# Synchronization and Channel Estimation for Filtered Multitone Modulation

Andrea M. Tonello and Fabio Rossi

Dipartimento di Ingegneria Elettrica Gestionale e Meccanica (DIEGM)
Università di Udine
Via delle Scienze 208 - 33100 Udine - Italy
phone: +39 0432 558 288 fax: +39 0432 558 251
e-mail: tonello@uniud.it

***Abstract*—This paper deals with synchronization and channel estimation in a filtered multitone (FMT) modulated system. An FMT based system differs from the popular OFDM scheme in the deployment of sub-channel shaping filters. Although the synchronization problem in OFDM is well understood, in an FMT system it presents several challenges. The approach herein presented is based on the deployment of appropriate training sequences. We first recover symbol/frame timing and carrier frequency in the time domain. Then we perform channel estimation in the frequency domain. We assume the deployment of simple linear sub-channel equalizers. We study the performance over several frequency selective fading channels as a function of the sub-channel equalizer complexity.**

***Keywords*—Channel estimation, Equalization, Filtered multitone modulation, OFDM, Synchronization.**

## I. Introduction

MULTICARRIER modulation has attracted great attention for application to wideband wireless channels. This is because it has the potentiality of simplifying the equalization task that becomes mandatory in wideband channels that experience severe frequency selectivity. Orthogonal frequency division multiplexing (OFDM) is probably among the most popular multicarrier modulation techniques. It is essentially based on multicarrier transmission with sub-channel pulses that have a rectangular impulse response. Its digital implementation comprises an M-point inverse fast Fourier transform (IFFT), where $M$ equals the number of sub-channels, followed by the insertion of a cyclic prefix. If the channel time dispersion is within the length of the cyclic prefix, simple one-tap equalization can be used. More general multicarrier schemes deploy sub-channel filters with time-frequency concentrated response. Under certain conditions they can be implemented by using an IFFT followed by low-rate sub-channel filtering. These schemes are referred to as filtered multitone modulated systems (FMT) [3]. If the sub-channels have disjoint frequency response, i.e., do not overlap in frequency, it is possible to avoid the inter-carrier interference (ICI), and get low inter-symbol interference (ISI) that can be corrected with simplified sub-channel equalization. FMT modulation has the potentiality of achieving better spectral efficiency than OFDM yet requiring sufficiently simple equalization [2], [8].

It is well known that the acquisition of the carrier frequency, and the frame (symbol) timing in OFDM must be performed very accurately, otherwise a loss of orthogonality between sub-channels is introduced, which translates into intercarrier and intersymbol interference. The problem is well understood and several robust techniques are presented in the literature, e.g., [6].

In an FMT system the symbol timing, and carrier frequency acquisition is still of great importance [1]. To allow for an efficient implementation of the receiver, and lower the equalizer complexity, accurate time/frequency offset knowledge has to be acquired possibly in the time domain, i.e., before running the receiver filter bank. This is because the efficient receiver implementation that is based on low-rate sub-channel matched filtering followed by a fast Fourier transform (FFT), requires knowledge of the synchronization parameters. Then, sub-channel equalization is performed in the frequency domain after the receiver filter bank, i.e., after the FFT receiver module. Several equalization approaches can be used such as linear equalization, decision feedback equalization [2], or optimal maximum likelihood sequence detection [8], [9]. In all cases the practical implementation requires training of the equalizer coefficients, or equivalently estimation of the channel impulse response. In this paper we consider the use of simple linear equalizers and we study the channel estimation, i.e., equalizer coefficients training, problem.

The overall system architecture is shown in Fig.1 assuming a "inefficient" implementation for easy of graphical representation. The transmitter is implemented with a synthesis filter bank that uses a real prototype pulse $g(nT)$. In particular, for the simulation results that we report in this paper, we consider a truncated square-root-raised cosine prototype pulse. Transmission is through a frequency selective fading channel.

