed energy, we obtain only infinite solutions for the transmission time and bandwidth. Therefore, it is interesting to relax the problem and to find a resource allocation solution when the goal is to transmit with energy that is close, but not identical, to the one obtained in the asymptotic regime.

3.3. Energy efficiency factor

Conventionally, the energy efficiency η_{EE} is expressed in bit/J [1]

$$\eta_{EE} = \frac{Q_i}{J_i} = \frac{C_i}{(2^{C_i} - 1)} \gamma_i, \quad (6)$$

where η_{EE} decreases monotonically with C_i and the communication is spectrally efficient for high C_i but energetically inefficient.

The energy efficiency η_{EE} depends on the channel gain and, in some cases, it can be dominated by the link budget. The trade-off between the capacity and the energy has been widely investigated. In this paper, we focus on the energy efficiency to derive new resource allocation algorithms. To do this, we start by defining the energy efficiency factor as the ratio between the required energy for a certain quantity of information transmission and the asymptotic energy limit. We use the definition proposed in [10, 11].

Definition 1. The energy efficiency factor β is the ratio between the asymptotic limit and the needed energy with finite time and frequency resource.

Using (3) and Proposition 2, the energy efficiency factor of channel i is therefore

$$\beta_i = \frac{J_{i,0}}{J_i} = \frac{Q_i \log_e 2}{(2^{\frac{Q_i}{B_i T_i}} - 1) B_i T_i}. \quad (7)$$

This energy efficiency factor is independent from the normalized SNR, i.e., independent from the channel gain. This means that this energy efficiency factor does not depend on the link budget and it characterizes the ability of a transmission to exploit the energy capacity of a channel. It is then an effective measure to compare different communication systems independently from the channel effect.

For a given time-bandwidth product, the energy efficiency factor depends on the amount of information bits and decreases with Q_i . However, this energy efficiency factor becomes independent of Q_i if the time used to send information grows linearly with the amount of information bits. The energy efficiency factor β_i verifies the following properties.

Property 1. $0 < \beta_i \leq 1$.

Property 2. $\lim_{B_i T_i \rightarrow +\infty} \beta_i = 1$.

Property 3. $\lim_{B_i T_i \rightarrow 0} \beta_i = 0$.

The transmission of a certain amount of information bits is then efficient if the energy efficiency factor is near to 1 and it is inefficient if the energy efficiency factor is near to 0. This energy efficiency factor is linked to the spectral efficiency with the following property.

Property 4. $\beta_i = \frac{C_i \log_e 2}{2^{C_i} - 1}$.

In this property, C_i is the spectral efficiency expressed in bit/s/Hz assuming to transmit over the period T_i . The spectral-energy efficiency factor trade-off is drawn in Fig. 1. For example, the energy efficiency factor $\beta_i = -3$ dB is obtained with the spectral efficiency $C_i = 1.8$ b/s/Hz. The energy efficiency factor of a communication system decreases with the spectral efficiency and the higher the spectral efficiency, the lower the energy efficiency is. This conclusion is the same as the one obtained with the conventional definition of energy efficiency η_{EE} .

The total energy efficiency factor β in a system using n parallel channels is not the sum of the energy efficiencies

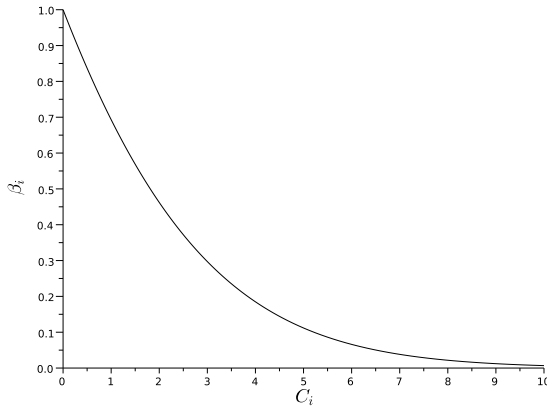


Figure 1. Energy efficiency factor β_i versus spectral efficiency C_i .

β_i but, with (4) and Proposition 3, it is

$$\beta = \frac{J_0}{J} = \frac{1}{\sum_{i=1}^n \frac{Q_i \max_j \gamma_j}{Q \gamma_i} \frac{1}{\beta_i}}. \quad (8)$$

Differently from the single channel case, β depends on the channel state but not in absolute terms. In fact, the channel influence is normalized by the best channel coefficient through the definition of γ_i .

Using the weighted harmonic mean and the mean inequalities, β verifies the following property.

Property 5.
$$\frac{\min_i \beta_i}{\frac{\max_i \gamma_i}{Q} \sum_{i=1}^n \frac{Q_i}{\gamma_i}} \leq \beta \leq \frac{\max_i \beta_i}{\frac{\max_i \gamma_i}{Q} \sum_{i=1}^n \frac{Q_i}{\gamma_i}}.$$

In a Gaussian channel, it follows that $\min_i \beta_i \leq \beta \leq \max_i \beta_i$, and if $\{Q_i\}_i$ are equally distributed then $\frac{\bar{\gamma}_H}{\max_i \gamma_i} \min_i \beta_i \leq \beta \leq \frac{\bar{\gamma}_H}{\max_i \gamma_i} \max_i \beta_i$ with $\bar{\gamma}_H$ the harmonic mean of $\{\gamma_i\}_i$.

By definition, β satisfies also Property 1. Property 3 can be extended to β : if only in one channel the product $B_i T_i$ goes to zero, then β goes also to zero. Property 2 is modified by the following property.

Property 6. Let $j = \arg \max_i \gamma_i$, then $\beta = 1$ if and only if $Q_j = Q$ and the product $B_j T_j$ goes to infinity.

This property says that if a very high energy efficiency factor is needed, then all the information bits must be transmitted through the best channel. In this case, only one channel is used. If a certain channel, not the best, is used, then the energy efficiency factor is limited.

