Synchronization Algorithms for Multiuser Filtered Multitone (FMT) Systems

Andrea M. Tonello and Francesco Pecile
DIEGM - Università di Udine
Via delle Scienze 208 - 33100 Udine - Italy
phone: +39 0432 558 288 - fax: +39 0432 558 251 - e-mail: tonello@uniud.it

***Abstract*—In this paper we consider the synchronization problem in a filtered multitone (FMT) modulated system. An FMT based system differs from the popular OFDM scheme in the deployment of sub-channel shaping filters. We assume a multiuser uplink scenario where sub-channels are partitioned among the active users. Users are asynchronous such that the received signals experience independent time offsets, carrier frequency offsets, and multipath fading. The approach herein presented is based on the deployment of training sequences. We recover symbol/frame timing and carrier frequency for each active user by exploiting the sub-channel separability with an iterative procedure. Then, we run simple linear sub-channel equalizers with RLS training. The robustness of the scheme in an asynchronous uplink scenario is evaluated.**

Keywords—Filtered multitone modulation, Multiuser systems OFDM, Synchronization.

I. INTRODUCTION

Multicarrier modulation has attracted great attention for application to wideband wireless channels. This is because it has the potentiality of simplifying the equalization task that becomes mandatory in wideband channels that experience severe frequency selectivity. Orthogonal frequency division multiplexing (OFDM) is probably among the most popular multicarrier modulation techniques. It is essentially based on multicarrier transmission with sub-channel pulses that have a rectangular impulse response. More general multicarrier schemes deploy sub-channel filters with time-frequency concentrated response. Under certain conditions they can be implemented by using an inverse fast Fourier transform (IFFT) followed by low-rate sub-channel filtering. These schemes are referred to as filtered multitone modulated systems (FMT) [1]. If the sub-channels have disjoint frequency response, i.e., they do not overlap in frequency, it is possible to avoid the inter-carrier interference (ICI), and get low inter-symbol interference (ISI) that can be corrected with simplified sub-channel equalization. FMT modulation has the potentiality of achieving better spectral efficiency than OFDM yet requiring sufficiently simple equalization [2]. Multiuser architectures are possible both with OFDM and FMT. They are simply obtained by partitioning the available tones (sub-channels) among the active users [3]-[5]. In the uplink, multiuser OFDM is severely affected by time misalignments and carrier frequency offsets that can be large in a cellular, high mobility environment. This is due to the fact that in conventional OFDM, sub-channels exhibit *sinc* like frequency response, therefore their orthogonality can be easily lost in the absence of precise synchronization [6]-[7]. In an asynchronous multiuser environment, increased robustness and better performance can be obtained with filtered multitone (FMT) modulation architectures where the sub-channels are shaped with appropriate time-frequency concentrated pulses.

In this paper we consider the synchronization, i.e., acquisition of the carrier frequency and the frame/symbol timing, in a filtered multitone (FMT) modulated system. Although the synchronization problem in OFDM is well understood [8]-[9], in an FMT system it presents several challenges. Synchronization algorithms for a single user FMT system where presented in [10]-[11]. In [10] a blind scheme has been considered. In [11] both time domain and frequency domain algorithms with training have been investigated.

In this paper we consider the multiuser uplink scenario. Detection is single user based. It works by running a filter bank that is matched to the sub-channels that are assigned to the desired user after compensation of the time and frequency offsets. We propose an iterative synchronization approach that exploits the sub-channel separability to estimate the users' time/frequency offset. It should be noted that to allow for an efficient implementation of the receiver, during the detection stage, accurate time/frequency offset compensation has to be done up-front, i.e., before running the receiver filter bank. This is because the efficient receiver implementation that is based on low-rate sub-channel matched filtering followed by a fast Fourier transform (FFT) [1] requires to compensate the time/frequency offset deploying a common estimate for all the sub-channels that are assigned to the user under consideration.

This paper is organized as follows. First, we describe the asynchronous multiuser FMT system model. Then, we propose a synchronization algorithm. Finally, we report performance results for an uplink scenario.

