Performance Limits of Multicarrier Based Systems in Fading Channels with Optimal Detection

Andrea M. Tonello

Bell Labs – Lucent Technologies

67 Whippany Rd - Whippany - NJ 07981

email: tonello@lucent.com - tonello@ieee.org

Abstract

We investigate the best attainable performance (performance limits) of multicarrier modulated signals in time-variant frequency selective fading channels when optimal maximum likelihood detection is deployed. It is found that filtered multicarrier modulation is a diversity transform that is capable of yielding coding gains and time/frequency diversity gains as a function of the sub-carrier spacing, and the sub-channel filter shape.

Keywords

Diversity, fading channels, filtered multitone modulation, matched filter bound, OFDM, optimal detection.

INTRODUCTION

In this paper we investigate the performance limits of multicarrier (MC) modulation over time-variant frequency selective fading channels. The basic principle behind MC modulation is to convert the information data symbols sequence at high rate into a number of sub-sequences at low rate. Each low rate sequence is transmitted through a sub-channel that is shaped with an appropriate filter centered on a given sub-carrier. When the sub-carriers are uniformly spaced and the sub-channel filters are identical, an efficient digital implementation is possible and generally referred to as filtered multitone modulation (FMT) [1]. Discrete multitone modulation (DMT) is a particular implementation that deploys rectangular time domain filters such that the sub-channel filtering operation is avoided. DMT is also referred to as orthogonal frequency division multiplexing (OFDM).

In general the frequency selectivity of the channel introduces intercarrier (ICI) and intersymbol (ISI) interference at the receiver side. The design of the sub-channel filters and the choice of the sub-carrier spacing in an FMT system aims at subdividing the spectrum in a number of sub-channels that do not overlap in the frequency domain, such that we can avoid the ICI and get low ISI contributions. In a DMT system the insertion of a cyclic prefix longer then the channel time dispersion is such that ISI and ICI are eliminated and the receiver simplifies to a simple one-tap equalizer per sub-channel. Clearly, the insertion of the cyclic prefix as well as the increase of the sub-carrier spacing translates into a spectral efficiency penalty. The channel temporal selectivity can also introduce ICI as a result of a loss of the sub-channels orthogonality. This happens when the channel is not static over the duration of the FFT block.

The presence of ISI and ICI is such that some form of multichannel equalization is required [5], [7]. We show that in the presence of a time-variant multipath channel the system can be represented with a discrete-time multiple-input multiple-output model. The multichannel impulse response is time-variant and exhibits non-null cross correlations in both time and frequency. Based on this model the optimal receiver searches for the maximum likelihood solution implementing a multichannel Viterbi algorithm.

We study the performance of the optimal receiver in terms of pairwise error probability in a time-variant frequency selective Rayleigh fading channel. For uncoded transmission, a limit on the best attainable performance is given by the probability of error achieved with ideal equalization, i.e., *matched filter performance bound* [11]. It is found that multitone modulation is a diversity transform that is capable of yielding coding and diversity gains as a function of the sub-channel filter impulse response, and the time-frequency characteristics of the channel.

The application of FMT modulation in asynchronous multiuser multitone systems and the extension of the optimal multitone detection scheme to that scenario is studied [5], [6], [8]-[10]. In such a scenario ISI, ICI, and MAI arise for the presence of independent, across users, time offsets, carrier frequency offsets, and multipath fading channels.

MULTITONE TRANSMITTER

A multicarrier modulated signal (complex lowpass representation) can be written as

$$x(t)=\sum_{k\in\mathcal{K}}\sum_{l\in\mathbb{Z}}a^k(lT_0)g(t-lT_0)e^{j2\pi f_k t} \tag{1}$$

where $a^k(lT_0)$ is the sequence of complex data symbols (e.g., M-QAM or M-PSK) transmitted on sub-channel k at rate $1/T_0$ with $T_0=NT$; $g(t)$ is a sub-channel shaping filter (*prototype filter*); $\mathcal{K}=\{0,\ldots,M-1\}$ is the set of sub-carrier indices k. The sub-channel carrier frequency is f_k, and in general $N\geq M$.