At the receiver, after RF demodulation, and analog-to-digital conversion, the sequence of samples experiences both a timing offset $\Delta t$, and a carrier frequency offset $\Delta f$ with respect to the transmitter reference. The FMT front-end (analysis filter bank) comprises a bank of sub-channel matched filters followed by sampling and (low rate) sub-channel equalization. The optimal analysis filter bank requires knowledge of the time offset, and frequency offset. Therefore, we propose to estimate such offsets in the time-domain (up-front) so that they can be compensated before passing the signal through the analysis filter bank. This idea was presented in [1]. However, in [1] we mostly focused on a blind approach. Instead, in this paper we propose a correlation approach that is based on transmission of appropriate training sequences. Once compensation is done, the analysis filter bank can be efficiently implemented as described in [3]. That is, we run serial-to-parallel conversion, low-rate sub-channel matched filtering (with pulses matched to the transmit pulses), and finally we run a fast Fourier transform.

We propose two training approaches for synchronization, and we study their impact in terms of introduced redundancy. To this respect, in an OFDM system, synchronization can be acquired, in principle, by exploiting the redundancy of the cyclic prefix, which translates into high efficiency. In an FMT system the amount of redundancy that is required for training is increased by the sub-channel filters memory.

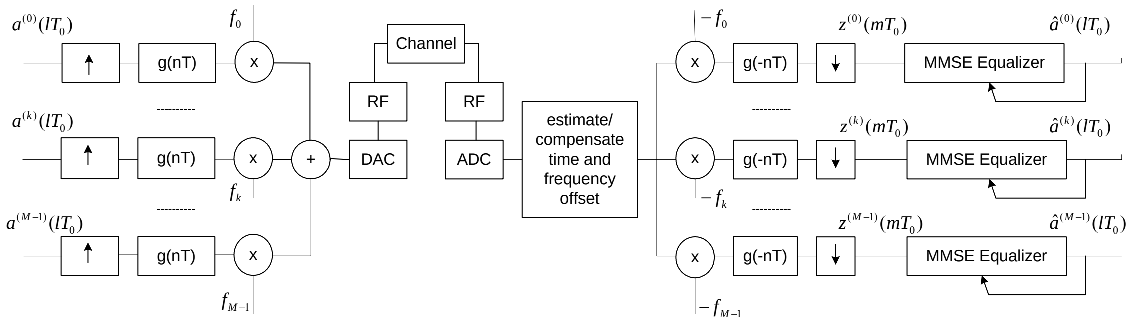


Fig. 1. FMT system with $T_0 = LT$, $f_k = k / (MT)$, and $L / M \geq 1$.

## II. SYSTEM MODEL

### A. Transmsitter

The complex baseband transmitted signal is obtained by a filter bank modulator with prototype pulse $g(nT)$, and sub-channel carrier frequency $f_k = k/(MT)$, $k = 0,...,M-1$, with $T$ being the transmission period (chip period). Therefore, it can be written as

$$x(nT) = \sum_{k=0}^{M-1} \sum_{l=-\infty}^{\infty} a^{(k)}(lT_0) g(nT - lT_0) e^{j\frac{2\pi}{M}nk} \tag{1}$$

where $a^{(k)}(lT_0)$ is the $k$-th sub-channel data stream that we assume to belong to the M-PSK/M-QAM constellation set and has rate $1/T_0$ with $T_0 = LT \geq MT$. The interpolation factor $L$ is chosen to increase the frequency separation between sub-channels, thus to minimize the amount of inter-carrier interference at the receiver side. The discrete time signal (1) is digital-to-analog converted, RF modulated, and transmitted over the air.

A possible efficient implementation of the transmitter that is based on polyphase filtering is described in [3]. Another, efficient implementation is described in [7].

### B. Channel

At the receiver side the signal is RF demodulated, and analog-to-digital converted. We assume a discrete time baseband channel model that includes the DAC and ADC filters. Its impulse response is

$$h(nT) = \sum_{p=0}^{N_P} \alpha_p \delta(nT - pT) \tag{2}$$

where the complex tap amplitudes $\alpha_p$, are assumed to be independent, zero-mean complex Gaussian (Rayleigh fading). The power delay profile is exponential with root-mean-square delay spread $\tau_{rms}$.