II. SYSTEM MODEL

The complex baseband transmitted signal $x^{(u)}(nT)$ of user u is obtained by a filter bank modulator with prototype pulse $g(nT)$ and sub-channel carrier frequency $f_k = k/(MT)$, $k = 0,...,M-1$, with T being the transmission period

$$x^{(u)}(nT) = \sum_{k=0}^{M-1} x^{(u,k)}(nT) \tag{1}$$

$$x^{(u,k)}(nT) = \sum_{l\in\mathbb{Z}} a^{(u,k)}(lT_0)g(nT - lT_0)e^{j2\pi f_k nT} \tag{2}$$

This work was partly supported by MIUR under project FIRB-PRIMO "Reconfigurable platforms for wideband wireless communications".

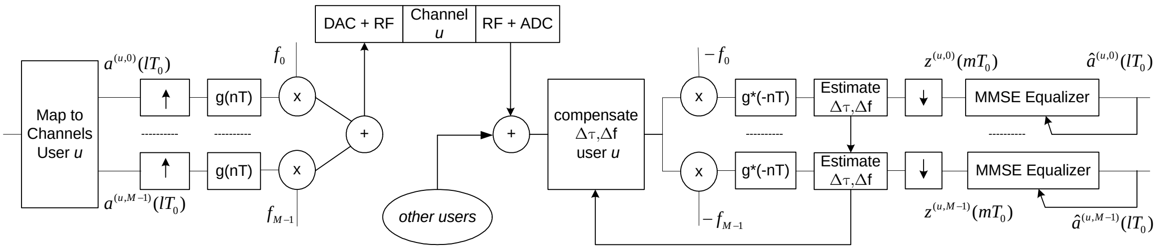

Fig. 1. Multiuser FMT system model with highlighted transmitter and receiver of user *u*.

where $a^{(u,k)}(lT_0)$ is the *k*-th sub-channel data stream of user *u* that we assume to belong to the M-PSK/M-QAM constellation set and that has rate $1/T_0$ with $T_0 = NT \geq MT$. The interpolation factor N is chosen to increase the frequency separation between sub-channels, thus to minimize the amount of inter-carrier interference (ICI) and multiple access interference (MAI) at the receiver side. If for instance we assume an ideal root-raised cosine pulse with roll-off α_1, the frequency guard equals $\alpha_2/(MT)$ with $\alpha_2 = 1 - M(1+\alpha_1)/N$. A possible efficient implementation of the transmitter that is based on polyphase filtering is described in [1]. The discrete time signal is digital-to-analog converted, RF modulated, and transmitted over the air. Distinct FMT sub-channels can be assigned to distinct users. In this case, the symbols are set to zero for the unassigned FMT sub-channels:

$$a^{(u,k)}(lT_0) = 0 \quad \text{for} \quad k \notin K_u \tag{3}$$

where K_u denotes the set of M_u sub-channel indices that are assigned to user u.

At the receiver, after RF demodulation, and analog-to-digital conversion, the discrete time received signal can be written as

$$\begin{aligned} y(\tau_i) = &\sum_{u=1}^{N_U}\sum_{k=0}^{M-1}\sum_{n\in\mathbb{Z}} x^{(u,k)}(nT) g_{CH}^{(u)}(\tau_i - nT - \Delta_\tau^{(u)}) e^{j\left(2\pi\,\Delta_f^{(u)}\tau_i + \phi^{(u)}\right)} \\ &+ \eta(\tau_i) \end{aligned} \tag{4}$$

where $\tau_i = iT + \Delta_0$, $i \in \mathbb{Z}$ and Δ_0 is a sampling phase. N_U is the number of users, $\Delta_\tau^{(u)}$ is the time offset of user *u*, $\Delta_f^{(u)}$ and $\phi^{(u)}$ are the carrier frequency and phase offset, $g_{CH}^{(u)}(t)$ is the fading channel impulse response of user *u*, and $\eta(\tau_i)$ is the additive Gaussian noise with zero mean contribution. Note that we assume the time/frequency offsets to be identical for all sub-channels that are assigned to a given user.