An efficient discrete-time implementation is possible when the sub-carriers are uniformly spaced, i.e., $f_k=k/T_1$ with $T_1=MT$. In this paper we consider the case $M=N$, i.e., T_0=T_1, such that the sub-carriers are minimally spaced and the discrete-time MC signal can be rewritten as

$$x(iT)=\sum_{k\in\mathcal{K}}\sum_{l\in\mathbb{Z}}a^k(lT_0)e^{j\frac{2\pi}{M}ki}g(iT-lT_0)\,. \tag{2}$$

If we define the *sub-channel transmit filter* as $g_T^k(t)=g(t)e^{j2\pi f_k t}$ we can rewrite (2) as follows

$$x(iT)=\sum_{k\in\mathcal{K}}\sum_{l\in\mathbb{Z}}a^k(lT_0)g_T^k(iT-lT_0)\,. \tag{3}$$

Proc. of IEEE Wireless Personal Multimedia Communications Symposium 2002, WPMC 02

Honolulu, Hawaii

October 27-30, 2002

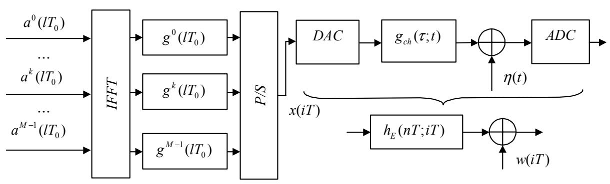

Fig. 1. Multitone lowpass transmission model.

Let $iT = nT + mT_0$ with n=0,…,M-1, m=-∞,…,∞, then the polyphase decomposition[1] of (2) yields

$$x^n(mT_0) = \sum_{l\in\mathbb{Z}}\sum_{k\in\mathcal{K}} a^k(lT_0)e^{j\frac{2\pi}{M}kn} g^n(mT_0 - lT_0) \qquad (4)$$

with $g^n(mT_0)$=g(mT_0+nT). Therefore, the discrete-time MC modulator (referred to as multitone modulator, MT, in the following) comprises the following steps: S/P conversion, IFFT, low rate filtering, P/S conversion.

CHANNEL

The MT signal (4) is D/A converted, RF modulated, and transmitted over the air. The received signal is RF demodulated, and A/D converted. Let $h_E(\tau;t)$ be the time-variant baseband impulse response that comprises the cascade of the analog filter in the D/A converter, the radio channel $g_{ch}(\tau;t)$, and the analog filter in the A/D converter. The analog filters in the D/A and A/D are assumed to approximate an ideal square-root raised cosine filter with Nyquist frequency $1/(2T)$. The propagation media is assumed time-invariant over the duration of the A/D filter. Thus, the sequence of samples at the output of the A/D converter can be written as $y(iT) = \sum_{n\in\mathbb{Z}} x(nT)h_E(iT - nT; iT) + w(iT)$ where $w(iT)$ is a sequence of i.i.d. Gaussian variables with zero-mean and variance N_0. If we define the equivalent *sub-channel receive filter* as $g_R^k(\tau;t) = \int_R g_T^k(\lambda)h_E(\tau-\lambda;t)d\lambda$ the broadband received signal can be written as the superposition of M narrowband signals:

$$y(iT) = \sum_{k\in\mathcal{K}}\sum_{l\in\mathbb{Z}} a^k(lT_0)g_R^k(iT - lT_0; iT) + w(iT)\,. \qquad (5)$$

OPTIMAL MULTITONE DETECTION

Under the hypothesis of the model (5), the optimal detector is based on the maximum likelihood principle, i.e., it seeks the data sequence $\{b^k(lT_0)\}$, k=0,…,M-1, l=-∞,…,∞, that minimizes the accumulated Euclidean distance

$$\Delta = \sum_{i\in\mathbb{Z}}\left| y(iT) - \sum_{l\in\mathbb{Z}}\sum_{k\in\mathcal{K}} b^k(lT_0)g_R^k(iT - lT_0; iT)\right|^2. \qquad (6)$$

The metric can be further partitioned as follows (neglecting constant additive terms)

$$\Delta \sim -\mathrm{Re}\left\{\sum_{l\in\mathbb{Z}}\sum_{k\in\mathcal{K}} b^{k^*}(lT_0)\left[2z^k(lT_0) - \sum_{l'\in\mathbb{Z}}\sum_{k'\in\mathcal{K}} b^{k'}(l'T_0)s^{k,k',l,l'}(l;l')\right]\right\} \quad (7)$$

$$z^k(lT_0) = \sum_{i\in\mathbb{Z}} y(iT)g_R^{k^*}(iT - lT_0; iT) \qquad (8)$$

$$s^{k,k',l,l'}(l;l') = \sum_{i\in\mathbb{Z}} g_R^{k^*}(iT - lT_0; iT)g_R^{k'}(iT - l'T_0; iT)\,. \quad (9)$$

