Space-Time Bit-Interleaved Coded Modulation with an Iterative Decoding Strategy

Andrea M. Tonello

DEI - Department of Electronics and Informatics - University of Padova
Via Gradenigo 6/A - 35131 Padova - Italy – e-mail: tonello@dei.unipd.it

Abstract

A space-time coding approach for wireless communications deploying multiple transmit and multiple receive antennas is presented. The approach is based on the concatenation of a convolutional encoder, a bit interleaver, and a space-time signal constellation mapper that combines multi-level/phase modulation with multiple transmit antennas. The decoding strategy follows an iterative (turbo-like) algorithm, where soft information is exchanged between a soft-in soft-out demapper and a soft-in soft-out convolutional decoder. It is applicable with one or more receive antennas, and shows that spectral efficient/reliable communications are possible with few iterations. We address the performance analysis in both block and fast flat Rayleigh fading in order to provide insight into the space-time code construction criteria for the approach that we propose. Finally, simulation results are reported for schemes with 2 bits/s/Hz and 4 bits/s/Hz.

1 Introduction

The severe attenuation disturbances (i.e. fading) of wireless channels often mandate the use of diversity techniques whenever reliable communications have to be granted. The basic principle of diversity is to provide the receiver with replicas of the transmitted signal that experience less attenuation. Diversity can be exploited in time, in frequency, and in space [1]. Recently, schemes deploying multiple transmit and multiple receive antennas gained a lot of attention since it was shown that the capacity of a wireless link can be largely increased with such an architecture [2]. A systematic approach, known as space-time coding, was presented in [3]. It considers the design of coding, modulation, transmit and receive diversities in a unified fashion.

Most of the known space-time coding schemes deploy trellis codes [3] or block codes [4]. A different approach to space-time coding is proposed in this paper. It is based on the serial concatenation of a convolutional encoder, a bit interleaver, and a space-time signal constellation mapper. The information bits are encoded with a convolutional encoder, then are appropriately interleaved and split into several parallel streams. Each stream is mapped into signal constellation points using multi-level/phase modulators (e.g. M-PSK, M- QAM modulators), and transmitted via a transmit antenna. Each receive antenna captures a linear superposition of the transmitted signals corrupted by noise. The resulting scheme is a space-time extension (i.e. to multiple transmit antennas) of the bit-interleaved coded modulation concept [5-6]. We refer to it as space-time bit-interleaved coded modulation, STBICM.

Part of this work was supported by LUCENT TECHNOLOGIES - BELL LABS.
The author is on leave from *Lucent Technologies*, Whippany Laboratory - NJ - USA.

Decoding is addressed for a frequency non-selective (i.e. flat) fading channel, and is based on the 'turbo' concept. This strategy was originally applied to the decoding of turbo codes [7-8], and then to various decoding problems such as iterative demapping/decoding of multi-level modulation [9-10], and iterative detection/equalization of coded M-DPSK signals [11]. The decoding problem of STBICM in frequency selective fading channels is addressed in [12]. In this paper we show that STBICM can be reliably decoded by using a feedback receiver structure where soft information is exchanged between a soft-in soft-out demapper, and a soft-in soft-out convolutional decoder. The decoding algorithm can be applied with one or more receive antennas, and yields significant performance gains by performing some iterations.

In the proposed space-time coded system, variable spectral efficiencies can be easily obtained by appropriate choice of the convolutional code rate, the number of transmit antennas, and the signal constellation mapper. However, in order to exploit both the spatial and the temporal diversities, the joint design of the code, the interleaver and of the space-time constellation mapper has great importance. We address the performance analysis of STBICM in both block and fast Rayleigh fading scenarios, and devise code construction criteria. Differently from [3], the construction of STBICM with iterative decoding aims to maximize the distance properties (Hamming and Euclidean) at a bit level rather than at a symbol level. The appropriate design of the convolutional code, the interleaver, and the bit-to-symbol mapping rule allows optimizing the coding gain, and fully exploiting the spatial and temporal diversities.

Several STBICM schemes with a spectral efficiency of 2 bits/s/Hz and 4 bits/s/Hz are described, and performance results from simulations are reported.

The paper is organized as follows. Section 2 introduces the STBICM concept. Section 3 describes the channel model. Section 4 addresses the decoding algorithm in flat fading. Performance analysis and code construction criteria are discussed in section 5. In section 6, simulation results for several schemes that achieve full diversity with low complexity are shown. Finally, the conclusions follow.