A. Receiver with A-Priori Knowledge of the Parameters

Here we consider a single user based FMT receiver where we first acquire time/frequency synchronization with each active user. Then, for each user, we compensate the time/frequency offsets, we run FMT demodulation via a bank of filters that is matched to the transmitter bank, and we sample the outputs at rate $1/T_0$. Assuming a priori knowledge of the time and frequency offsets (that have in practice to be estimated as described in what follows), the sequence of samples at the front-end filter output of FMT sub-channel k of user u that compensates the time offset $\Delta_\tau^{(u)}$, and the frequency offset $\Delta_f^{(u)}$ of only user u, can be written as

$$\begin{aligned} z_{a-priori}^{(u,k)}(mT_0) &= \sum_{i\in\mathbb{Z}} y(iT + \Delta_\tau^{(u)}) g^*(iT - mT_0) e^{-j2\pi\left(f_k + \Delta_f^{(u)}\right)iT} \\ &= \sum_{u'=1}^{N_U}\sum_{k'=0}^{M-1}\sum_{l\in\mathbb{Z}} a^{(u',k')}(lT_0) g_{EQ}^{(u',k'),(u,k)}(mT_0 - lT_0; mT_0) + \eta^{(u,k)}(mT_0) \end{aligned} \tag{5}$$

where $\eta^{(u,k)}(mT_0)$ is the sequence of filtered noise samples, and where

$$\begin{aligned} g_{EQ}^{(u',k'),(u,k)}(lT_0; mT_0) &= \sum_{n\in\mathbb{Z}}\sum_{i\in\mathbb{Z}} g(nT + lT_0 - mT_0) g^*(iT - mT_0) \\ &\times e^{j2\pi\left(f_{k'}nT - f_k iT\right) + j2\pi\left(\Delta_f^{(u')} - \Delta_f^{(u)}\right)iT + j\phi^{(u'),(u)}} g_{CH}^{(u')}(iT - nT - \Delta_\tau^{(u')} + \Delta_\tau^{(u)}), \end{aligned} \tag{6}$$

with $\phi^{(u'),(u)} = \phi^{(u')} + 2\pi\Delta_f^{(u')}\Delta_\tau^{(u)}$, is the multi-channel time-variant impulse response between the input sub-channel of indices (u', k') and the output sub-channel of indices (u,k).

We can rewrite (5) as follows

$$\begin{aligned} z_{a-priori}^{(u,k)}(mT_0) &= \sum_{l\in\mathbb{Z}} a^{(u,k)}(lT_0) g_{EQ}^{(u,k),(u,k)}(mT_0 - lT_0; mT_0) \\ &\quad + ICI^{(u,k)}(mT_0) + MAI^{(u,k)}(mT_0) + \eta^{(u,k)}(mT_0) \end{aligned} \tag{7}$$

such that we can highlight, in general, the fact that the sub-channel filter output of index k may suffer from ISI, ICI (from the sub-channels of index $k' \neq k$ that are assigned to user u), and MAI (from all sub-channels that belong to the other users) as a consequence of frequency overlapping sub-channels, time/frequency offsets, and channel time/frequency selectivity. When the sub-channels do not overlap, e.g., with ideal root-raised-cosine pulses with appropriate sub-carrier spacing, and the carrier frequency offsets of all users do not exceed half the frequency guard the ICI and MAI components are zero. Some intersymbol interference over each sub-channel may be present and can be counteracted with sub-channel equalization. Now let us model the channel with a T-spaced tap delay line

$$g_{CH}^{(u)}(iT) = \sum_{p\in P} \alpha_p^{(u)} \delta(iT - pT) \tag{8}$$

where $\alpha_p^{(u)}$ are the channel tap amplitudes, P is the set of integer tap delays. Further, let us assume the time offsets to be integer multiples of *T*. Then, the sub-channel output of user *u* can be written as follows