Property 7. The energy efficiency factor β that can be obtained using the only one channel i is upper bounded by

$$\beta \leq \frac{\gamma_i}{\max_j \gamma_j}.$$

3.4. Discussion

As it is shown in Section 3.3, the defined energy efficiency factor β is a relative measure. This measure characterizes the ability of a communication system to exploit what is feasible for a given channel from an energetic point of view. The conventional definition of energy efficiency given in (6) is a key measure in the green communication context to evaluate the energy costs. However, it should be noted that it depends on the link budget: the same channel capacity C_i and bandwidth B_i lead to different energy efficiencies η_{EE} depending on the channel gain and noise level. On the contrary, the energy efficiency factor definition used in this paper and given in (7) is more appropriate to evaluate the performance of a certain allocation of resources and to measure its intrinsic energetic capacity, as it will be discussed below.

4. PROBLEM FORMULATION

Our interest is not to minimize the energy needed to transmit Q information bits because this problem is solved with the use of infinite transmission time or with an infinite bandwidth. The goal is to transmit Q bits in a finite time and finite bandwidth with the efficiency factor β , as stated in Problem 1 below.

Problem 1. Find $\{Q_i, T_i\}_{i=1}^n$ such that

$$\sum_{i=1}^n J_i = \frac{J_0}{\beta}, \quad (9)$$

$$\sum_{i=1}^n Q_i = Q, \quad (10)$$

$$\forall i \in [1, n] Q_i \geq 0, \quad (11)$$

$$\forall i \in [1, n] T_i \geq 0. \quad (12)$$

The variables are $\{Q_i, T_i\}_{i=1}^n$ in this problem formulation. There are two possible other formulations with variables $\{Q_i, J_i\}_{i=1}^n$ or $\{J_i, T_i\}_{i=1}^n$. Note that despite the

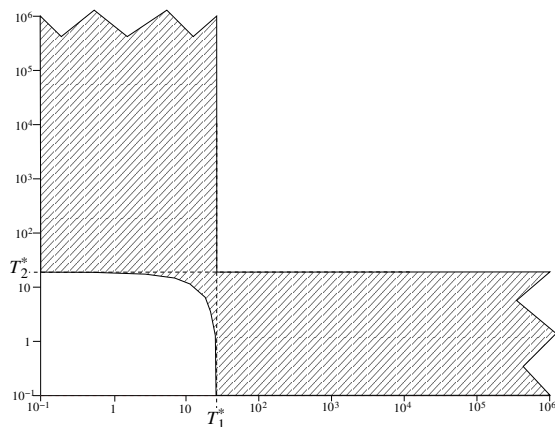


Figure 2. Feasible set $\{T_1, T_2\}$ for the 2-channel case.

fact that all three sets of variables $\{Q_i, T_i, J_i\}_{i=1}^n$ must be non-negative, the constraints (11)–(12) are sufficient.

Let us analyze the case of two channels, $n = 2$, with β such that Property 7 is satisfied for both channels. In this case, all the information bits Q can be transmitted in channel 1 or in channel 2. The feasible set of parameters $\{Q_i\}_i$ is defined by $Q_2 = Q - Q_1$, with $Q_1 \in [0, Q]$. The feasible set of parameters $\{J_i\}_i$ is defined by $J_2 = \frac{J_0}{\beta} - J_1$, with $J_1 \in [0, \frac{J_0}{\beta}]$. On the contrary, there is no expression for the feasible set $\{T_i\}_i$. The region of feasible $\{T_i\}_i$ can be obtained by simulation solving Problem 1 and an example is given Fig. 2. In this figure, the feasible set corresponds to the dashed region and T_i^* is the solution of (3) with $J_i = \frac{J_0}{\beta}$ and $Q_i = Q$ for $i \in \{1, 2\}$. For $T_1 \in [T_1^*, +\infty)$, T_2 varies from 0 to T_2^* and for $T_2 \in [T_2^*, +\infty)$, T_1 varies from 0 to T_1^* .

In the following, we firstly analyze some specific cases, i.e., we add a further constraint to Problem 1 so that we simplify and reduce the set of solutions. Such constraints are given by uniform time and uniform bit allocations over the channels as well as uniform spectral efficiencies. The strength of these cases is that they lead to a simpler transmitter or receiver structure.

4.1. Uniform time and uniform information bit allocations

Let us assume that all the channels transmit the same information bits $Q_i = \frac{Q}{n}$ during the same time $T_i = T$ and in the same bandwidth $B_i = B$. In this case, the transmitter does not need to know the channel conditions to allocate the information over the n channels. The problem

is then: find T such that

$$TB(2^{\frac{Q}{nTB}} - 1) \sum_{i=1}^n \frac{1}{\gamma_i} = \frac{J_0}{\beta}. \quad (13)$$

From Property 6, it is clear that (13) cannot be solved for very high values of β . The transmission is feasible under certain conditions on β as discussed in the following.

Proposition 4. Under uniform time and information bit allocations, the energy efficiency factor β is reachable if and only if

$$\beta \times \max_{i \in [1, n]} \gamma_i \leq \frac{n}{\sum_{i=1}^n \frac{1}{\gamma_i}}.$$

Proof

See Appendix C. \square

With a more flexible allocation, empty channels can be allowed. In this case, the energy efficiency factor is always feasible with a variable number of loaded channels. This case is analyzed in the next paragraph.

4.2. Uniform spectral efficiencies

Let α be the spectral efficiency per channel, i.e. $\alpha = C_i$ for all i , with the same bandwidth $B_i = B$. With this constraint in practical systems, the same modulation scheme is used for all channels. With uniform spectral efficiencies, Q_i depends on T_i as $Q_i = \alpha B T_i$ for all i . Then, the number of variables can be reduced from $2n$ down to n . With $n = 2$ the solutions are defined by $X_2 = X - X_1$ where X_i is Q_i , J_i or T_i , and X is Q , $\frac{J_0}{\beta}$ or $\frac{Q}{\alpha}$, respectively. With n higher than 2, the solutions are hyper planes.

5. TIME MINIMIZATIONS

The problem we are now interested in is not to find all the feasible points in the region of solutions of Problem 1 but only particular points that minimize some defined temporal quantities. Specifically, Problem 1 is reformulated so that we obtain the solution that minimizes the maximum transmission time or the average occupancy time, as defined in the following.