0-7803-6507-0/00/$10.00 ©2000 IEEE

2 Transmission Model and STBICM Concept

We consider a wireless communication system comprising $N_t \geq 2$ transmit antennas and $N_r \geq 1$ receive antennas. At the transmitter (figure 1) the information bit stream b_i is first convolutionally encoded and then bit-interleaved to produce the bit stream d_i. The interleaved bit stream is S/P converted into N_t streams d_i^t ($t=1,...,N_t$, $i=-\infty,...,\infty$). Each parallel stream is mapped (modulated) into complex constellation points x_k^t ($t=1,...,N_t$, $k=-\infty,...,\infty$) belonging to a multi-phase/level signal set (i.e. M-PSK or M-QAM signal sets). Each antenna simultaneously transmits the modulated symbols. Ater S/P conversion the sub-sequence of bits that at time kT (with T symbol period) is mapped into N_t channel symbols can be written as $\underline{d}_k = [d_k^{1,1} ... d_k^{1,N} ... d_k^{N_t,1} ... d_k^{N_t,N}]$, where $N=log_2M$, and M is the modulator order. Furthermore, mapping is in general defined by a rule such that the vector of transmitted symbols at time kT is $\underline{x}_k = [x_k^1 ... x_k^{N_t}]^T = \mu(\underline{d}_k)$.

The purpose of the bit interleaver is twofold. First, it is used to de-correlate the fading channel and maximize the diversity order of the system. Second, it removes the correlation in the sequence of convolutionally coded bits, and this is an essential condition for the iterative decoding algorithm that we propose in section 4. We emphasize that no orthogonality constraint is imposed on the antenna constellations, and that with ideal interleaving independent bits are mapped into antenna constellation points.

The spectral efficiency of the STBICM resulting scheme is $R=R_cN_tlog_2M$ bits/s/Hz, with R_c convolutional encoder rate. Different spectral efficiencies can be easily obtained by appropriate choice of the convolutional encoder rate, of the modulation order, and of the number of transmit antennas. Criteria for the construction of "good" space-time bit-interleaved codes are discussed in section 5. In particular we show that the performance depends on certain Hamming and Euclidean distance properties of the convolutional code and the bits-to-symbols mapping rule.

3 Channel Model

One or more receive antennas capture the signals transmitted by the N_t antennas. Assuming a flat fading channel, the sequence of T-spaced samples at the r-th antenna matched filter output can be written as

$$y_k^r = \sqrt{E_s} \sum_{t=1}^{N_t} h_k^{r,t} x_k^t + n_k^r . \qquad (1)$$

In (1) $h_k^{r,t}$ is the equivalent channel impulse response of the link between the t-th transmit antenna and the r-th receive antenna, at time kT; n_k^r is a sequence of i.i.d. complex Gaussian variables with zero mean and variance $N_0/2$ per dimension. We further make the following

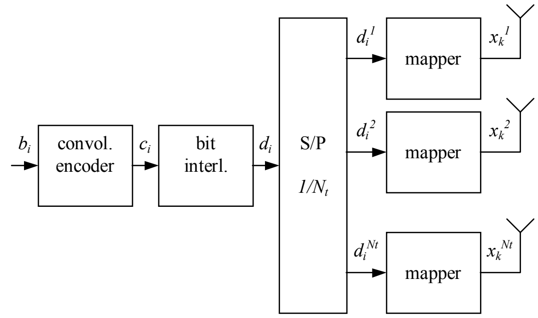

Figure 1: STBICM base band transmitter.

assumptions. The channel taps are complex Gaussian distributed with zero mean (Rayleigh fading). The channel impulse responses of different antenna links are independent. Both fast fading (i.e. temporal uncorrelated fading coefficients) and block fading (i.e. static fading coefficients over a block of transmitted symbols, but independent over blocks) are considered. The symbol constellation and the channel profile are normalized: $E[|x_k^t|^2]=1$, $t=1,...,N_t$, and $E[|h_k^{r,t}|^2]=1$, $r=1,...,N_r$, $t=1,...,N_t$. Thus, the average signal-energy-to-noise-ratio can be defined as

$$SNR = \frac{E_s}{N_0} \sum_{r=1}^{N_r} \sum_{t=1}^{N_t} E[|x_k^t|^2]\, E[|h_k^{r,t}|^2] = \frac{E_s N_t N_r}{N_0} . \qquad (2)$$

Finally, in vector notation (1) can be written as

$$\underline{y}_k = \begin{bmatrix} y_k^1 \\ ... \\ y_k^{N_r} \end{bmatrix} = \begin{bmatrix} h_k^{1,1} & ... & h_k^{1,N_t} \\ ... & ... & ... \\ h_k^{N_r,1} & ... & h_k^{N_r,N_t} \end{bmatrix} \begin{bmatrix} x_k^1 \\ ... \\ x_k^{N_t} \end{bmatrix} + \begin{bmatrix} n_k^1 \\ ... \\ n_k^{N_r} \end{bmatrix} . \qquad (3)$$

Thus, $\underline{y}_k = \underline{H}_k \underline{x}_k + \underline{n}_k$.