$$z_{a-priori}^{(u,k)}(mT_0) = \sum_{l\in\mathbb{Z}} a^{(u,k)}(lT_0) \beta^{(u,k)}(mT_0 - lT_0) + \eta^{(u,k)}(mT_0) \tag{9}$$

$$\beta^{(u,k)}(lT_0) = \sum_{p\in P} \alpha_p^{(u)} e^{-j2\pi f_k pT + j\phi^{(u),(u)}} \kappa_g(lT_0 - pT)$$

where $\kappa_g(pT)$ is the prototype pulse autocorrelation. Clearly, (9) holds true when we do not have ICI and MAI, and perfect compensation of the time/frequency offset. Without compensation of the time/frequency offset, the desired user front-end output exhibits increased amounts of ISI, ICI, and MAI. Therefore, a key issue is the estimation of the synchronization parameters so that they can be compensated before passing the signal through the analysis filter bank.

III. Iterative Synchronization

In the synchronization stage we estimate the time/frequency offsets of each user. This is accomplished with a training based approach, i.e., each user transmits a frame that comprises a known training data portion $a_{TR}^{(u,k)}(lT_0)$, $k \in K_u$ $l = 0,...,N_{TR}-1$. The training sequence is assumed to be random and to have good autocorrelation. Then, the estimation of the time/frequency offsets is done for each user with an iterative procedure (see also the Appendix) where we first filter the received signal with a bank of filters that is matched to the transmit filter bank. Second, the time offset and the frequency offset of the user are estimated. Third, we re-run the filter bank by now pre-compensating with the estimated time/frequency offsets. The procedure can be repeated iteratively.

Therefore, at the first iteration (when we do not have any a priori knowledge of the time/frequency offset of the desired user) we run the following filter bank for the channels of user u

$$z_{it=1}^{(u,k)}(nT) = \sum_{i\in\mathbb{Z}} y(iT)g^*(iT-nT)e^{-j2\pi f_k iT}. \quad (10)$$

The outputs are used to compute the following correlation metric that uses the known training symbols

$$P_{it}^{(u,k)}(n) = \sum_{l=0}^{N_{TR}-K-1}\left[Z_{it}^{(u,k)}(lT_0;nT)^* Z_{it}^{(u,k)}(lT_0+KT_0;nT)\right] \quad (11)$$

$$Z_{it}^{(u,k)}(lT_0;nT) = z_{it}^{(u,k)}\left(lT_0+nT\right)\frac{a_{TR}^{(u,k)}(lT_0)^*}{|a_{TR}^{(u,k)}(lT_0)|^2}. \quad (12)$$

Metric (11) is used to locate the training sequence and to estimate the time offset of sub-channel k of user u as follows

$$\widehat{\Delta_{\tau,it}^{(u,k)}} = T \arg\max_n \left\{\left|P_{it}^{(u,k)}(n)\right|^2\right\}, \quad (13)$$

while it is used to estimate the frequency offset as follows

$$\widehat{\Delta_{f,it}^{(u,k)}} = \frac{1}{2\pi KT_0}\arg\left\{P_{it}^{(u,k)}(n_{\max})\right\} \quad n_{\max} = \widehat{\Delta_{\tau,it}^{(u,k)}}/T. \quad (14)$$

At this point, we compute the average values (across the assigned sub-channels)

$$\widehat{\Delta_{\tau,it}^{(u)}} = \frac{1}{M_u}\sum_{k\in K_u}\widehat{\Delta_{\tau,it}^{(u,k)}} \qquad \widehat{\Delta_{f,it}^{(u)}} = \frac{1}{M_u}\sum_{k\in K_u}\widehat{\Delta_{f,it}^{(u,k)}}. \quad (15)$$

The frequency offset estimation holds for $|\Delta f| < 1/(2KT_0)$. Further, the value $K \geq 1$ is chosen to minimize the variance of the estimator.