5.1. Definitions and new problems

Even if the key parameter is the energy efficiency factor β , it is important to minimize the time needed to transmit the information bits for a given energy efficiency factor. We first define the transmission time.

Definition 2. The transmission time is the maximum value in the set $\{T_i\}_i$ for $i \in [1, n]$

$$T_M = \max_{i \in [1, n]} T_i = \|T_1, \dots, T_n\|_\infty.$$

The transmission time T_M is the time used to transmit the information bits. Based on this definition, the new problem is stated as follows.

Problem 2. Minimize the transmission time T_M under the constraints (9)–(12).

We now define another important quantity called the average channel occupancy time.

Definition 3. The average channel occupancy time is the average of the transmission periods T_i with $i \in [1, n]$

$$T_S = \frac{1}{n} \sum_{i=1}^n T_i = \|T_1, \dots, T_n\|_1.$$

The average channel occupancy time measures the average channel busy time for a given transmission of information bits. It is important to minimize the transmission time for a given energy efficiency factor so that we free one or more channels as soon as possible for other users or other communication systems. Therefore, the new problem can be stated as follows.

Problem 3. Minimize the average channel occupancy time T_S under the constraints (9)–(12).

In summary, Problems 2 and 3 find the specific solutions of Problem 1 that minimize T_M and T_S , respectively.

5.2. Transmission time minimization

Before providing the solution of Problem 2, we need the following intermediate result.

Lemma 1. Under an energy efficiency factor constraint and uniform spectral efficiencies, the minimization of the transmission time T_M leads to the uniform allocation of the time and uniform allocation of the information bits over a given subset of channels.

Proof

See Appendix D. \square

The minimization of the transmission time under a uniform spectral efficiency constraint leads to finding the channel subset which minimizes the time T in (13). With n channels, there are $2^n - 1$ possible subsets. The search of the subset of channels can be reduced to a number lower than n . In fact, let

$$f : \sum_i \gamma_i^{-1} \mapsto T \quad (14)$$

be the function such that $T = f(\sum_i \gamma_i^{-1})$ satisfies (13). This function is a decreasing function. Then, the optimal subset of channels is defined by the highest $\sum_i \gamma_i^{-1}$ and at most n comparisons are needed as explained in Algorithm 1.

Algorithm 1.

- 1: Sort γ_i in decreasing order: $\gamma_{\pi(1)} \geq \gamma_{\pi(2)} \geq \dots \geq \gamma_{\pi(n)}$
- 2: Set $k = 1$
- 3: Evaluate the cost function $f(\cdot)$ over the channel indexes $\{\pi(1), \dots, \pi(k)\}$
- 4: If $f(\pi(1), \dots, \pi(k))$ is feasible, i.e. T exists, then $k \mapsto k + 1$ and go to step 3 while $k \leq n$
- 5: $\{\gamma_{\pi(1)}, \dots, \gamma_{\pi(k-1)}\}$ is the optimal channel subset

Algorithm 1 can also be used to solve the water-filling problem of Proposition 1.

Let \mathcal{I} be the optimal subset obtained with Algorithm 1. Then, the energy efficiency factor is related to the spectral efficiency α and the subset of channels \mathcal{I} as reported by the following corollary.

Corollary 1. Under uniform spectral efficiency α , the minimization of the transmission time to transmit Q information bits (Problem 2) leads to the energy efficiency factor

$$\beta = \frac{\alpha \log_e 2}{2^\alpha - 1} \frac{|I|}{\sum_{i \in \mathcal{I}} \frac{\max_{j \in [1, n]} \gamma_j}{\gamma_i}}.$$

Proof

It is obtained by using (13) with the sum over the subset \mathcal{I} of channel and Lemma 1. \square

The expression of β in Corollary 1 can be compared to the one in Property 4, and the bound in Proposition 4 remains valid by summing over the channels in \mathcal{I} instead over $[1, n]$. Corollary 1 can also be compared to (8). The relation between β_i in (8) and α in Corollary 1 is

$$\beta_i = \frac{\alpha \log_e 2}{2^\alpha - 1}. \quad (15)$$

The uniform spectral efficiency constraint leads to a uniform energy efficiency factor over the channels. The total energy efficiency factor is then equal to the energy efficiency factor per channel weighted by the harmonic mean of the ratio $\frac{\gamma_i}{\max_j \gamma_j}$, with $i \in \mathcal{I}$ whereas $j \in [1, n]$. The energy efficiency factor depends on the subset \mathcal{I} of channels used and it depends also on all the n channels through the maximal normalized SNR $\max_j \gamma_j$. However, since the subset \mathcal{I} is composed of the best channels, then $\max_j \gamma_j$ is by definition included in this subset.

To compute the minimum time of transmission (Problem 2) for the transmission of a given quantity of information bits Q , the following lemma is also needed.

Lemma 2. Under the energy efficiency factor constraint and the information bit allocation constraint, the minimization of the transmission time T_M for the bit allocation $\{Q_i\}_{i=1}^n$ over n parallel and independent channels leads to a uniform channel time allocation.

Proof

It is based on the proof of Lemma 1. See Appendix E. \square

If one time T_i is higher than all other times, it is then possible to relocate energy from another channel to channel i to reduce the time T_i without any change in the allocation of information bits and with the same total energy.

The solution of Problem 2 can now be established with the following theorem.

Theorem 1 (Solution of Problem 2). Under an energy efficiency factor constraint, the minimization of the transmission time of Q bits over n parallel and independent channels leads to

$$\begin{aligned} Q_i^* &= 0, & \lambda \gamma_i &\leq 1, \\ Q_i^* &= B_i T_M^* \log_2(\lambda \gamma_i), & \lambda \gamma_i &> 1, \end{aligned}$$

with λ such that $\sum_{i=1}^n Q_i^* = Q$ and the minimal transmission time T_M^* is the solution of

$$\begin{aligned} \exp \left(\frac{Q \log_e 2}{T_M^* \sum_{i \in \mathcal{I}} B_i} - \frac{\sum_{i \in \mathcal{I}} B_i \log_e \gamma_i}{\sum_{i \in \mathcal{I}} B_i} \right) \\ = \frac{J_0}{\beta T_M^* \sum_{i \in \mathcal{I}} B_i} + \frac{\sum_{i \in \mathcal{I}} \frac{B_i}{\gamma_i}}{\sum_{i \in \mathcal{I}} B_i}, \end{aligned}$$

with $\mathcal{I} = \{i | i \in [1, n] \cap \lambda \gamma_i \leq 1\}$.