### C. Received Signal

The discrete time received signal (at rate $1/T$) in the presence of a time offset[1] $\Delta t = \varepsilon_t T$, $\varepsilon_t \in \mathbb{Z}$, and a carrier frequency offset $\Delta f = \varepsilon_f / (MT)$, $\varepsilon_f \in \mathbb{R}$, can be written as

$$r(nT) = e^{j(2\pi \Delta f nT + \phi)} \sum_{i=-\infty}^{\infty} x(iT) h(nT - \Delta t - iT) + \eta(nT). \tag{3}$$

Without loss of generality in the following we assume the phase $\phi$ to be zero. In (3) $\eta(nT)$ is a sequence of i.i.d. zero mean complex Gaussian random variables. Typically, a front-end matched filter bank is used. In the absence of knowledge of the time, and frequency offsets, the $\hat{k}$-th sub-channel receiver filter is matched to the sub-channel transmit pulse. Thus, we obtain at its output (in the absence of noise)

$$z^{(\hat{k})}(mT) = \sum_{k=0}^{M-1} \sum_{l=-\infty}^{\infty} a^{(k)}(lT_0) g_{EQ}^{(k,\hat{k})}(lT_0, mT) \tag{4}$$

$$\begin{aligned} g_{EQ}^{(\hat{k},k)}(lT_0, mT) = \sum_{n=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} e^{j\frac{2\pi}{M}[ik - n(\hat{k} - \varepsilon_f)]} \\ \times g(iT - lT_0) g(nT - mT) h(nT - \varepsilon_t T - iT). \end{aligned} \tag{5}$$

Now if we sample (4) at sub-channel symbol rate $1/T_0$, we obtain that no intersymbol interference (ISI), and no intercarrier interference (ICI) arise only when synchronization is ideal, the channel is not time dispersive, and the prototype pulse autocorrelation yields a Nyquist pulse, e.g., when we use ideal square root raised cosine pulses. In general we can distinguish among the following situations:

- $\Delta t = \Delta f = 0$: If the sub-channels are frequency separated, we still get some ISI because each sub-channel may not have perfectly flat frequency response. Further. note that when we implement the prototype pulse with a FIR filter, sidelobs are present (yielding ICI) and the Nyquist criterion on each sub-channel is not perfectly fulfilled (yielding ISI).

- $\Delta f = 0$, $\Delta t \neq 0$: We get some extra ISI due to a wrong sampling phase, and increased ICI when the sub-channels are not frequency separated.

- $\Delta f \neq 0$, $\Delta t = 0$: We get some ISI and ICI when the sub-channels are not frequency separated, or $\Delta f$ exceeds the guard frequency between sub-channels.

To minimize the amount of interference we need to compensate both the frequency offset, and the time offset, i.e., perform synchronization. Then, to cancel the residual ISI we run sub-channel equalization.

## III. TIME-FREQUENCY SYNCHRONIZATION

In this paper we propose to acquire time and frequency synchronization in the time domain, i.e., we do it before running the receiver filter bank. We use a correlation approach, where first we estimate timing, and then the carrier frequency offset. Compensation is performed. Then we run the receiver analysis filter bank, which can be done using the efficient FFT based architecture [3]. To acquire synchronization in the time domain we basically have three approaches as described below.

[1] A fractional time offset can be included in the equivalent channel impulse response.

## D. Blind Synchronization

In blind synchronization we assume no known training sequence. However, we assume to insert zeros, i.e., power drops in the transmitted signal. Then at the receiver side a correlation approach is used to estimate both timing and carrier frequency as described in [1]. One drawback with this approach is that we need to insert other training sequences or pilot symbols for channel estimation purposes or equalizer training.