To proceed let us define the following index relations[2] $m = k + lM - 1$, $l(m) = m \text{ div } M$, $k(m) = m \bmod M$ for $k = 0,...,M-1$, $l = -\infty,...,\infty$, $m = -\infty,...,\infty$. Then,

$$\Delta \sim -\sum_{m\in\mathbb{Z}} \mathrm{Re}\left\{ b_m^*\left[2z_m - \sum_{m'\in\mathbb{Z}} b_{m'}s_{m,m'}\right]\right\} \qquad (10)$$

where $b_m = b^{k(m)}(l(m)T_0)$, $z_m = z^{k(m)}(l(m)T_0)$, and $s_{m,m'} = s^{k(m),k(m')}(l(m)T_0, l(m')T_0)$. Since $s_{m,m'} = s_{m',m}^*$ we can rewrite (10) as follows

$$\Delta \sim -\sum_{m\in\mathbb{Z}} \mathrm{Re}\left\{ b_m^*\left[2z_m - b_m s_{m,m} - 2\sum_{m'>0} b_{m-m'}s_{m,m-m'}\right]\right\}. \quad (11)$$

It follows that the search of the maximum likelihood transmitted sequence can be implemented with a Viterbi algorithm. The transition metric is defined as $\Delta_m = -\mathrm{Re}\left\{ b_m^*\left[2z_m - b_m s_{m,m} - 2\sum_{m'>0} b_{m-m'}s_{m,m-m'}\right]\right\}$. The search algorithm sequentially processes the z-parameters.

Detection with Tapped-Delay Line Channel Model

To proceed we assume a tapped-delay line channel model, $h_E(\tau;t) = \sum_p \alpha(p;t)\delta(\tau - \tau_p)$. The receive sub-channel impulse response reads $g_R^k(\tau;t) = \sum_p \alpha(p;t)g_T^k(\tau - \tau_p)$. If we further assume $\tau_p = pT$, and we denote with $\mathcal{P}$ the set of tap indices p, the discrete-time z and s parameters can be re-written as follows,

$$\begin{aligned} z^k(lT_0) = \sum_{n\in\mathcal{K}} e^{-j\frac{2\pi}{M}nk} \sum_{m\in\mathbb{Z}} g^*(nT + mT_0 - lT_0)\cdot \\ \cdot \sum_{p\in\mathcal{P}} y(nT + pT + mT_0)\alpha^*(p; nT + pT + mT_0) \end{aligned} \qquad (12)$$

$$\begin{aligned} s^{k,k',l,l'}(l;l') = \sum_{i\in\mathbb{Z}}\sum_{p,p'\in\mathcal{P}} \alpha^*(p;iT)\alpha(p';iT)e^{-j\frac{2\pi}{M}k(i-p)} e^{j\frac{2\pi}{M}k'(i-p')} \cdot \\ \cdot g^*(iT - lT_0 - pT)g(iT - l'T_0 - p'T) \end{aligned} \qquad (13)$$

Therefore, the optimal detector structure can be depicted as in Fig. 2. Note that the front-end part resembles a rake combiner where the channel taps are coherently combined in an adaptive fashion. If the channel is time-invariant over the duration of the prototype filter (quasi-static fading), the computation of the z-parameters simplifies to (see Fig. 3)

$$\begin{aligned} z^k(lT_0) \approx \sum_{n\in\mathcal{K}} e^{-j\frac{2\pi}{M}nk} \sum_{m\in\mathbb{Z}} g^*(nT + mT_0 - lT_0)\cdot \\ \cdot \sum_{p\in\mathcal{P}} \alpha^*(p; lT_0)y(nT + pT + mT_0) \end{aligned}. \quad (14)$$

---

[1] The polyphase decomposition is here defined in the time domain as a serial to parallel conversion of a high rate signal $x(iT)$, i=-∞,…,∞ into M low rate signals $x^n(mT_0)$=$x(mT_0+nT)$, T_0=MT, n=0,…,M-1, m=-∞,…,∞.

[2] We denote with (a div b) and (a mod b) the integer division and the reminder of the integer division.