Proof

See Appendix F. \square

It is now possible to transmit Q information bits under the energy efficiency factor β with the minimal time of transmission. The optimal subset \mathcal{I} is obtained using Algorithm 1. The result in Theorem 1 can be compared to Proposition 1.

Corollary 2. The bit allocation $\{Q_i\}_{i=1}^n$ which minimizes the transmission time (Problem 2) under an energy efficiency factor constraint is the bit allocation which minimizes the energy under the minimal transmission time.

Proof

The Karush-Kuhn-Tucker (KKT) conditions of Proposition 1 with the solution of Theorem 1 are identical to the KKT conditions of Theorem 1 with the solution of Proposition 1. Then, the solutions are also identical. \square

Corollary 2 provides another method to obtain the solution of Theorem 1. The minimum transmission time of Theorem 1 can be obtained using Lemma 2, Proposition 1 and an iterative algorithm, such as the bisection or secant method.

5.3. Average channel occupancy time minimization

We focus now on Problem 3. The goal now is to minimize the mean of T_i , $i \in [1, n]$ such that (9)–(12) are satisfied. Unfortunately, a closed form expression can not be derived in the general case of unequal bandwidths B_i . If all the channel bandwidths B_i are equal to B , then the following theorem provides a simple solution.

Theorem 2 (Solution of Problem 3 with $B_i = B$). Under an energy efficiency factor constraint and with $B_i = B$ for all i , the minimization of the average channel occupancy time to transmit Q information bits over multiple parallel and independent channels (Problem 3) leads to transmit over the best channel. The minimal average channel occupancy time T_S^* satisfies

$$\beta \left(e^{\frac{Q \log_e 2}{n B_i T_S^*}} - 1 \right) - \frac{Q \log_e 2}{n B_i T_S^*} = 0,$$

with $i = \arg \max_{j \in [1, n]} \gamma_j$.

Proof

See Appendix G. \square

The best way to minimize the channel occupancy, and therefore to minimize the average channel occupancy time, is to transmit the information bits in the best channel. This result is valid only if all the channel bandwidths are equal. Numerical examples show that it is not the case when there are different channel bandwidths. For example, let $n = 2$, $Q = 100$, $\beta = 1/2$, $\{\gamma_1, \gamma_2\} = \{2, 1\}$. With $B_1 = B_2 = 1$, and using the Lambert function $W(x)$, the minimal average channel occupancy time is

$$T_S^* = -\frac{Q \ln 2}{2W(-\beta e^{-\beta}) + \beta} = 27.6. \quad (16)$$

With $B_2 = 10$, the configuration $\{Q_1, Q_2\} = \{49, 51\}$ with $\{J_1, J_2\} = \{0.991 \frac{J_0}{\beta}, 0.009 \frac{J_0}{\beta}\}$ leads to $T_S = 24$ which is lower than T_S^* . Contrarily to Theorem 1, Theorem 2 can therefore not be applied with unequal bandwidths.

5.4. Numerical examples

To show some numerical results, we consider an orthogonal frequency division multiplexing (OFDM) system with 1024 channels (sub-carriers) each having identical bandwidth B_i and a total transmission bandwidth of 20 MHz. Only $n = 998$ sub-carriers are used in the bandwidth [0.5;20] MHz. As an example of frequency selective channel response, we assume the frequency response of a typical power line communication link as described by the 15-path Zimmermann channel model [14]. The noise is assumed white Gaussian with a power spectral density of -110 dBm/Hz. The transmission of $Q = 1$ Mbit is assumed.

With all these assumptions, the asymptotic limit of energy needed to transmit 1 Mbit of information is (Proposition 3)

$$J_0 = \frac{10^6 \log_e 2}{\max_i \gamma_i} \approx 140 \mu\text{J}. \quad (17)$$

Now, in Fig. 3, we show the transmission time T_M and the average channel occupancy time T_S as a function of the energy efficiency factor β when the resource allocation algorithms associated to Lemma 1 and Theorems 1 and 2 are applied. In particular,

- Lemma 1: this algorithm leads to the minimum transmission time T_M under uniform channel spectral efficiencies. The Algorithm 1 is used to calculate the optimal resource allocation;
- Theorem 1: this algorithm provides the minimum transmission time without any spectral efficiency constraint;
- Theorem 2: this algorithm minimizes the average channel occupancy time.

As expected, the minimum T_S is obtained when Theorem 2 is applied. However, if we apply this resource allocation the transmission time T_M will be increased. Theorem 1 minimizes the transmission time as expected but with the inconvenience that the average channel occupancy time T_S is not minimized. The solution obtained with Lemma 1 is an intermediate solution that provides a transmission time close to the minimum transmission time.

When β goes to 1, $T_M = nT_S$ and both T_M and T_S go to infinity. All resource allocation solutions become identical since a single subcarrier is used.

We now compare these results to a conventional best effort communication where the objective is to maximize the bitrate. In this case, the bitrate is maximized under a power spectrum density constraint of -50 dBm/Hz, which is a peak power constraint. The solution leads to $T_M(\text{best effort}) = T_S(\text{best effort}) = 9.4$ ms to transmit $Q = 1$ Mbit. Consequently, the energy efficiency factor can be computed and it becomes equal to $\beta(\text{best effort}) = -56$ dB. Therefore, if we compare this result with the one presented in Fig. 3, we notice that to transmit 1 Mbit of information the conventional rate maximization solution requires about 10 times less time but 53 dB of more energy compared to the energy efficient algorithms that target an efficiency of $\beta = 0.5$ (-3 dB).