## E. Cyclic Training Approach

In cyclically prefixed OFDM, synchronization is accomplished by exploiting the redundancy introduced by the cyclic prefix. This translates into a received signal that exhibits a cyclic correlation, i.e., samples at a given distance $PT$ are identical besides the phase rotation $\phi = 2\pi\Delta f PT$ introduced by the frequency offset. Therefore, as proposed in [6], the time offset and the frequency offset can be estimated via a correlation approach. A similar method can be used also in an FMT system as suggested already in [1], provided that the received signal has the cyclic property. The difficulty here is that in an FMT architecture we have the transmit filters that add a memory effect. This problem can be solved as described below.

We assume to implement the prototype pulse with an FIR filter with $N_g$ T-spaced taps (chips). In order to obtain the cyclic property we have to feed the *k*-th transmit filter with a training sequence of length $N_{TR}$ of identical data symbols $a_{TR}^{(k)}(lT_0) = a_{TR}^{(k)}$. Training sequences of distinct sub-channels can be different. For now, let us assume an ideal channel. Then, it is possible to obtain an FMT signal $x_{TR}(nT)$ that exhibits a periodicity equal to $PT$ (Fig. 2), i.e., $x_{TR}(nT) = x_{TR}(nT + PT)$, over a finite interval of $I$ chips that satisfies the following relation:

$$I = LN_{TR} - step(N_g, L) = N_{RIP}P + W \tag{6}$$

where

$$step(N_g, L) = \begin{cases} N_g - L & \text{if } N_g > L \\ 0 & \text{if } N_g \le L \end{cases} \tag{7}$$

$N_{RIP}$ is the number of identical blocks of duration $PT$ each, and $W < P$. Further, the periodicity $P$ has to fulfil the relation $P = \text{l.c.m.}(M, L)$ where l.c.m. denotes the least common multiple.

It follows that the length of the sub-channel training sequence can be chosen equal to

$$N_{TR} = \left\lceil \frac{N_{RIP}P + W + step(N_g, L)}{L} \right\rceil \tag{8}$$

where $\lceil \cdot \rceil$ denotes the ceiling function. To keep the redundancy at minimum, we choose $N_{RIP} = 1$, we fix $W = P/2$, and we determine $N_{TR}$ according to (8). Now, $W$ is updated to fulfil (6), and it clearly satisfies $W < P$.

To determine the synchronization parameters we can run a correlation over an observation window of $W$ samples. In the presence of a tapped delay line channel as that in (2), the received signal still has the cyclic property with periodicity $PT$, but the observation window shrinks to $W' = W - N_P$. In such a scenario, timing and carrier synchronization is accomplished via the following metrics

$$\begin{aligned} P(d) &= \sum_{n=0}^{W'-1} r^*\big((d+n)T\big) r\big((d+n+P)T\big) \\ R(d) &= \sum_{n=0}^{W'-1} \left| r\big((d+n+P)T\big) \right|^2 \end{aligned} \tag{9}$$

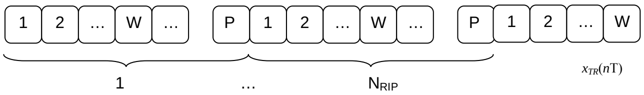


Fig. 2. Cyclic property in the training FMT signal.

$$\widehat{\Delta t} = T \arg\max_d \left\{ \frac{|P(d)|^2}{R^2(d)} \right\} \quad \widehat{\Delta f} = \frac{1}{2\pi PT} \text{angle}\left\{ P\left( \frac{\widehat{\Delta t}}{T} \right) \right\} \tag{10}$$

where the frequency offset estimation holds for $|\Delta f| < 1/(2PT)$.

This method yields good synchronization performance both in terms of timing and carrier frequency estimation. The major drawback of this approach is that the amount of redundancy that we need to send is high. Further, since a given sub-channel training sequence has all identical symbols, the performance of the channel estimator that is required for sub-channel equalization and that uses such training symbols is poor.