Once the estimates above are computed, we re-run the receiver filter bank. However, now the filter bank can exploit the a priori knowledge of the time/frequency offsets. Thus, for this new iteration we compute

$$z_{it+1}^{(u,k)}(mT_0) = \sum_{i\in\mathbb{Z}} y(iT+\widehat{\Delta_{\tau,it}^{(u)}})g^*(iT-mT_0)e^{-j2\pi\left(f_k+\widehat{\Delta_{f,it}^{(u)}}\right)iT}. \quad (16)$$

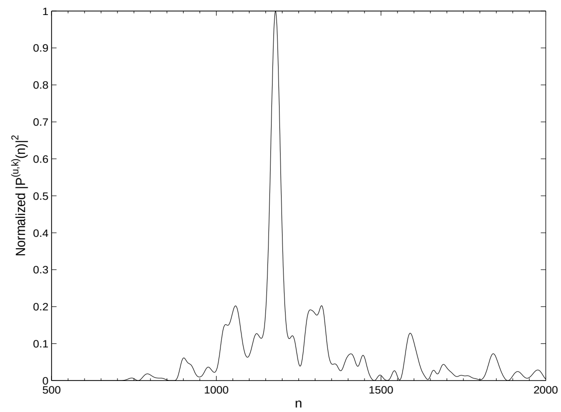

Fig. 2. Example of normalized metric (11) for a realization with 32 asynchronous users, and with random 4-PSK training of length 15.

TABLE I
EXAMPLE OF SYSTEM PARAMETERS

4-PSK	Modulation
$M = 32$	Number of tones of FMT modulator
$N_U = 32$	Maximum number of users per frame
$\alpha_1 = 0.2$	Roll-off root-cosine pulse
$T_0 = 40T$	FMT sub-channel symbol period
$R_{TOT} = M/T_0 = 0.8/T$	Aggregate symbol rate (in symb/s)

Now, using (16) we can recompute the metrics (13)-(14) in an iterative fashion.

Finally, we point out that if we assume that the uplink users transmit their frames by taking as a reference the downlink frame timing, the time offsets are equal to the two way propagation delay. Thus, the base station has an approximate knowledge of where the uplink frames are. Consequently, the correlation metric (11) needs to be run over a confined time window.

IV. Detection and Channel Estimation

Once we have obtained the estimates $\widehat{\Delta_\tau^{(u)}}$ and $\widehat{\Delta_f^{(u)}}$, we can compensate the received signal, run a bank of sub-channel filters that are matched to the sub-channel transmit filters and sample the outputs at rate $1/T_0$. It should be noted that this filterbank can be implemented with an efficient polyphase implementation that is based on running low rate sub-channel filtering, and an M-point FFT. Even, if synchronization is ideal, due to the channel time dispersion, the sub-channel filter output exhibits some residual ISI as shown in (9). This can be counteracted with some form of equalization. Herein, we consider a simple minimum mean square error (MMSE) linear equalizer that operates over sub-channel samples at rate $1/T_0$. Further, to limit complexity we assume the equalizer to have a small number of taps N_{EQ} (up to 3 taps in our simulations). The equalizer taps are determined by minimizing the MSE between its output and the desired symbol. Training of the equalizer can be done using either the least mean square (LMS) or the recursive least square (RLS) algorithm, first over the known training sequence symbols $a_{TR}^{(u,k)}(lT_0)$, and then in a data decision directed mode. We have tested both the LMS and the RLS algorithms, and we have found (as it is well known) that the latter has a much superior convergence speed.