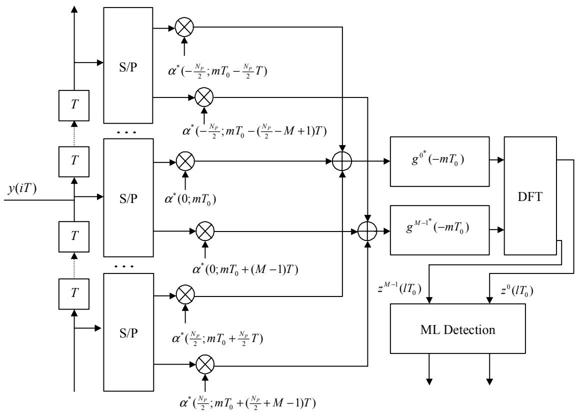

Fig. 2. Optimal MT detector in time-variant multipath channel

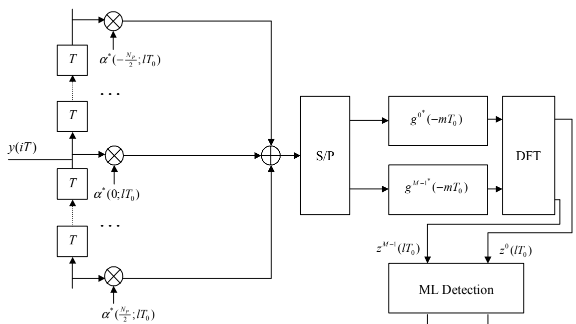

Fig. 3. Optimal MT detector in quasi-static multipath channel

DMT WITH CYCLIC PREFIX

In DMT with cyclic prefix M parallel data streams are passed through an IDFT. Then a cyclic prefix of length μ is inserted. Conventional demodulation is accomplished by disregarding the cyclic prefix and applying an M-point DFT. Assuming a time-invariant channel with N_P+1 taps and $\mu \ge N_P$+1, the DFT outputs are given by $\hat{z}^k(lT_0)=a^k(lT_0)H^k(lT_0)+n^k(lT_0)$, with $n^k(lT_0)$ being i.i.d. Gaussian variables with zero mean, and $H^k(lT_0)=\sum_p \alpha(p;lT_0)e^{j\frac{2\pi}{M}pk}$. Let us consider BPSK data symbols, and let the channel taps to be zero mean Gaussian (Rayleigh fading). Then, the probability of error is upper bounded as follows [3]

$$P_e(\gamma)\le\frac{1}{1+\gamma}. \quad (15)$$

with $\gamma=E_s/N_0$. Note that with this conventional detection approach no frequency diversity exploitation is achieved. If the channel is time-variant over the duration of the FFT block, then ICI is introduced. If we assume a flat fading channel with typical Jakes' temporal correlation the probability of error is bounded as follows [5]

$$P_e\le P_e\left(\frac{E_S}{N_0+E_S I}\right)\quad I=1-\frac{1}{M^2}\left(M+\sum_{k=1}^{M-1}2(M-k)J_0(2\pi f_d Tk)\right). \quad (16)$$

Note that for relatively high Doppler spreads a significant error floor is introduced.

PERFORMANCE OF OPTIMAL MT DETECTION

We study the performance in terms of *pairwise error probability* (PEP) that is defined as the probability that the optimal MT detector decides erroneously in favor of the sequence $\{b^k(lT_0)\}$ when $\{a^k(lT_0)\}$ was transmitted indeed. Under the hypothesis of perfect knowledge of the channel state information, an upper bound of the PEP is given by

$$P(\boldsymbol{a}\to\boldsymbol{b}\mid\boldsymbol{s})\le e^{-\frac{d^2(\boldsymbol{a},\boldsymbol{b})}{4N_0}}. \quad (17)$$

It is obtained with the standard approximation to the Gaussian tail function [3]. The *pairwise error event distance* is

$$d^2(\boldsymbol{a},\boldsymbol{b})=\sum_{i\in\mathbb{Z}}\Big|\sum_{l\in\mathbb{Z}}\sum_{k\in\mathcal{K}}\underbrace{[a^k(lT_0)-b^k(lT_0)]}_{e^k(lT_0)}g_R^k(iT-lT_0;iT)\Big|^2. \quad (18)$$