4 Iterative Decoding of STBICM in Flat Fading

Decoding of STBICM is addressed here for a flat fading environment. The strategy is based on two individually optimal steps that can be iteratively repeated (figure 2).

In the first step (referred to as demapping) from the channel samples we compute the a posteriori log-likelihood ratios of the coded and interleaved bits,

$$L(d_i^t) = \log[P(d_i^t = +1) / P(d_i^t = -1)] , \qquad (4)$$

for each transmit branch $t=1,...,N_t$. Since in general we have samples from an array of receive antennas, and we want to compute soft values for the bits that are transmitted on overlapping channels, this module can be interpreted as a multiple-soft-in multiple-soft-out a posteriori probabilities calculator (MIMO-APP). In the second step (referred to as decoding) the a posteriori log-likelihood ratios of the coded bits are P/S converted, deinterleaved, and fed to a soft-in soft-out convolutional decoder that is implemented according to the maximum a posteriori (MAP) algorithm [8]. The convolutional decoder provides both the log-likelihood ratios of the information bits $L(b_i)$, and new/improved log-likelihood ratios of the coded bits $L(c_i)$. Following the turbo decoding principle [7-8] extrinsic log-likelihood ratios of the coded bits are computed by subtracting the decoder inputs from the decoder outputs, $L_e(c_i)=L(c_i)-L_a(c_i)$. This is to minimize the correlation with previously computed soft values. The extrinsic values are interleaved, S/P converted, and fed back to the demapper where they are used in a new iteration as an estimate of the a priori log-likelihood ratios of the coded bits on each transmit branch, $L_a(d_i^t)$. Extrinsic information is also computed at the demapper output, $L_e(d_i^t)=L(d_i^t)-L_a(d_i^t)$. By repeating several times the above procedure, the performance of the system is greatly improved. In the final iteration the decoded sequence of information bits is obtained by making hard decisions on $L(b_i)$.

To proceed we need some more notation. Let $d_k^{t,m}$ be the bit that at time instant k is mapped into the m-th bit position of the constellation symbol of transmit antenna t ($m=1,\ldots,N$, $k=1,\ldots,N_s$, $t=1,\ldots,N_t$). If we fix bit $\hat{d}_k^{t,m}$, the set of all possible symbol vectors having bit $\hat{d}_k^{t,m}=b$, $b=\pm 1$, is $\mathrm{X}(\hat{d}_k^{t,m}=b)=\{\underline{x}_k=\mu(\underline{d}_k \,|\, \hat{d}_k^{t,m}=b)\}$. The cardinality of such a set is $2^{N_t N-1}$. With this notation, the log-likelihood ratios in (4), i.e. $L(\hat{d}_k^{t,m})$, conditioned on the channel state information are computed as

$$\ln\frac{P(\hat{d}_k^{t,m}=+1\,|\,\underline{y}_k,\underline{H}_k)}{P(\hat{d}_k^{t,m}=-1\,|\,\underline{y}_k,\underline{H}_k)}=\ln\frac{\sum_{\underline{\tilde{x}}_k\in \mathrm{X}(\hat{d}_k^{t,m}=+1)} p(\underline{\tilde{x}}_k,\underline{y}_k,\underline{H}_k)}{\sum_{\underline{\tilde{x}}_k\in \mathrm{X}(\hat{d}_k^{t,m}=-1)} p(\underline{\tilde{x}}_k,\underline{y}_k,\underline{H}_k)}. \quad (5)$$

The product of the conditioned channel probability density function, and the a priori probability of the symbol vector yields the joint probabilities in (5). Thus, under the AWGN assumption

$$p(\underline{\tilde{x}}_k,\underline{y}_k,\underline{H}_k)=Ae^{-\frac{1}{N_0}\sum_{r=1}^{N_r}\left|y_k^r-\sqrt{E_s}\sum_{l=1}^{N_t}h_k^{r,l}\tilde{x}_k^l\right|^2+\frac{1}{2}\sum_{l=1}^{N_t}\sum_{i=1}^{N}\tilde{d}_k^{l,i}L_a(d_k^{l,i})}, \quad (6)$$

where A is a constant. In (6) $\tilde{x}_k^l$ and $\tilde{d}_k^{l,i}$ are respectively the elements of the symbol vector $\underline{\tilde{x}}_k$, and the bits that are mapped to such a vector. Furthermore, the a priori log-likelihoods ratios $L_a(d_k^{l,i})$ are summed in (6) due to the assumption of independence among the interleaved bits.