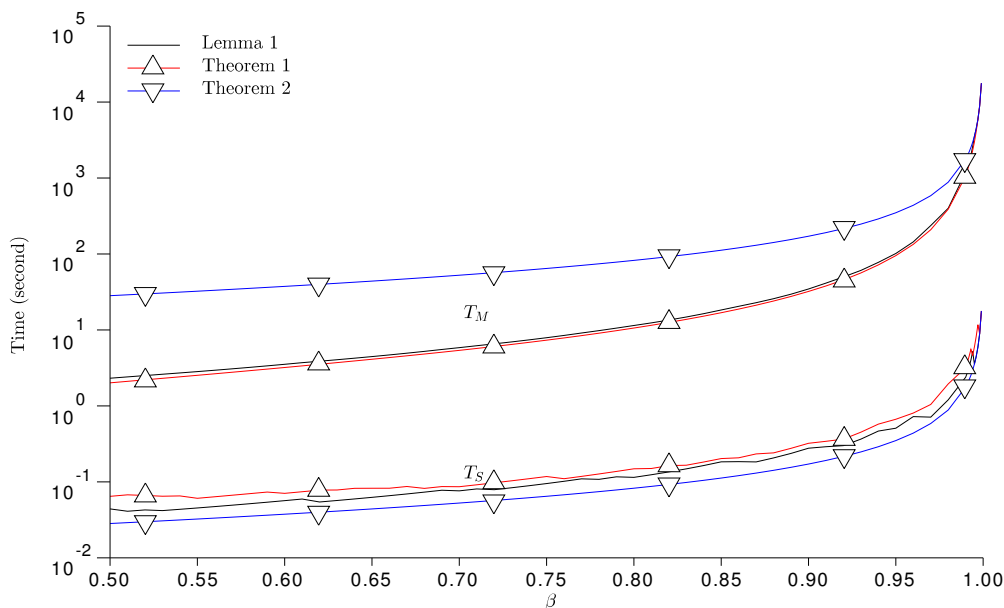


Figure 3. Time in second, T_M and T_S , needed for the transmission of 10^6 information bits for 3 transmission strategies versus energy efficiency factor β .

6. MULTI-OBJECTIVE OPTIMIZATION

The transmission time T_M and the average channel occupancy time T_S have been minimized under an energy efficiency factor constraint. If the minimization of T_M is not more important than T_S , it will be interesting to know if there exists a Pareto frontier for the transmission time and average channel occupancy time optimization problem. This Pareto frontier is an important frontier because it defines equivalent configurations for the multi-objective optimization problem.

The variables T_M and T_S are defined in \mathbb{R}_+ but they can not take all values in \mathbb{R}_+^2 for a given energy efficiency factor β . A qualitative example of feasible set of variables $\{T_M, T_S\}$ is given in Fig. 4 where three particular points, A , B , and C , and two half-lines, d_1 and d_2 are highlighted. Previous results have shown that $T_M \geq T_M(B)$ with $T_M(B)$ being the minimal transmission time given by Theorem 1, and $T_S \geq T_S(C)$ with $T_S(C)$ being the minimal average channel occupancy time given by Theorem 2. Note that Theorem 2 is valid only with equal bandwidths, which is the case treated hereafter. Other

bounds on T_M and T_S as discussed below can also be specified.

Property 8. n times the average channel occupancy time nT_S is lower bounded by the transmission time, i.e. $\sum_i T_i \geq \max_i T_i$.

Property 9. The average channel occupancy time T_S is upper bounded by the transmission time, i.e. $\frac{1}{n} \sum_i T_i \leq \max_i T_i$.

These two properties allow us to draw the half-lines d_1 and d_2 in Fig. 4. All couples $\{T_M, T_S\}$ are above d_1 and below d_2 . The half-line d_1 is defined by $nT_S = T_M$ with $T_M \geq T_M(C)$ and the half-line d_2 by $T_S = T_M$ with $T_M \geq T_M(B)$. Another bound is given by the line segment AB where $T_M(A) = T_M(B)$, $T_S(A) = T_M(B)$ and $nT_S(B) = |\mathcal{I}| \times T_M(B)$ with \mathcal{I} given by Theorem 1. An other bound can also be defined in the region of feasible solutions. It corresponds to the curve segment BC in Fig. 4. It is defined as follows.

Definition 4. For every point $\{T_M, T_S\}$ in the curve segment BC and for every transmission time $T_M \in [T_M(B), T_M(C)]$, the average channel occupancy time T_S

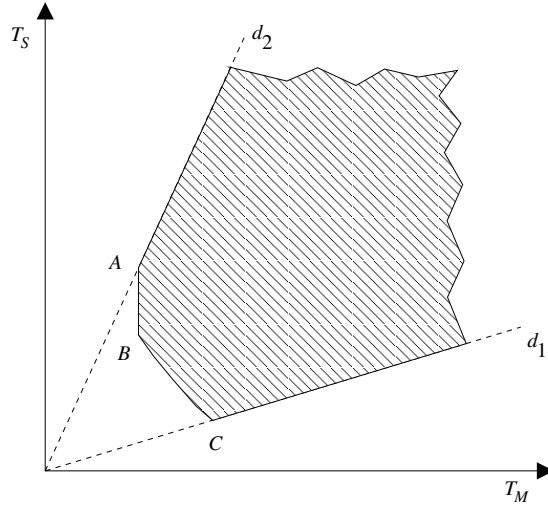


Figure 4. Feasible set of variable $\{T_M, T_S\}$.

is

$$T_S(BC) = \min \frac{1}{n} \sum_i T_i,$$

with $T_i \in [0, T_M]$, $\sum_i Q_i = Q$ and $\sum_i J_i = \frac{J_0}{\beta}$.

By definition BC is a Pareto front. The following theorem specifies the values of the points within this front.

Theorem 3. For every point $\{T_M, T_S\}$ in the Pareto frontier (BC) , $T_i \in \{0, \tilde{T}, T_M\}$ such that the sum of T_i is minimal and with no more than only one T_j equal to \tilde{T} where \tilde{T} is the solution of

$$\begin{cases} \sum_{\substack{i \in \mathcal{I} \\ i \neq j}} B_i T_M \log_2 \alpha \gamma_i + B_j \tilde{T} \log_2 \alpha \gamma_j = Q, \\ \sum_{\substack{i \in \mathcal{I} \\ i \neq j}} B_i T_M (\alpha - \frac{1}{\gamma_i}) + B_j \tilde{T} (\alpha - \frac{1}{\gamma_j}) = \frac{J_0}{\beta}, \end{cases}$$

and where j is the index of the poorest channel in \mathcal{I} .