Using the s-parameters definition we obtain

$$d^2(\boldsymbol{a},\boldsymbol{b})=\sum_{l,l'\in\mathbb{Z}}\sum_{k,k'\in\mathcal{K}}e^{k^*}(lT_0)e^{k'}(l'T_0)s^{k,k'}(l,l'). \quad (19)$$

Matched Filter Bound

In the case of uncoded transmission single error events are possible, i.e., the detected sequence may differ only in one data symbol. If we assume a single error event, the performance evaluation corresponds to the evaluation of the matched filter performance bound (MFB) [11]. That is, the probability of error when perfect equalization is achieved. Let us assume the single error event to occur on sub-channel $\bar{k}$ and time instant $\bar{l}T_0=0$. Further, let us assume a tapped delay line channel model. Then, the error event distance (19) can be written as follows

$$\begin{aligned} d^2_{MFB}(\bar{k}) = D_e\sum_{i\in\mathbb{Z}}\sum_{p,p'\in\mathcal{P}} e^{j\frac{2\pi}{M}\bar{k}(p-p')}\alpha^*(pT;iT)\cdot \\ \cdot\alpha(p'T;iT)g^*(iT-pT)g(iT-p'T) \end{aligned} \quad (20)$$

with $D_e=|e^{\bar{k}}(0)|^2$ being the squared Euclidean distance between the transmitted and detected data symbol. For instance, with BPSK modulation $D_e=4E_S$. Note that in general (20) is a function of the sub-channel index. Using matrix notation we can write

$$d^2_{MFB}(\bar{k})=D_e\sum_{i\in\mathbb{Z}}\boldsymbol{\alpha}_i^H\mathbf{G}_i\boldsymbol{\alpha}_i=D_e\boldsymbol{\alpha}^H\mathbf{G}\boldsymbol{\alpha} \quad (21)$$

$$\begin{aligned} \boldsymbol{\alpha}_i &= [\alpha(-N_P/2;iT)e^{j\frac{\pi}{M}\bar{k}N_P},\ldots,\alpha(N_P/2;iT)e^{-j\frac{\pi}{M}\bar{k}N_P}]^T \\ \mathbf{g}_i &= [g(iT+N_PT/2,\ldots,g(iT-N_PT/2)] \\ \mathbf{G}_i &= \mathbf{g}_i^H\mathbf{g}_i \\ \boldsymbol{\alpha} &= [\boldsymbol{\alpha}^T_{-(L+N_P)/2},\ldots,\boldsymbol{\alpha}^T_{(L+N_P)/2}]^T \\ \mathbf{G} &= diag\{\mathbf{G}_{-(L+N_P)/2},\ldots,\mathbf{G}_{(L+N_P)/2}\}. \end{aligned} \quad (22)$$

where we have assumed a number of channel taps equal to N_P+1, and a prototype pulse of duration L+1. Assuming the channel taps to be zero mean Gaussian (Rayleigh fading), we can rewrite the normal quadratic form (21) as (see the Appendix)

$$d_{MFB}^2(\bar{k}) = \sum_i \lambda_i |\beta_i|^2 \quad (23)$$

where λ_i are the eigenvalues of the matrix $D_e\mathbf{RG}$ with $\mathbf{R} = E[\boldsymbol{\alpha}\boldsymbol{\alpha}^H]$, and β_i are i.i.d. complex Gaussian variables with zero mean and unit variance. Averaging (17) over the distributions of $|\beta_i|^2$ (exponential) a bound on the PEP is then found

$$P_{e,MFB} \leq \prod_i (1+\frac{\lambda_i}{4N_0})^{-1} \leq (\frac{E_S}{4N_0})^{-d} \prod_{\lambda_i \neq 0} (\frac{\lambda_i}{E_S})^{-1}. \quad (24)$$

where d is the number of nonzero eigenvalues. From the analysis of (24) we can draw the following remarks:

A. FMT modulation can be interpreted as a diversity transform. It performs time or spectrum spreading as a function of the prototype filter and the sub-carrier spacing.
B. FMT modulation with optimal detection yields both a diversity and a coding gain over an uncoded single carrier system deployed on a flat Rayleigh fading channel. The diversity gain d equals the number of nonzero eigenvalues of the matrix $D_e\mathbf{RG}$, while the product of the nonzero eigenvalues gives the coding gain.
C. The diversity gain satisfies the bound $0 \leq d \leq \min\{rank(\mathbf{R}), rank(\mathbf{G})\}$. If the channel is frequency selective but time-invariant then $0 \leq d \leq N_P + 1$. If the channel is frequency non-selective but time-variant then $0 \leq d \leq L+1$. Therefore, a sub-channel bandwidth expansion potentially increases the frequency diversity gain, while a sub-channel bandwidth compression (pulse duration expansion) increases the time diversity gain.