At the first pass through the demapper no a priori information on the coded bits is assumed, thus it is set to zero. In the following iterations, the a priori log-likelihood ratios of the bits of each transmit antenna branch are approximated from the decoder outputs. This a priori knowledge helps improve the metric quality, and de-couple

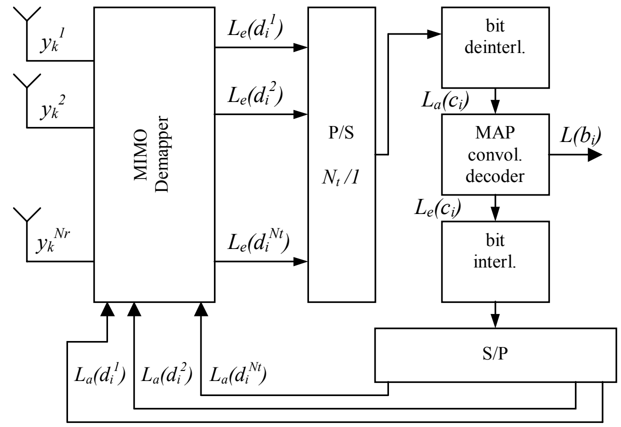

Figure 2: Iterative STBICM base band receiver.

the signals that are simultaneously transmitted. In the single transmit - single receive antenna case, (5) simplifies to the computation of the log-likelihoods as shown in [9-10].

Finally, from a complexity standpoint (5) can be simplified by the known max-Log approximation, i.e. the log of the sum of exponentials is approximated by the largest exponent in the exponentials.

5 Performance Analysis for STBICM Construction

Consider a block of transmitted coded symbols of length $N_t\,N_s$. Since at each time instant there are N_t simultaneous transmissions, we can format such a sequence into the matrix $\underline{X}=[\underline{x}_1\ldots\underline{x}_{N_s}]$ of size N_t by N_s. Let instead $\hat{\underline{X}}$ be the sequence of symbols the receiver decides in favor of.

An upper bound on both the average bit error probability and frame (block) error rate probability can be computed from a weighted sum of the pairwise error probability $P(\underline{X}\rightarrow\hat{\underline{X}})$, i.e. the probability of deciding in favor of $\hat{\underline{X}}$ when $\underline{X}$ is transmitted [1], [5]. The pairwise error probability (PEP) depends upon the decoding strategy. Iterative decoding consists, basically, of a first step where the bits are demapped using feedback information from the MAP convolutional decoder [8] that is deployed at the second step. Convolutional decoding yields the maximum-likelihood solution, since no a priori information on the information bits is known.

If $\underline{c}$ and $\hat{\underline{c}}$ are the coded bit sequences associated respectively to the symbol sequences $\underline{X}$ and $\hat{\underline{X}}$, then $P(\underline{c}\rightarrow\hat{\underline{c}})=P(\underline{X}\rightarrow\hat{\underline{X}})$. Let the block of coded bits be generated by a rate $R_c=1/n$ convolutional encoder. The coded block has length $N_c=nN_b=N_tN_sN$ bits, with N_b input information bits. The decoder decides in favor of the codeword $\hat{\underline{c}}$ that maximizes the accumulated path metric [8]:

$$\hat{\underline{c}}=\arg\max_{\underline{\tilde{c}}}\{\sum_{i=1}^{N_b}\sum_{l=1}^{n}\tilde{c}_i^l L_a(c_i^l)\}=\arg\max_{\underline{\tilde{c}}}\{\sum_{i=1}^{N_b}\sum_{l=1}^{n}\pi_a(\tilde{c}_i^l)\}. \quad (7)$$

The path metric can be equivalently expressed using either the log-likelihood ratios $L_a(c_i^l)$, or the log-likelihoods $\pi_a(\tilde{c}_i^l)=\ln P(c_i^l=\tilde{c}_i^l)$. Since there is a one to one correspondence between the sequences of coded bits $\underline{\hat{c}}$ and the sequence of coded and interleaved bits $\underline{\hat{d}}$, then the decoder decides in favor of $\underline{\hat{c}}$ given that the sequence $\underline{c}$ was indeed transmitted if the following is verified