## F. Random (PN) Training Approach

Another possible approach is to use pseudo-random training sequences followed by a correlation approach between the received signal and the known transmitted FMT signal $x_{TR}(nT)$. With pseudo-random training sequences the performance of the channel estimator is quite improved compared to the cyclic training approach. In principle the approach would be optimal if we knew the channel response and we run correlation with the equivalent training signal that is obtained by the convolution of the channel with the known transmitted signal. Instead, we propose to run a correlation with $x_{TR}(nT)$ itself. However, it should be noted that since the training sequences are in general preceded and followed by unknown data symbols, due to the memory of the prototype pulses we do not exactly know the transmitted signal. A possibility is to correlate the received signal with the training signal that would be obtained in the absence of (all zero) adjacent data symbols. Another possibility is to choose the training sequence length in a way such that we can identify a window of transmitted chips that depend only on the training data. This is described in what follows.

Now, let $a_{TR}^{(k)}(lT_0)$, $l = 0, ..., N_{TR} - 1$, be the training sequence that we send on sub-channel $k$. We assume it to be pseudo-random with symbols that belong to the PSK signal set. It can be proved that the observation window over which the transmitted signal depends only on the training symbols has duration $WT$ with

$$W = LN_{TR} - step(N_g, L). \tag{11}$$

We can determine the sub-channel training sequence length as

$$N_{TR} = \left\lceil \frac{W + step(N_g, L)}{L} \right\rceil. \tag{12}$$

Then, we recalculate $W$ to satisfy (11). That is, we have to send a training sequence of length $N_{TR}$ that satisfies (12) if we want an observation window of length $W$ chips that depends only on the training symbols.

Now, we propose the use of the following synchronization metrics

$$\begin{aligned} P(d) &= \sum_{n=0}^{W-K-1} r^*\big((d+n)T\big) x_{TR}(nT) r\big((d+n+K)T\big) x_{TR}^*((n+K)T) \\ R(d) &= \sum_{n=0}^{W-K-1} \left| x_{TR}(nT) \right|^2 \left| x_{TR}\big((n+K)T\big) \right|^2 \end{aligned} \tag{13}$$

$$\widehat{\Delta t} = T \arg\max_{d} \left\{ \frac{|P(d)|^2}{R^2(d)} \right\} \quad \widehat{\Delta f} = \frac{1}{2\pi KT} angle \left\{ P\left( \frac{\widehat{\Delta t}}{T} \right) \right\} \quad (14)$$

where the frequency offset estimation holds for $|\Delta f| < 1/(2KT)$. We point out that the value $K \geq 1$ is chosen to minimize the variance of the estimator.

This method requires less redundancy compared to the cyclic training method. Note that although the training sequence length may appear high, we use it also for training of the sub-channel equalizer.

## IV. Detection and Channel Estimation

Once we have estimated $\Delta t$, and $\Delta f$ in the time domain, we compensate the received signal, and we run a bank of sub-channel filters that are matched to the sub-channel transmit filters. Indeed, if we want to sample the filter bank output at rate $1/T_0$, we can use an efficient polyphase implementation that is based on running low rate sub-channel filtering, and an M-point FFT. Even, if synchronization is ideal, due to the channel time dispersion, the sub-channel filter output exhibits some residual ISI. This can be counteracted with some form of equalization. Herein, we consider a simple minimum mean square error (MMSE) linear equalizer [5] that operates over sub-channel samples at rate $1/T_0$. Further, to limit complexity we assume the equalizer to have a small number of taps $N_{EQ}$ (up to 5 taps in our simulations).

### G. Fine Sub-Channel Timing in the Frequency Domain

If timing is only coarse, i.e., within $\pm T_0$ of the true value, it is beneficial to determine a fine sampling phase, and improve the performance of the equalizer that operates over samples at rate $1/T_0$. This goal can be accomplished by first running the time domain synchronizer that we have described above. Then, we apply a frequency domain synchronizer (we use this terminology because we now determine timing at the output of the analysis filter bank). In other words, we run the receiver filter bank at high speed and we obtain, at each sub-channel filter output, samples at chip rate $1/T$. Clearly, this is required only for a brief period of time that corresponds to about the training signal duration. After that, for data detection we can operate at low rate $1/T_0$.