It is interesting to note that some analogy exists with the analysis of the PEP in space-time coded systems [4]. However, in the system that we consider, coding, i.e., the multi-tone transform, takes place across sub-channels and not across antennas. To get insight we evaluate $P_{e,MFB}$ in the next sections assuming first a time-invariant frequency selective channel and then a time-variant flat fading channel.

Time-Invariant Frequency Selective Channel

If the channel is time-invariant frequency selective we obtain

$$d_{MFB}^2(\bar{k}) = D_e \boldsymbol{\alpha}_0^H \sum_{i \in \mathbb{Z}} \mathbf{G}_i \boldsymbol{\alpha}_0 \quad (25)$$

with $(\sum_i \mathbf{G}_i)_{p,p'} = \kappa(p-p') = \sum_i g^*(iT) g(iT + pT - p'T)$ being the prototype pulse autocorrelation. In a time invariant channel, $rank\{\mathbf{R}\} \leq (N_P + 1)$. If the channel taps are uncorrelated, $\mathbf{R}_i = diag\{\Omega_{-N_P/2}, ..., \Omega_{N_P/2}\}$, and $d = \mathrm{rank}\{\mathbf{G}_i\}$.

If we choose a *rectangular* prototype pulse $g(n) = 1/\sqrt{M}\,\mathrm{rect}(n/M)$, then

$$\kappa(p) = 1 - |p|/M \quad if \quad |p| \leq M; \quad 0 \quad otherwise. \quad (26)$$

If we choose an ideal band limited prototype pulse $g(n) = 1/\sqrt{M}\,\mathrm{sinc}(n/M)$, then

$$\kappa(p) = \mathrm{sinc}(p/M). \quad (27)$$

If we deploy a Gaussian prototype pulse $g(n) = \sqrt{\sigma/M/\sqrt{\pi/2}}\, e^{-(\sigma n/M)^2}$ with $\sigma = f_{3dB}\pi\sqrt{2/\ln 2}$, then

$$\kappa(p) = e^{-\frac{1}{2}(\frac{\sigma}{M}p)^2} \quad (28)$$

To proceed, let us assume a time-invariant channel with an exponential power delay profile, i.e., a channel with $N_P + 1$ rays that are independent, zero mean Gaussian with power $E[|\alpha(p)|^2] \sim e^{-\rho|p|}$, $-N_P/2 \leq p \leq N_P/2$. We choose $\rho = 0.01$, and N_p={10, 50}. In Fig. 4 we report the matched filter probability of error bound when we deploy a *rect* pulse (DMT), a *sinc* pulse (FMT), and a Gaussian pulse (FMT-G) with BPSK modulation. Now let us assume to fix the overall transmission bandwidth $1/T$. Then, for a fixed number of sub-carriers M, DMT with optimal detection yields better performance than FMT. This is because the DMT pulse is not strictly band limited. That is, a sub-channel bandwidth expansion yields increased frequency diversity gains. If we deploy a Gaussian filter we can expand the sub-channel bandwidth by increasing f_{3dB} (in Fig. 4, $f_{3dB} = 0.33$, and $f_{3dB} = 0.9$ are shown).

If we fix the prototype filter and the transmission bandwidth, lower diversity gains are found when we increase the number of sub-carriers. This is because with a large number of sub-carriers the sub-channels become flat and only residual ISI remains on each sub-channel. Therefore, the frequency diversity gain is maximized when we deploy single carrier modulation with spectrum expansion (in Fig. 4 we plot also the bound for FMT with one single carrier).