$$\sum_{k=1}^{N_s}\sum_{t=1}^{N_t}\sum_{m=1}^{N}\pi_e(\hat{d}_k^{t,m})-\sum_{k=1}^{N_s}\sum_{t=1}^{N_t}\sum_{m=1}^{N}\pi_e(d_k^{t,m})\geq 0\,. \quad (8)$$

where $\pi_e(\hat{d}_k^{t,m})$ are the extrinsic log-likelihoods provided by the demapper. At a given decoding iteration, these are computed from the channel samples and the a priori log-likelihoods provided by the decoder at the previous iteration. Thus, (besides a constant factor)

$$\pi_e(\hat{d}_k^{t,m}=\pm 1)=\ln\sum_{\underline{\tilde{x}}_k\in X(\hat{d}_k^{t,m}=\pm 1)} e^{-\frac{1}{N_0}\sum_{r=1}^{N_r}|y_k^r-\sum_{l=1}^{N_t}\sqrt{E_s}h_k^{r,l}\tilde{x}_k^l|^2+\sum_{l=1}^{N_t}\sum_{\substack{i=1\\(l,i)\neq(t,m)}}^{N}\ln P_a(\tilde{d}_k^{l,i})} \quad (9)$$

Equation (9) shows that the extrinsic information on a given bit is evaluated from the knowledge of the a priori probability, $P_a(\tilde{d}_k^{l,i})=e^{\tilde{d}_k^{l,i}L_a(d_k^{l,i})}/(1+e^{\tilde{d}_k^{l,i}L_a(d_k^{l,i})})$ [8], of all the other bits that are transmitted at the same time.

To proceed and gain insight, we assume to deploy a genie demapper that has perfect extrinsic knowledge. In other words the feedback from the decoder is assumed exact. Then (9) simplifies to

$$\pi_e(\hat{d}_k^{t,m})=-\frac{1}{N_0}\sum_{r=1}^{N_r}|n_k^r+\sqrt{E_s}h_k^{r,t}(x_k^t-\hat{x}_k^t)|^2\,. \quad (10)$$

With this assumption demapping corresponds to computation of the log-likelihoods for a 2-ary constellation whose two symbols belong to a subset of all possible symbols of antenna t. They differ only on bit $\hat{d}_k^{t,m}$, and are given by $\hat{x}_k^t=[\mu(d_k^{1,1},...,\hat{d}_k^{t,m}=\pm 1,...,d_k^{N_t,N})]_t$.

From (8) and (10) the PEP now becomes (11)

$$P\left(\sum_{r=1}^{N_r}\sum_{k=1}^{N_s}\sum_{t=1}^{N_t}\sum_{m=1}^{N}[|n_k^r+\sqrt{E_s}h_k^{r,t,m}(\mu(d_k^{t,m})-\mu(\hat{d}_k^{t,m}))|^2-|n_k^r|^2]\geq 0\right)$$

where we have assumed, in general, that each bit is transmitted through an independent fading channel due to the bit interleavers and to independent antenna mapping. For easy of notation $\mu(\hat{d}_k^{t,m})=[\mu(\underline{d}_k\,|\,\hat{d}_k^{t,m})]_t$. A Chernoff bound [1] on the pairwise error probability conditioned on the channel state information can be computed, yielding

$$P(\underline{c}\rightarrow\underline{\hat{c}}\,|\,\underline{H})\leq e^{-\frac{E_s}{4N_0}\sum_{r=1}^{N_r}\sum_{k=1}^{N_s}\sum_{t=1}^{N_t}\sum_{m=1}^{N}|h_k^{r,t,m}|^2|\mu(d_k^{t,m})-\mu(\hat{d}_k^{t,m})|^2}\,. \quad (12)$$

Finally, to obtain the average pairwise error probability, we need to average (12) over the CSI statistics.

5.1 Average PEP in Fast Rayleigh Fading

Consider $h_k^{r,t,m}$ to be a sequence (over indices k, r, t, m) of i.i.d. zero mean, power one, complex Gaussian variables. The pdf of their amplitude $a_k^{r,t,m}=|h_k^{r,t,m}|$ is Rayleigh distributed with pdf $p(a)=2ae^{-a^2}$, $a\geq 0$. Averaging (12) then yields

$$P(\underline{X}\rightarrow\underline{\hat{X}})\leq\prod_{r=1}^{N_r}\prod_{k=1}^{N_s}\prod_{t=1}^{N_t}\prod_{m=1}^{N}\frac{1}{1+\frac{E_s}{4N_0}|\mu(d_k^{t,m})-\mu(\hat{d}_k^{t,m})|^2}\,. \quad (13)$$