V. PERFORMANCE RESULTS

In this section we report BER performance for the system whose parameters are reported in Table I. The number of sub-channels is $M = 32$ and the prototype pulse is a truncated root-raised-cosine pulse with roll-off $\alpha_1 = 0.2$ and duration $29T_0$ which yields good sub-channel separation. The interpolation factor is $N = 40$ such that we obtain a frequency guard, assuming ideal perfectly confined pulses with bandwidth $(1+\alpha_1)/T_0$, equal to $0.05/T_0 = 0.04/(MT)$. If we assume a transmission bandwidth equal to 10 MHz the frequency guards equal 12.5 kHz. The channel is assumed to be Rayleigh faded with an exponential power delay profile with taps that have average power $\Omega_p \sim e^{-pT/(\gamma T_0)}$ with $\gamma = \{0.025,\ 0.1\}$. The profile is truncated at -20 dB. With a bandwidth of 10 MHz we have that $\gamma T_0 = \{0.1,\ 0.4\}\ \mu s$. The sub-channel training sequences are random 4-PSK sequences with length 15 symbols. We further choose K=3. The sub-channel MMSE equalizer has 1 or 3 taps and deploys practical training with the RLS algorithm over the known training sequences. The data portion also uses 4-PSK modulation.

In Fig. 3 we assume a single user that is allocated to all 32 sub-channels. We plot BER as a function of a fixed frequency offset assuming a time offset equal to $4T_0$. The curve labelled as uncompensated assumes perfect timing acquisition but the frequency offset is not compensated. The curve shows that the BER dramatically increases if we do not perform compensation of the frequency offset. The curves labelled as compensated assume practical estimation and compensation of both the time offset and the frequency offset with the algorithm of Section III. In this case the BER performance sensitivity to the offsets is low. In particular note that for large frequency offsets running a second iteration for the estimation of the parameters improves significantly the performance. In this case the BER curve remains flat for all frequency offsets that we have considered and it is practically identical to the BER that can be achieved with perfect synchronization. For the channel in the left plot of Fig. 3 one tap equalization is sufficient. However, in the right plot of Fig. 3 where we assume a larger delay spread, 3 taps equalization performs better than single tap equalization.

In Fig. 4-5 we assume a fully loaded system with 4 and 32 users that are allocated in an interleaved fashion respectively to 8 or a single sub-channel. The users are asynchronous with independent uniformly distributed time/frequency offsets respectively in $[0, 2T_0]$ and $[-\Delta_{f,\max}, \Delta_{f,\max}]$. They have equal power and independently faded channels. Now, looking at the left plot of Fig. 4-5 where the channel is mildly frequency selective the performance with 4 or 32 users is similar. For the 32 users case we do not see any significant gain by using two iterations in the synchronization stage. For the 4 users case there is some improvement for large frequency offsets.

Comparing the right plots of Fig. 4 and 5 where the channel is more frequency selective we see that actually the 4 users system performs worse (although not significantly) than the 32 users system. This is justified by the fact that when the channel is highly frequency selective, the optimal sampling phase may vary across the FMT sub-channels. Thus, while in the 32 users case timing is acquired independently across the sub-channels, in the 4 users case the 8 sub-channels of a given user have a

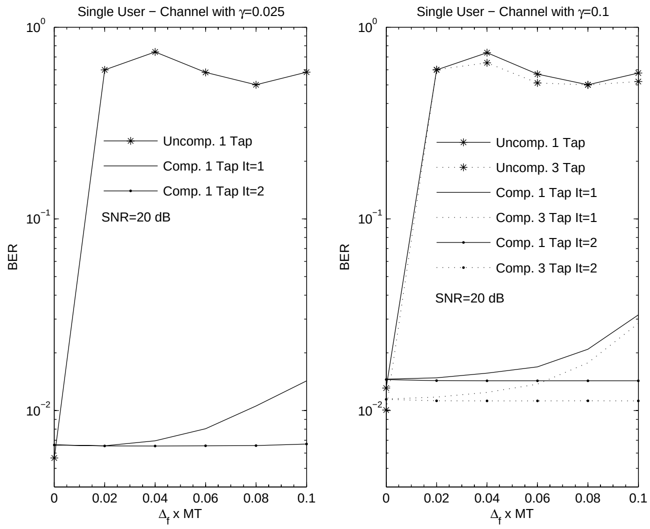

Fig. 3. BER as a function of constant frequency offset with one user. 1-3 tap RLS equalizer, and 1-2 iterations at the synchronization stage.