For sub-channel *k*, the fine timing phase is obtained by locking on the peak of the correlation between the receive sub-channel filter output at rate $1/T$, and the known sub-channel training sequence $a_{TR}^{(k)}(lT_0)$ as follows

$$P(d) = \sum_{l=0}^{N_{TR}-1} \left[ z^{(k)}\left(dT + lT_0\right) a_{TR}^{(k)*}(lT_0) \right] \quad (15)$$

$$\widehat{\Delta t_k} = T \arg\max_{d} \left\{ |P(d)|^2 \right\}. \quad (16)$$

This is very similar to what is conventionally done in single carrier systems [4]. In our scenario, it is done on a sub-channel base. However, we further average the sub-channel timing estimates to obtain the value $\widehat{\Delta t} = \sum_{k=0}^{M-1} \widehat{\Delta t_k}$ /M . This has to be done to allow the efficient FFT based filter bank implementation [3] during the data detection stage.

### H. Adaptive MMSE Sub-Channel Equalizer

Once we have determined the sub-channel sampling phase, we do sampling at rate $1/T_0$ and we run an MMSE equalizer (for each sub-channel) whose taps are determined to minimize the MSE between its output and the desired symbol [5], i.e., $J = E[|z_{EQ}^{(k)}(lT_0) - a^{(k)}(lT_0)|^2]$. Training of the equalizer can be done using either the least mean square (LMS) or the recursive least square (RLS) algorithm, first over the known training sequence symbols $a_{TR}^{(k)}(lT_0)$, and then in a data decision directed mode. We have tested both the LMS and the RLS algorithms, and we have found (as it is well known) that the latter has a much superior convergence speed [5].

## V. Simulation Results

### I. System Parameters

We have assumed a Rayleigh faded channel with a *rms* delay spread of 50 ns or 100 ns. The number of sub-channels is $M = 32$, the interpolation factor is $L = 36$, and the prototype pulse is a squared root raised cosine pulse implemented with a FIR filter with $N_g = 515$ chips. The sub-channel training sequence consists of $N_{TR} = 25$ random 4-PSK symbols. If we assume a transmission bandwidth equal to $1/T = 20$ MHz, the sub-channel spacing is equal to 625 kHz. The carrier frequency offset has been chosen smaller or equal to $0.01/(MT) = 6.25$ kHz which should be the case for a commercial oscillator with a precision of 1 p.p.m. and carrier frequency up to 5 GHz.

### J. Synchronizer Performance

In Fig. 3 and 4 we report histograms of the synchronization performance of the algorithms based on cyclic training with $N_{TR}$=36 symbols, and on PN training assuming $N_{TR}$=25 symbols and the parameter $K = 250$. We assume coarse timing of $\pm 2T_0$, and a frequency offset $\Delta f = 0.01/(MT)$. Both the time-domain and frequency-domain algorithms are used. The figures show that the probability of obtaining the correct time phase is very high. Further, the standard deviation is equal to 0.57*T* for both channels and training schemes. The frequency offset estimator works better with the cyclic approach. The residual error is low yielding a standard deviation of 17*e*-4/(*MT*) for $\tau_{rms} = 50$ ns, and 41*e*-4/(*MT*) for $\tau_{rms} = 100$ ns with PN training, and of 35*e*-5/(*MT*) with cyclic training for both channels.

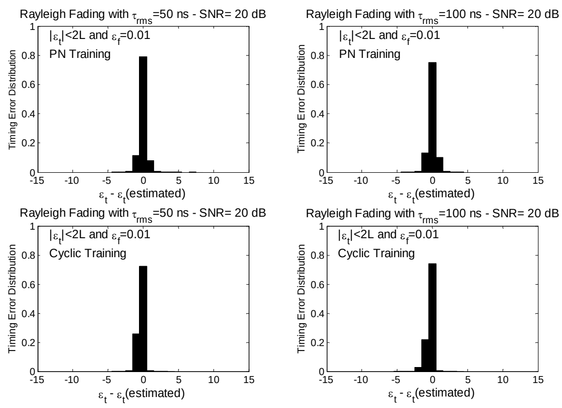


Fig. 3. Histogram of synchronization timing error.