5.2 Average PEP in Block Rayleigh Fading

If we assume constant fading over the entire block of N_s symbols, i.e. $|h_k^{r,t,m}|=|h^{r,t}|$ for $k=1,...,N_s$, $m=1,...,N$, then averaging (12) yields

$$P(\underline{X}\rightarrow\underline{\hat{X}})\leq\prod_{r=1}^{N_r}\prod_{t=1}^{N_t}\frac{1}{1+\frac{E_s}{4N_0}\sum_{k=1}^{N_s}\sum_{m=1}^{N}|\mu(d_k^{t,m})-\mu(\hat{d}_k^{t,m})|^2}\,. \quad (14)$$

5.3 Code Construction Criteria in Fast Rayleigh Fading

Let $d_H^{k,t,m}$ be the *Hamming distance* between bits $d_k^{t,m}$ and $\hat{d}_k^{t,m}$ of the interleaved error event sequences. Further, let $d_E^{k,t,m}=|\mu(d_k^{t,m})-\mu(\hat{d}_k^{t,m})|^2$ be the associated *squared Euclidean distance*. Such a distance verifies

$$d_E^{k,t,m}\geq d_H^{k,t,m}d_{E,\min}^{t,m}\,, \quad (15)$$

where $d_{E,\min}^{t,m}$ is the *minimum squared Euclidean interdistance* among all possible symbol pairs that differ only on bit $\hat{d}_k^{t,m}$. Finally, let $d_H^{t,m}(\underline{c},\underline{\hat{c}})=\sum_{k=1}^{N_s}d_H^{k,t,m}$ be the *bit-branch Hamming distance* between the coded sequences $\underline{c}$ and $\underline{\hat{c}}$ computed over the sub-portions that are transmitted over antenna branch t, and mapped to bit position m in the symbols. Then, substituting (15) in (13)

$$P(\underline{X}\rightarrow\underline{\hat{X}})\leq\prod_{t=1}^{N_t}\prod_{m=1}^{N}(1+\frac{E_s}{4N_0}d_{E,\min}^{t,m})^{-N_r d_H^{t,m}(\underline{c},\underline{\hat{c}})}\,. \quad (16)$$

If $d_H(\underline{c},\underline{\hat{c}})$ is the *total Hamming distance* between $\underline{c}$ and $\underline{\hat{c}}$, then for a sufficiently high SNR (16) simplifies to

$$P(\underline{X}\rightarrow\underline{\hat{X}})\leq(\frac{E_s}{4N_0})^{-N_r d_H(\underline{c},\underline{\hat{c}})}\prod_{t=1}^{N_t}\prod_{m=1}^{N}(d_{E,\min}^{t,m})^{-N_r d_H^{t,m}(\underline{c},\underline{\hat{c}})}\,, \quad (17)$$

showing that the PEP exhibits a diversity advantage equal to $G_d=N_r d_H(\underline{c},\underline{\hat{c}})$, and a coding gain equal to $G_c=D^{-N_r}$ with $D=\prod_{t=1}^{N_t}\prod_{m=1}^{N}(d_{E,\min}^{t,m})^{d_H^{t,m}(\underline{c},\underline{\hat{c}})}$. Thus, in fast

Rayleigh fading under the exact feedback assumption the following code construction criteria are devised

- To maximize the diversity gain the Hamming distance $d_H(\underline{c},\hat{\underline{c}})$ has to be maximized for any pair of coded bit sequences.
- To maximize the coding gain the product distance D has to be maximized for any pair of coded sequences. This implies the joint maximization of the bit-branch Hamming distances and the choice of an appropriate bit-to-symbol mapping rule with high minimum squared Euclidean interdistance.

For instance, with Gray mapping and 4-PSK modulation the equality holds in (15), and (17) simplifies to

$$P(\underline{X}\to\hat{\underline{X}})\le\left(\frac{E_s}{2N_0}\right)^{-N_r d_H(\underline{c},\hat{\underline{c}})}. \tag{18}$$

In this case a standard maximum *free Hamming distance* convolutional code maximizes the diversity performance.