Proof

See Appendix H. □

As in previous cases, we are left with the determination of the subset \mathcal{I} for which $T_i \neq 0$. A modified version of Algorithm 1 can be used, where the optimal subset is the one which provides the minimal average channel

occupancy time, and, as in Theorem 1,

$$\begin{cases} Q_i = B_i T_i \log_2 \alpha \gamma_i, \\ J_i = B_i T_i (\alpha - \frac{1}{\gamma_i}) \end{cases}$$

for all $i \in \mathcal{I}$. The cardinality of \mathcal{I} varies from the value given by Theorem 1, in the point B , to 1, in the point C . \tilde{T} varies from 0 to T_M .

Thus, we have found the Pareto frontier BC . All points in this frontier are equivalent from the multi-objective optimization point of view.

To show some numerical results, we consider the example of Section 5.4, with $\beta = 0.5$. The minimal transmission time T_M is given by Theorem 1 and it is equal to 2.01 s, see Fig. 3. This configuration leads to $T_S = 64.5$ ms, see Fig. 3, and the corresponding point in Fig. 4 is B . The minimal channel occupancy time is given by Theorem 2 and it is equal to 28.3 ms. The corresponding point in Fig. 4 is C with $T_M(C) = 28.3$ s. Then, in this Pareto frontier, T_M varies from 2.01 s to 28.3 s and T_S varies from 28.2 ms to 64.5 ms. In this example, the range of variation of T_M is higher than the range of variation of T_S . In point C , $T_M(C) = nT_S(C)$, with $n = 998$, because only one channel is used. In point B , only 3.21 % of the channels are used then $T_M(B) = 3.21 \% \times T_S(B)$. Note that in point A , all the channels are used and $T_S(A) = T_M(A) = T_M(B)$.

7. CONCLUSION

Without any transmission time constraint, the minimization of the energy needed to transmit an amount of information leads to an infinite time of transmission. Conventional energy efficiency formulations deal with the minimization of the transmit energy and lead to the use of an infinite transmission time or bandwidth. In this paper, we have instead formulated resource allocation problems with a given energy constraint near to the asymptotic energy limit that is achievable with a finite bandwidth and time of transmission. To solve such problems, we have defined the energy efficiency factor as the ratio between the energy required in the asymptotic regime and the energy required to transmit a certain amount of information with limited (time, bandwidth) resources. With this new energy

metric definition, we have studied the resource allocation problem in parallel independent Gaussian channels from an information theoretic point of view. We have investigated the minimization of the transmission time and of the average channel occupancy time under an energy efficiency factor constraint. Finally, the Pareto front for the joint transmission time and average channel occupancy time optimization problem has been derived. The proposed resource allocation algorithms provide a method to implement energy efficient communication systems.

A. PROOF OF PROPOSITION 1

The problem is to find the information bit allocation that minimizes (4), i.e.

$$\{Q_i\}_i = \arg \min \sum_{i=1}^n \left(2^{\frac{Q_i}{B_i T_i}} - 1 \right) \frac{B_i T_i}{\gamma_i}. \quad (18)$$

under the constraints $\sum_{i=1}^n Q_i = Q$ and $Q_i \geq 0, \forall i$

The function $Q_i \mapsto J_i(Q_i)$ is convex. The problem is then a convex optimization problem and the Lagrangian is

$$\mathcal{L} = \sum_{i=1}^n J_i(Q_i) + \lambda' \left(Q - \sum_{i=1}^n Q_i \right) - \sum_{i=1}^n \mu_i Q_i. \quad (19)$$

The bit allocation $\{Q_i^*\}_{i=1}^n$ that satisfies the KKT conditions

$$\left\{ \begin{array}{l} \sum_{i=1}^n Q_i^* = Q, \\ \mu_i \geq 0, \forall i \in [1, n], \\ Q_i^* \geq 0, \forall i \in [1, n], \\ \mu_i Q_i^* = 0, \forall i \in [1, n], \\ \frac{\log_e 2}{\gamma_i} 2^{\frac{Q_i^*}{B_i T_i}} - \lambda' - \mu_i = 0, \end{array} \right. \quad (20)$$

is optimal. Let $\lambda' = \lambda \log_e 2$. If $\mu_i = 0$ then $Q_i^* = B_i T_i \log_2(\lambda \gamma_i) \geq 0$ and $\lambda \gamma_i \geq 1$. If $\mu_i \neq 0$ then $Q_i^* = 0$ and $\lambda \gamma_i \leq 1$.

B. PROOF OF PROPOSITION 3

The problem is to find the information bit allocation that minimizes the transmitted energy, i.e.

$$J_0 = \min_{\left\{ \sum_{i=1}^n Q_i = Q, Q_i \geq 0 \forall i \right\}} \sum_{i=1}^n J_{i,0}. \quad (21)$$

Assume there exists one j such that $\gamma_j > \gamma_i \forall i \neq j$, then

$$\sum_{i=1}^n J_{i,0} = \frac{\log_e 2}{\gamma_j} Q + \sum_{\substack{i=1 \\ i \neq j}}^n \left(\frac{\log_e 2}{\gamma_i} - \frac{\log_e 2}{\gamma_j} \right) Q_i \quad (22)$$

is minimal if and only if $Q_i = 0$ for all $i \neq j$. If two, or more, channels are maximal, i.e. $\gamma_{j_1} = \gamma_{j_2} > \gamma_i$ for all $i \notin \{j_1, j_2\}$, then the total information Q can be split between these channels j_1 and j_2 without any restriction. Otherwise, only one channel carries all the information Q and there is one single solution.