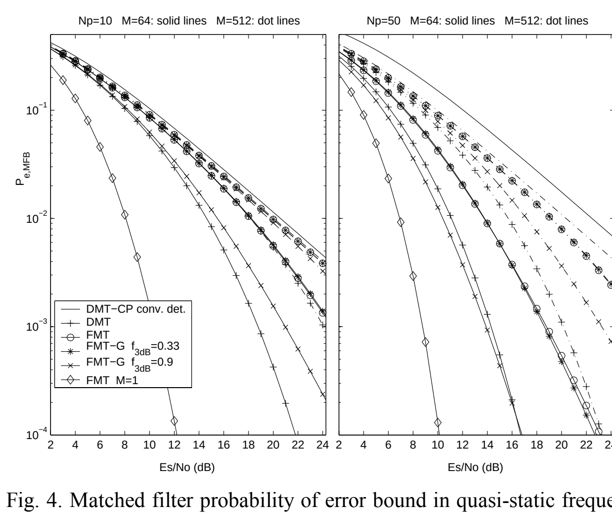

Fig. 4. Matched filter probability of error bound in quasi-static frequency selective Rayleigh fading. Exponential power delay profile, N_P +1 rays, M sub-carriers. Rectangular sub-channel pulse (DMT), Sinc sub-channel pulse (FMT), Gaussian sub-channel pulse (G-FMT). Performance of conventional detection of DMT with cyclic prefix is also shown.

Finally, note that if we deployed DMT with a cyclic prefix of length $N_P + 1$, conventional detection would allow to perfectly counteract the ISI channel. However, no diversity gains would be found and a SNR penalty would be determined by the insertion of the cyclic prefix as Fig. 4 shows.

Time-Variant Frequency Non-Selective Channel

If we assume the channel to be frequency nonselective (flat) but time-variant, we obtain

$$d_{MFB}^2(\bar{k}) = D_e \sum_{i \in \mathbb{Z}} |\alpha(0;iT)|^2 |g(iT)|^2 . \qquad (29)$$

If we assume a completed correlated Rayleigh fading channel, i.e., quasi-static, then there is only one nonzero eigenvalue ($\lambda = 4E_s$ with BPSK), and the bound on the pairwise error probability becomes $P_{e,MFB} \le (1 + E_S / N_0)^{-1}$. If we assume a completed uncorrelated channel, then the eigenvalues are $\lambda_i = 4E_S |g(iT)|^2$ and the pairwise error probability becomes $P_{e,MFB} \le \prod_i (1 + |g(i)|^2 E_S / N_0)^{-1}$. We can understand that to maximize the temporal diversity gain we have to deploy a prototype filter that yields a sub-channel bandwidth compression (longer duration pulse). Therefore, FMT yields better performance than DMT in fast Rayleigh fading channels. If we deploy a Gaussian pulse, the maximization of the temporal diversity calls for smaller cut-off frequencies.

In Fig. 5 we show the error probability bound assuming a typical flat fading Jakes' channel model whose temporal correlation is $R(i,j) = J_0(2\pi f_d T(i-j))$. If we fix the prototype filter and the transmission bandwidth, higher diversity gains are found when we increase the number of subcarriers. FMT yields better performance than DMT but worse than Gaussian FMT with f_{3dB}=0.1. With conventional detection of DMT a significant error floor is introduced.

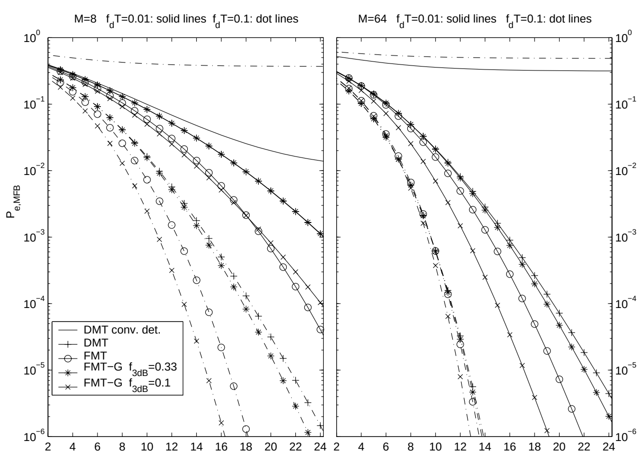

Fig. 5. Matched filter probability of error bound in time-variant frequency non-selective Rayleigh fading. *M* sub-carriers, f_dT normalized Doppler. Rectangular sub-channel pulse (DMT), Sinc pulse (FMT), Gaussian pulse (G-FMT). Performance of conventional detection of DMT is also shown.