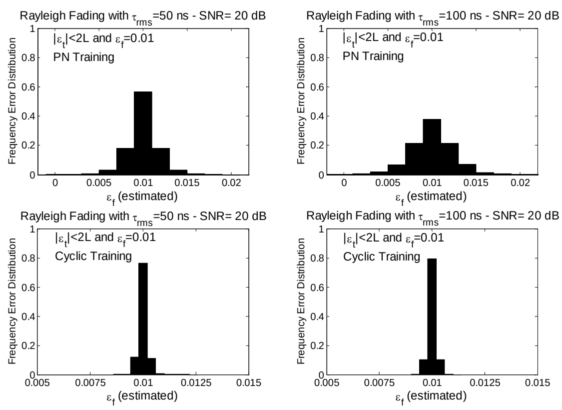


Fig. 4. Histogram of synchronization frequency error.

### K. Equalizer Performance

In Fig. 5 we report the BER performance of the MMSE based equalization scheme assuming ideal synchronization. We see that there is some gain as we increase the number of equalizer taps for the channel with 100 ns delay spread. The performance of the RLS based equalizer that uses the 25 PN training symbols, is very good compared to the curves with ideal channel knowledge.

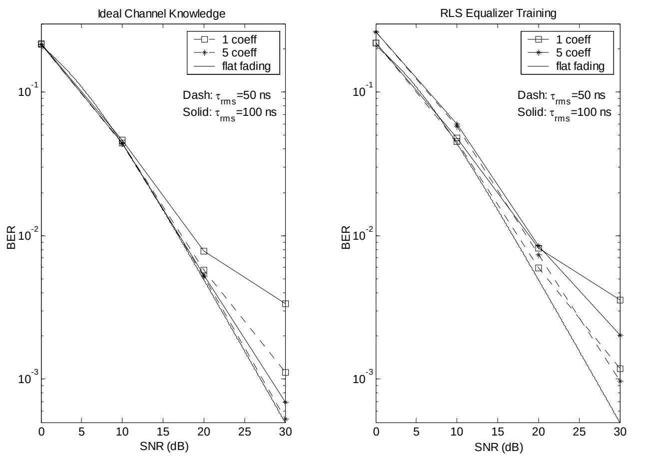


Fig. 5. Ideal and practical equalizer performance for different number of equalizer coefficients (taps).

### L. Overall Receiver Performance

In Fig. 6 and Fig. 7 we report the BER performance as a function of the time offset and the frequency offset for a fixed SNR of 20 dB. The sub-channel equalizers deploy practical training with the RLS algorithm. The curves show that the BER dramatically increases if we do not perform compensation of the time and the frequency offset. However, the BER performance is very good when we use the pseudo-random training and compensation method that we have described assuming 25 symbols. Note that the BER performance with compensation remains practically constant for all offsets considered. Further, for low frequency offsets (Fig. 7) the non-compensated curves exhibit lower BER. The intersection point is determined by the estimator resolution as a function of its standard deviation. For some situations the 1 tap practical equalizer performs better that the 5 taps practical equalizer. This is partly due to the fact that the quality of the RLS algorithm decreases as the number of taps increases.

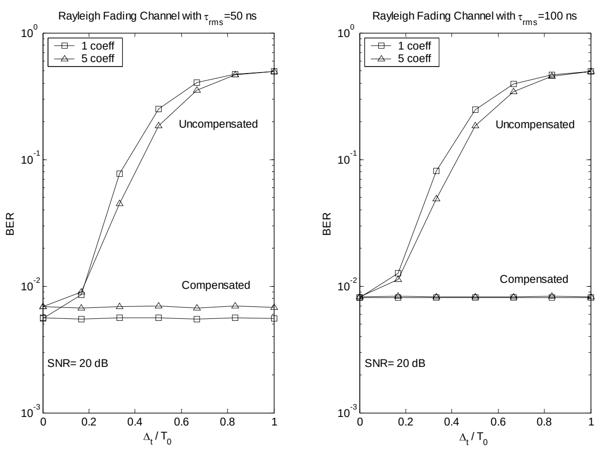


Fig. 6. BER performance as a function of time offset.