C. PROOF OF PROPOSITION 4

Under a uniform channel transmission period allocation and a uniform information bit allocation, the minimal needed energy is obtained with infinite transmission time, as it is stated by Proposition 2,

$$\lim_{T \rightarrow \infty} TB \left(2^{\frac{Q}{nTB}} - 1 \right) \sum_{i=1}^n \frac{1}{\gamma_i} = \sum_{i=1}^n \frac{Q \log_e 2}{n \gamma_i}. \quad (23)$$

The transmission with energy efficiency factor β is feasible if and only if the minimal needed energy (23) is lower than $\frac{J_0}{\beta}$, i.e.,

$$\sum_{i=1}^n \frac{Q \log_e 2}{n \gamma_i} \leq \frac{Q \log_e 2}{\beta \max_i \gamma_i}. \quad (24)$$

In this case, the transmission is then feasible otherwise the target energy efficiency factor is not achievable. Note that the transmission is feasible in finite time in the case of strict inequality.

D. PROOF OF LEMMA 1

Lemma 1 is proven in the general case without any constraint on the information quantity. The variables $\{J_i\}_{i=1}^n$ are considered instead of $\{T_i\}_{i=1}^n$. The Lagrangian of the convex optimization problem is

$$\mathcal{L} = \max_i T_i + \lambda \frac{J_0}{\beta} - \sum_{i=1}^n (\lambda + \mu_i) J_i, \quad (25)$$

with

$$T_i = \frac{\gamma_i J_i}{B_i (2^\alpha - 1)}. \quad (26)$$

The infinite norm is not differentiable but it is the limit of the p -norm for $p \rightarrow \infty$. We then use this p -norm and the roots of the derivative of the Lagrangian are

$$J_i^{p-1} = \frac{\left(\sum_{j=1}^n \left(\frac{\gamma_j}{B_j (2^\alpha - 1)} \right)^p \right)^{1-\frac{1}{p}}}{p \left(\frac{\gamma_i}{B_i (2^\alpha - 1)} \right)^p} (\lambda + \mu_i). \quad (27)$$

As in Appendix A, the KKT conditions are used and the solution is independent of p

$$J_i^* = \frac{J_0}{\beta} \frac{\frac{B_i}{\gamma_i}}{\sum_{j \in \mathcal{I}} \frac{B_j}{\gamma_j}} \quad (28)$$

for all i in $\mathcal{I} = \{i \in [1, n] | \mu_i = 0\}$. Using (26) and (28), it follows

$$T_i^* = \frac{J_0}{\beta} \times \left(\sum_{j \in \mathcal{I}} \frac{(2^\alpha - 1) B_j}{\gamma_j} \right)^{-1}, \quad (29)$$

and T_i is independent of i . With $B_i = B$ for all i ,

$$Q_i^* = \alpha B T_i^* = \frac{J_0}{\beta} \frac{\alpha}{2^\alpha - 1} \frac{1}{\sum_{j \in \mathcal{I}} \gamma_j^{-1}} \quad (30)$$

and Q_i^* is then independent of i .

These results have been obtained without any constraint over α and $\{Q_i\}_i$. It then remains valid with the constraint $\sum_i Q_i = Q$. Note that the uniform time allocation is also valid even if all B_i are not equal.

E. PROOF OF LEMMA 2

In Appendix D it has been proven that for all $\{Q_i\}_{i=1}^n$ and with uniform channel capacities, the optimal transmission periods $\{T_i^*\}_{i=1}^n$ that minimize the transmission time are independent of i . This result can be extended without channel capacity constraint. Using (3), let f_i be the function such that $f_i(T_i) = J_i$. This function is monotonic and convex. The Lagrangian of the problem is given by (25) replacing T_i by $f_i^{-1}(J_i)$ instead of (26). The solution T_i^* is then

$$T_i^* = \frac{J_0}{\beta} \frac{1}{\sum_{j \in \mathcal{I}} \left(2^{\frac{Q_j}{B_j T_j^*}} - 1 \right) \frac{B_j}{\gamma_j}} \quad (31)$$

and $T_i^* = T_M$ for all $i \in \mathcal{I}$.

F. PROOF OF THEOREM 1

Let f be the function

$$f(T, \{Q_i\}_{i=1}^n) = \sum_{i=1}^n \left(2^{\frac{Q_i}{B_i T}} - 1 \right) \frac{B_i T}{\gamma_i} = \frac{J_0}{\beta}. \quad (32)$$

This function is convex and decreasing along the T axis. The inverse function is then convex and

$$f^{-1}\left(f(T, \{Q_i\}_{i=1}^n), \{Q_i\}_{i=1}^n\right) = T. \quad (33)$$

The Lagrangian of the problem is

$$\begin{aligned} \mathcal{L} = & f^{-1}\left(\frac{J_0}{\beta}, \{Q_i\}_{i=1}^n\right) + \lambda' \left(Q - \sum_{i=1}^n Q_i \right) \\ & - \sum_{i=1}^n \mu_i Q_i \end{aligned} \quad (34)$$

and

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial Q_i} = & \frac{\frac{\log_e 2}{2} \frac{Q_i}{B_i T}}{\gamma_i} \\ & \frac{\sum_{j=1}^n \frac{B_j}{\gamma_j} \left(2^{\frac{Q_j}{B_j T}} \left(\frac{Q_j \log_e 2}{B_j T} - 1 \right) + 1 \right)}{\left(2^{\frac{Q_j}{B_j T}} - 1 \right) \frac{B_j}{\gamma_j}} \\ & - \lambda' - \mu_i. \end{aligned} \quad (35)$$

Solving the KKT conditions with \mathcal{I} being the index subset such that if $i \in \mathcal{I}$ then $\mu_i = 0$, the optimal time T^* is the

solution of

$$\exp\left(\frac{Q \log_e 2}{T^* \sum_{i \in \mathcal{I}} B_i} - \frac{\sum_{i \in \mathcal{I}} B_i \log_e \gamma_i}{\sum_{i \in \mathcal{I}} B_i}\right) \quad (36)$$

$$= \frac{J_0}{\beta T^* \sum_{i \in \mathcal{I}} B_i} + \frac{\sum_{i \in \mathcal{I}} \frac{B_i}{\gamma_i}}{\sum_{i \in \mathcal{I}} B_i}.$$

To reduce the complexity of the root finding, we prove that the root of the function exists and is unique. To do this, let $g(x)$ be the function corresponding to the previous equation with $Tx = 1$ and

$$g(x) = e^{ax-b} - cx - d. \quad (37)$$

The study of this function shows that it is convex and $\lim_{x \rightarrow \infty} f(x) = +\infty$. The inequality between the weighted arithmetic mean and the weighted geometric mean leads to $g(0) \leq 0$ with equality if and only if $\gamma_i = \gamma_j$ for all i and j : the solution T^* of (36) exists and is unique. Using (35), the optimal Q_i^* is

$$Q_i^* = B_i T^* \log_2(\gamma_i \lambda) \quad (38)$$

for $i \in \mathcal{I}$ and with λ such that $\sum_{i \in \mathcal{I}} Q_i^* = Q$.