APPENDIX: Normal Quadratic Form

Let us consider the following central quadratic form

$$\Lambda = \boldsymbol{\alpha}^H \mathbf{G} \boldsymbol{\alpha} \qquad (30)$$

where $\boldsymbol{\alpha}$ is a vector of complex Gaussian random variables with zero mean and covariance $\mathbf{R} = E[\boldsymbol{\alpha}\boldsymbol{\alpha}^H]$ while $\mathbf{G}$ is an Hermitian matrix of size $L \times L$. Let $\mathbf{C}\mathbf{C}^H = \mathbf{R}$ be the Cholesky factorization of $\mathbf{R}$, and $\mathbf{U}$ be the unitary eigenvector matrix that diagonalizes $\mathbf{C}^H\mathbf{G}\mathbf{C}$, i.e., $\boldsymbol{\Gamma} = \mathbf{U}\mathbf{C}^H\mathbf{G}\mathbf{C}\mathbf{U}^{-1} = diag\{\lambda_1, ..., \lambda_L\}$, [2]. Then, we obtain

$$\Lambda = \boldsymbol{\beta}^H \mathbf{U}^{-1^H} \mathbf{C}^H \mathbf{G}\mathbf{C}\mathbf{U}^{-1} \boldsymbol{\beta} = \boldsymbol{\beta}^H \boldsymbol{\Gamma} \boldsymbol{\beta} = \sum_{i=1}^{L} \lambda_i |\beta_i|^2 \qquad (31)$$

where $\boldsymbol{\beta} = \mathbf{U}\mathbf{C}^{-1}\boldsymbol{\alpha}$ is a vector of independent zero mean Gaussian variables with unit variance. The eigenvalues of $\mathbf{C}^H\mathbf{G}\mathbf{C}$ are the same as the eigenvalues of $\mathbf{R}\mathbf{G}$. Therefore, we do not need to compute the Cholesky factorization.

CONCLUSIONS

We have proposed optimal maximum likelihood detection of multicarrier modulated signals in fading channels. With such a detection approach both the frequency and the temporal diversity of the channel can be exploited. Conventional detection of DMT signals with cyclic prefix is indeed simple but sub-optimal: it is unable to exploit the frequency diversity and to cope with fast time-variant channels.

The analysis of the pairwise error probability shows that multitone modulation can be interpreted as a diversity transform. The maximization of the coding and diversity gains calls for the optimization of the prototype filter impulse/frequency response and the sub-carrier spacing (number of sub-carriers for fixed overall bandwidth) under the constraint of a given time-frequency channel characteristic.

REFERENCES

[1] G. Cherubini, E. Eleftheriou, S. Olcer, J. Cioffi, "Filter bank modulation techniques for very high-speed digital subscriber lines", *IEEE Commun. Magazine*, pp. 98-104, May 2000.

[2] R. Horn, C. Johnson, "Matrix analysis", *Cambridge Univ. Press*, 1988.

[3] J. Proakis, "Digital Communications", Mc Graw Hill, 1995.

[4] V. Tarokh, N. Seshadri, R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction", *IEEE Trans. Info. Theory, March 1998*, pp. 744-765.

[5] A. Tonello, "Multicarrier multiuser asynchronous communications", *Doctor of research degree thesis*, University of Padova, Italy, December 2001.

[6] A. Tonello, S. Pupolin, "Discrete multi-tone and filtered multi-tone architectures for broadband asynchronous multiuser communications", *Proc. of WPMC 01*, Aalborg, pp. 461-466, September 2001.

[7] A. Tonello, "On turbo equalization of interleaved space-time codes", *Proc. of IEEE VTC 01 Fall*, Atlantic City, pp. 887-891, October 2001. To appear also in *IEEE Trans. on Comm*.

[8] A. Tonello, S. Pupolin, "Performance of single user detectors in multitone multiple access asynchronous communications", *Proc. of IEEE VTC 02 Spring*, pp. 199-203, May 2002.

[9] A. Tonello, "Multiuser detection and turbo multiuser decoding in asynchronous multitone multiple access systems", *Proc. of IEEE VTC 02 Fall*, Vancouver, September 2002.

[10] A. Tonello, "Multiuser detection/decoding in asynchronous multitone multiple access systems", *Proc. of WPMC 02*, Honolulu, October 2002.

[11] G. Ungerboeck, "Adaptive maximum likelihood receiver for carrier-modulated data transmission systems", *IEEE Trans. on Comm.*, pp. 624-636, May 1974.