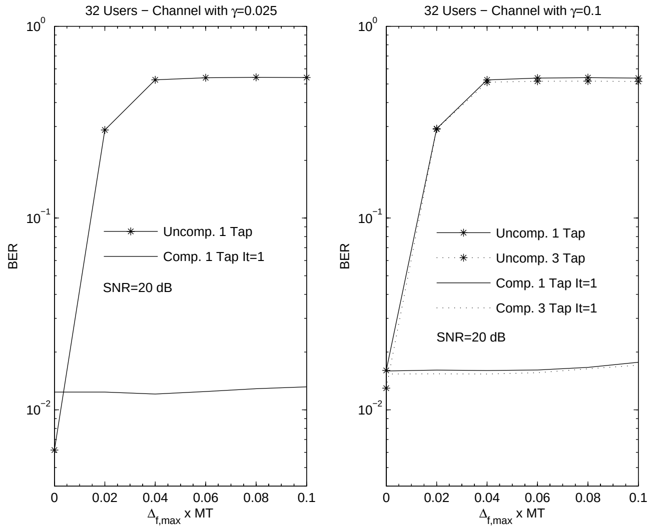

Fig. 4. Average BER with 32 asynchronous users as a function of the maximum frequency offset. 1-3 tap RLS equalizer.

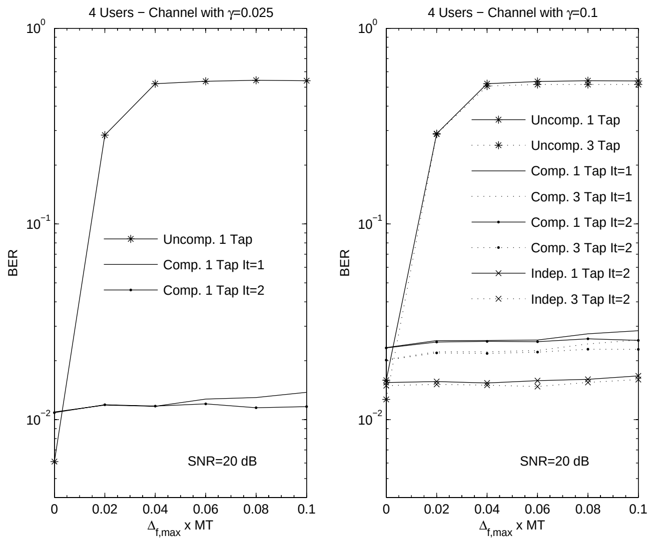

Fig. 5. Average BER with 4 asynchronous users as a function of the maximum frequency offset. 1-3 tap RLS equalizer, and 1-2 synchronization iterations.

common sampling phase that is used for all 8 samplers at the input of the T_0 spaced equalizers. This argument is confirmed by looking at the curves of Fig. 4 that are labeled as Indep. (independent) where estimation and compensation is done independently across the 8 sub-channel of a given user. We point out that in this case complexity increases because we need to run independent matched filters also during the detection stage.

VI. Conclusions

In this paper we have discussed a training based synchronization approach for multiuser FMT systems. It is based on an iterative correlation approach that exploits the separability of sub-channel signals that belong to different uplink users. We have considered simple MMSE adaptive equalization with an RLS algorithm and we have reported results as a function of the number of equalizer taps. The overall receiver performance has been shown to demonstrate the effectiveness of the algorithms and their robustness to a wide range of time/frequency offsets and channel frequency selectivity.