G. PROOF OF THEOREM 2

Let $i = \arg \max_j \gamma_j$ and T_S be the average channel occupancy time. If all the information bits Q are transmitted through the channel i , then

$$\frac{J_0}{\beta} = \left(2^{\frac{Q}{n B T_S}} - 1\right) \frac{n B T_S}{\gamma_i}, \quad (39)$$

which proves the last part of the theorem. Let j be another channel, with $Q_j = Q - Q_i$ and $T_j = n T_S - T_i$. Using the Taylor series expansion of the exponential function, we obtain

$$J_i + J_j - \frac{J_0}{\beta} = \frac{1}{\gamma_i} \sum_{p=1}^{\infty} \frac{\log_e^p 2}{p! B^{p-1}} \left(\frac{Q_i^p}{T_i^{p-1}} + \frac{\gamma_i}{\gamma_j} \frac{Q_j^p}{T_j^{p-1}} - \frac{(Q_i + Q_j)^p}{(T_i + T_j)^{p-1}} \right). \quad (40)$$

But

$$\frac{Q_i^p}{T_i^{p-1}} + \frac{Q_j^p}{T_j^{p-1}} - \frac{(Q_i + Q_j)^p}{(T_i + T_j)^{p-1}} \geq 0 \quad (41)$$

for all $p \in \mathbb{N}$, $\{Q_i, Q_j, T_i, T_j\} \in \mathbb{R}_+^4$ and with equality when $Q_i T_j = Q_j T_i$. Then,

$$J_i + J_j - \frac{J_0}{\beta} \geq 0 \quad (42)$$

with equality if and only if $\gamma_i = \gamma_j$ and $Q_j T_i = Q_i T_j$. This means that the target energy $\frac{J_0}{\beta}$ can not be reached if the channels that transmit information are not the ones with the highest γ_i . Consequently, the average channel occupancy time is minimized if all the information bits Q are transmitted through the best channel.

H. PROOF OF THEOREM 3

The optimization problem is derived from Definition 4. With the new constraint $T_i \leq T_M$, $\forall i$, the Lagrangian of this problem is

$$\mathcal{L} = \frac{1}{n} \sum_i T_i + \sum_i o_i (T_M - T_i) - \sum_i \nu_i T_i - \sum_i \xi_i Q_i + \lambda \left(Q - \sum_{i=1}^k Q_i \right) + \mu \left(\frac{J_0}{\beta} - \sum_{i=1}^k J_i \right). \quad (43)$$

The KKT conditions are

$$\left\{ \begin{array}{l} \sum_i Q_i = Q \\ \sum_i \left(2^{\frac{Q_i}{B_i T_i}} - 1 \right) \frac{B_i T_i}{\gamma_i} = \frac{J_0}{\beta} \\ \nu_i T_i = o_i (T_M - T_i) = \xi_i Q_i = 0 \\ \frac{2^{\frac{Q_i}{B_i T_i}}}{2^{\frac{Q_i}{B_i T_i}}} + \frac{\lambda + \xi_i}{\gamma_i} = 0 \\ \frac{B_i}{\gamma_i} \left(2^{\frac{Q_i}{B_i T_i}} \left(\frac{Q_i \ln 2}{B_i T_i} - 1 \right) + 1 \right) + \frac{1 - \nu_i - o_i}{\mu} = 0 \end{array} \right. \quad (44)$$

If $\nu_i = o_i = \xi_i = 0$ then only one point satisfies the KKT conditions. Let \tilde{T} be this point, j the corresponding channel index, \mathcal{I} the index subset such that $\mathcal{I} = \{i | Q_i \neq 0\}$

0} and $\alpha = -\frac{\lambda}{\mu}$, then

$$\begin{cases} \sum_{i \in \mathcal{I}} B_i T_M \log_2 \alpha \gamma_i + B_j \tilde{T} \log_2 \alpha \gamma_j = Q, \\ \sum_{i \in \mathcal{I}} B_i T_M (\alpha - \frac{1}{\gamma_i}) + B_j \tilde{T} (\alpha - \frac{1}{\gamma_j}) = \frac{J_0}{\beta} \end{cases} \quad (45)$$

and

$$\begin{aligned} \tilde{T} &= \frac{J - T_M \sum_{i \in \mathcal{I}} B_i (\alpha - \frac{1}{\gamma_i})}{B_j (\alpha - \frac{1}{\gamma_j})} \\ &= \frac{Q - T_M \sum_{i \in \mathcal{I}} B_i \log_2 (\alpha \gamma_i)}{B_j \log_2 (\alpha \gamma_j)}. \end{aligned} \quad (46)$$

Only one α satisfies the previous constraints and it is the solution of (46). Thus, $T_j = \tilde{T}$ and $T_i = T_M$ for all $i \in \mathcal{I}$ with $i \neq j$, otherwise $T_i = 0$. Unfortunately, neither the index i nor the index j is provided by the equations. But the average channel occupancy time is minimized, as the transmission time, if the channels are fully exploited. Then, the best channels among \mathcal{I} are such that $T_i = T_M$ and the poorest one is such that $T_j = \tilde{T}$.

The case where $\nu_i \neq 0$ or $\xi_i \neq 0$ leads to $T_i = 0$ or $Q_i = 0$ and the index i is outside the subset \mathcal{I} . The case $o_i \neq 0$ leads to $T_i = T_M$ which is taken into account in the definition of the subset \mathcal{I} .

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