5.4 Code Construction Criteria in Block Rayleigh Fading

Let $d_E^t(\underline{c},\hat{\underline{c}})=\sum_{k=1}^{N_s}\sum_{m=1}^{N}|\mu(d_k^{t,m})-\mu(\hat{d}_k^{t,m})|^2$ be the sum of the *squared Euclidean distances* computed on the sub-sequences transmitted over antenna *t*. It verifies

$$d_E^t(\underline{c},\hat{\underline{c}})\ge\sum_{m=1}^{N}d_H^{t,m}(\underline{c},\hat{\underline{c}})d_{E,\min}^{t,m}. \tag{19}$$

Thus, the pairwise error probability is upper bounded by

$$P(\underline{X}\to\hat{\underline{X}})\le\prod_{r=1}^{N_r}\prod_{t=1}^{N_t}\frac{1}{1+\frac{E_s}{4N_0}\sum_{m=1}^{N}d_H^{t,m}(\underline{c},\hat{\underline{c}})d_{E,\min}^{t,m}}. \tag{20}$$

and for high SNRs

$$P(\underline{X}\to\hat{\underline{X}})\le\left(\prod_{t=1}^{N_t}\frac{E_s}{4N_0}\sum_{m=1}^{N}d_H^{t,m}(\underline{c},\hat{\underline{c}})d_{E,\min}^{t,m}\right)^{-N_r}. \tag{21}$$

Thus in block Rayleigh fading under the exact feedback assumption the following applies

- A diversity order of at most N_tN_r is achieved. This is obtained when for any pair of coded bit sequences the Hamming distance computed over their antenna sub-portions differs from zero for all antennas.
- To maximize the coding gain the product distance $D=\prod_{t=1}^{N_t}\sum_{m=1}^{N}E_s/4N_0 d_H^{t,m}(\underline{c},\hat{\underline{c}})d_{E,\min}^{t,m}$ has to be maximized for any pair of coded sequences. An appropriate mapping increases such a product distance.

For instance, with Gray mapping and 4-PSK modulation (21) simplifies to

$$P(\underline{X}\to\hat{\underline{X}})\le\left(\prod_{t=1}^{N_t}\frac{E_s}{2N_0}d_H^t(\underline{c},\hat{\underline{c}})\right)^{-N_r}, \tag{22}$$

where $d_H^t(\underline{c},\hat{\underline{c}})$ is the Hamming distance computed on the sub-sequences that are transmitted over antenna *t*. In this case, convolutional codes with maximum *free Hamming distance per branch* are a good choice. These codes may not have maximum compound free distance.

5.5 Further Performance Considerations

The STBICM design criteria derived in the previous sub-sections rely on the assumption of exact feedback. We can think that the proposed soft feedback iterative decoding algorithm asymptotically mimics the exact feedback decoder. The error introduced by the practical decoder translates into some magnification of the PEP. Remarkably we found that the slope of the frame error rate curves (see next section) evaluated by simulations matches the diversity order predicted by the theory. We point out that although improved coding gains can be found with mappings different from Gray, demapping and feedback at the initial iterations can be more unreliable, affecting the overall performance. Finally, it should be noted that the block fading model of section 5.2 applies to a static fading environment where interleaving is limited within each block of coded data. Coded blocks are then sent sufficiently apart in time or frequency. Interleaving across distinct blocks is capable of decorrelating the channel to the limit represented by the fast fading scenario.

6 STBICM Schemes and Simulation Results

Using the criteria in section 5, several STBICM schemes have been designed for an optimal tradeoff between complexity and performance. Frame error rate from simulations is reported for both block and fast fading.

For reference, the transmit delay diversity scheme with 4-PSK modulation is also considered (referred to as TXDEL2). This scheme is basically a repetition code as pointed out in [3] with a spectral efficiency of 2 bits/s/Hz.

<u>*Scheme 1: 2 bits/s/Hz - 2 tx antennas - 1 and 2 rx antennas*</u>

Blocks of 260 bits are convolutionally encoded with a 4 states rate 1/2 code. Two bits are used for tail termination. The code polynomials in octal notation are (7,5). Bits from the first polynomial are sent to the first antenna, while bits from the second polynomial are sent to the second antenna. Two random interleavers of length 260 are used before Gray mapping to 4-PSK symbols on each antenna. No inter-frame interleaving is considered. Thus no channel decorrelation results in block fading. With this scheme the free Hamming distance of the code is 5. The free Hamming distance on antenna one is 3, while on antenna two is 2. The diversity order in block fading with one receive antenna is 2. We refer to this scheme as STBICM2. From figure 3 in block fading at FER=10^{-2}, STBICM2 with 1 rx antenna gains 7 dB from the first pass through the decoder (it=0) to the second (it=1). The overall gain with 4 passes (it=3) over TXDEL2 is 3 dB. When deploying 2 rx antennas there is a diminished return from iterations, although the gain over TXDEL2 is still 3 dB. In fast fading the overall gain over TXDEL2 is 12 dB with 1 rx antenna and 7 dB with 2 rx antennas.

Scheme 2: 2 bits/s/Hz - 4 tx antennas - 2 rx antennas

Similarly to scheme 1, however we use a rate 1/4 convolutional code with polynomials (7,5,7,5), i.e. one per antenna. The free Hamming distance is 10. The diversity order in block fading is 4 with one receive antenna.