Appendix

The synchronization algorithm in Section III has been derived as a result of the following observations. First, if the carrier frequency offsets fulfil the relation $\Delta_f^{(u)} T_0 \sim 0$, i.e., they are much smaller than the sub-channel bandwidth, (10) can be written in correspondence of the training sequence as

$$\begin{aligned} z_{it=1}^{(u,k)}(lT_0+nT) &= e^{j2\pi\Delta_f^{(u)} lT_0} a_{TR}^{(u,k)}(lT_0) h_{EQ}^{(u,k)}(nT) \\ &\quad + I^{(u,k)}(lT_0;nT) + \eta^{(u,k)}(lT_0+nT) \end{aligned} \tag{17}$$

for $l = 0,...,N_{TR-1}$, $n \in \mathbb{Z}$, and for a certain sub-channel equivalent impulse response $h_{EQ}^{(u,k)}(nT)$. If we further assume the interference term that comprises ISI, ICI, and MAI components, to be small, the metric (11) has the peak in correspondence of the training sequence (assuming it to have good autocorrelation properties). That is, for $n = n_{\max}$, we obtain

$$P_{it}^{(u,k)}(n_{\max}) = e^{j2\pi\Delta_f^{(u)} KT_0} \left| h_{EQ}^{(u,k)}(n_{\max}T) \right|^2 (N_{TR} - K) + \hat{\eta}^{(u,k)}(n_{\max}T). \tag{18}$$

Thus, metrics (13)-(14) follow. The term K is chosen to minimize the variance of the estimator. In particular to take into account the presence of the ISI, it has to be, possibly, such that KT_0 is larger than the sub-channel time dispersion.

Clearly, the above approximations are improved when we possess an a priori knowledge of the carrier frequency offset. That is, when we have obtained a first estimate of the carrier frequency offset, we can re-run the filter bank that compensates for it as described by (16). In other words, we iteratively refine the knowledge of the carrier frequency offset, which allows to center at the appropriate frequency the receive filter bank of the desired user.

References

[1] G. Cherubini, E. Eleftheriou, S. Ölçer, "Filtered Multitone Modulation for Very High-Speed Digital Subscribe Lines", *IEEE JSAC*, pp. 1016-1028 , June 2002.

[2] A. Tonello, "Performance Limits for Filtered Multitone Modulation in Fading Channels", to appear in IEEE *Trans. on Wireless Comm.*

[3] A. Tonello, S. Pupolin, "Discrete Multi-Tone and Filtered Multi-Tone Architectures for Broadband Asynchronous Multi-User Communications", *Proc. of WPMC 01*, Aalborg, Denmark, pp. 461–466, September 2001.

[4] A. Tonello, "Asynchronous Multicarrier Multiple Access: Optimal and Sub-optimal Detection and Decoding", *Bell Labs Tech. Journal* vol. 7 n. 3, pp. 191-217, 2003.

[5] A. Tonello, "A Concatenated Multitone Multiple Antenna Air-Interface for the Asynchronous Multiple Access Channel", *subm. to IEEE JSAC*, Jan. 2005.

[6] S. Kaiser, W. Krzymien, "Performance Effects of the Uplink Asynchronism is a Spread Spectrum Multi-carrier Multiple Access System", *European Trans. on Telec*., pp. 399-406, July-August 1999.

[7] A. Tonello, N. Laurenti, and S. Pupolin, "Analysis of the Uplink of an Asynchronous DMT OFDMA System Impaired by Time Offsets, Frequency Offsets, and Multi-path Fading", *Proc. of IEEE VTC 00-Fall*, Boston, pp.1094–1099, September 2000.

[8] T. Schmidl, D. Cox, "Robust Frequency and Timing Synchronization for OFDM", *IEEE Trans. Comm*., pp. 1613-1621, December 1997.

[9] J. van de Beek, P.O. Borjesson, M. Boucheret, et al. "A time and Frequency Synchronization Scheme for Multiuser OFDM", *IEEE JSAC*, pp. 1900-1914, Nov. 1999.

[10] A. Assalini, A. Tonello, "Time-Frequency Synchronization in Filtered Multitone Modulation Based Systems", *Proc. of WPMC 03*, Yokosuka, Japan, pp. 221-225, October 2003.

[11] A. Tonello, F. Rossi, "Synchronization and Channel Estimation for Filtered Multitone Modulation", *Proc. of WPMC 2004*, Abano Terme, pp. 590-594, Sept. 2004.