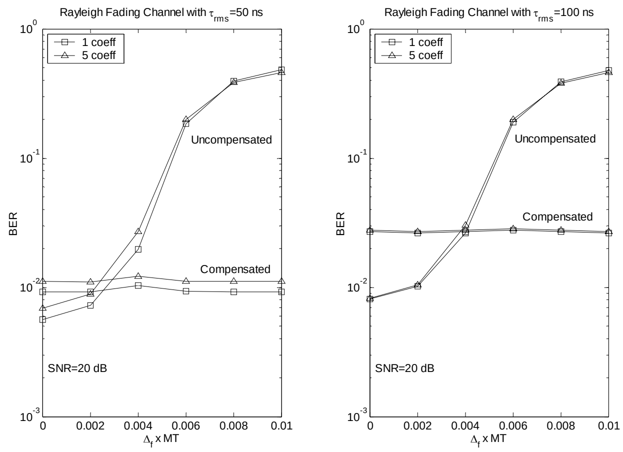


Fig. 7. BER performance as a function of frequency offset.

## VI. Conclusions

In this paper we have presented two training based synchronization approaches that operate in the time domain before running the receiver filter bank. A first method uses training sequences such that the received signal exhibits a cyclic property. A second method uses pseudo random training sequences. The second one is attractive for its performance, for its relatively low redundancy, and for the fact that we can re-use the training sequences in the frequency domain, i.e., after the sub-channel matched filter, both to acquire finer timing and to train an adaptive equalizer. We have considered simple MMSE adaptive equalization with an RLS algorithm and we have reported results as a function of the number of equalizer taps. The overall receiver performance has been shown to demonstrate the effectiveness of the algorithms and their robustness to a wide range of time/frequency offsets and channel frequency selectivity.

## References


[1] A. Assalini, A. Tonello, "Time-Frequency Synchronization in Filtered Multitone Modulation Based Systems", *Proc. of WPMC 03*, Yokosuka, Japan, pp. 221-225, October 2003.

[2] N. Benvenuto, S. Tomasin, L. Tomba, "Equalization Methods in OFDM and FMT Systems for Broadband Wireless Communications", *IEEE Trans Comm*., pp.1413–1418, Sept.2002.

[3] G. Cherubini, E. Eleftheriou, S. Oelser, "Filtered Multitone Modulation for Very High-Speed Digital Subscribe Lines", *IEEE JSAC*, pp. 1016-1028 , June 2002.

[4] H. Meyr, M. Moeneclaey, S. Fechtel, *Digital Communication Receivers*, Wiley 1997.

[5] J. Proakis, *Digital Communications,* Mc Graw Hill 1995.

[6] T. Schmidl, D. Cox, "Robust Frequency and Timing Synchronization for OFDM", *IEEE Trans. Comm*., pp. 1613-1621, December 1997.

[7] A. Tonello, R. Vitenberg, "An Efficient Wavelet Based Filtered Multitone Modulation Scheme", *Proc. of WPMC 04*, Abano Terme, Italy, September 2004.

[8] A. Tonello, "Performance Limits for Filtered Multitone Modulation in Fading Channels", to appear in IEEE *Trans. on Wireless Comm.,* submitted April 2003.

[9] A. Tonello, "Asynchronous Multicarrier Multiple Access: Optimal and Sub-optimal Detection and Decoding", *Bell Labs Technical Journal* vol. 7 n. 3, pp. 191-217, February 2003.

[10] A. Tonello, S. Pupolin, "Discrete Multi-Tone and Filtered Multi-Tone Architectures for Broadband Asynchronous Multi-User Communications", *Proc. of WPMC 01*, Aalborg, Denmark, pp. 461–466, September 2001.