From figure 4, STBICM2 with 8 iterations gains over TXDEL2 3.5 dB in block fading and 5.5 dB in fast fading. The improvements from 4 to 8 iterations are marginal.

Scheme 3: 4 bits/s/Hz - 4 tx antennas - 2 rx antennas

Blocks of 260 information bits are encoded as in scheme 1. However, the bits at the output of the encoder are parsed into four antenna sub-streams. 4-PSK modulation is still deployed. The diversity order in block fading is 3 with one receive antenna.

Figure 4, shows that it is possible to double the spectral efficiency and lose only 1.5 dB in block fading over TXDEL2 that still deploys 4 tx and 2 rx antennas. In fast fading it is actually possible to gain 1.5 dB over TXDEL2.

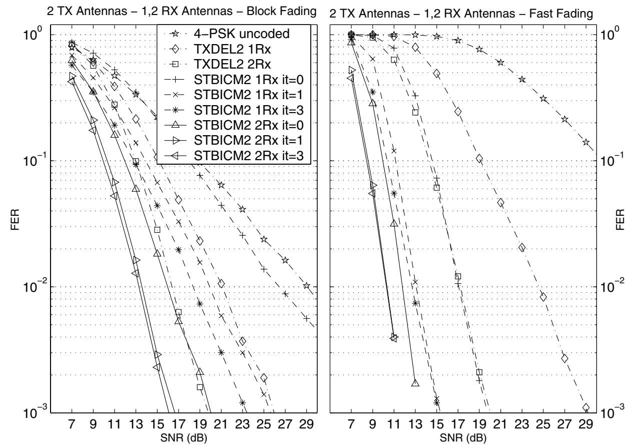

Figure 3: FER performance. Frames of length 260 bits. TXDEL2: transmit delay at 2 bits/s/Hz with 2 tx and 1, 2 rx antennas (*dash-dot*). STBICM2: scheme 1 at 2 bits/s/Hz with 2 tx-1rx (*dashed*) and 2 tx-2 rx (*solid*) antennas.

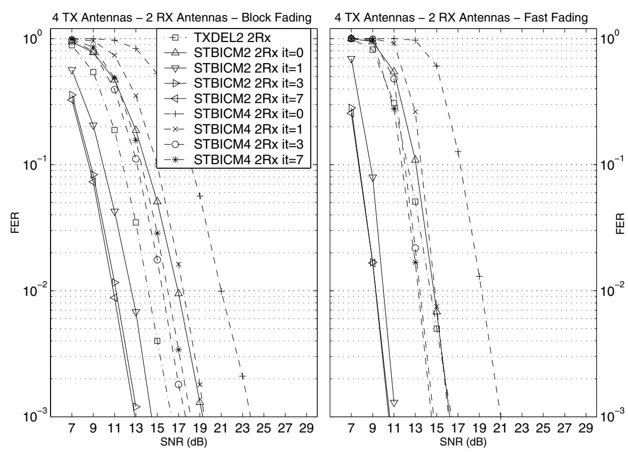

Figure 4: FER performance. Frames of length 260 bits. TXDEL2: transmit delay at 2 bits/s/Hz with 4 tx-2 rx antennas (*dash-dot*). STBICM2: scheme 2 at 2 bits/s/Hz with 4 tx-2 rx antennas (*solid*). STBICM4: scheme 3 at 4 bits/s/Hz with 4 tx-2 rx antennas (*dashed*).

Conclusions

This paper investigates a novel space-time coding approach based on space-time bit-interleaved convolutionally coded modulation with multiple transmit antennas. An iterative demapping/decoding strategy is proposed for flat fading channels with perfect knowledge of the channel state information. Performance analysis is attacked under the assumption of exact feedback from the decoder. As a result bounds on the pairwise error probability are found for both block and fast Rayleigh fading, and code construction criteria are devised. The diversity order and the coding gain depend on the Hamming distance of certain coded bit sub-sequences. Further, appropriate bit mappings can enlarge the product Euclidean distance of the code and yield higher coding gains.

Performance results from simulations of several STBICM schemes show that in terms of diversity gain there is agreement with theory, and that high coding gains are achieved over the transmit delay diversity scheme [3]. The gains are more pronounced in the fast fading scenario, and are achieved with few decoding iterations. Application of STBICM in frequency selective fading is studied in [12].

References

[1] J. G. Proakis, "Digital communications", NY: McGraw-Hill, 1995.

[2] G. Foschini, M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", *Wireless Personal Comm., 1998*, pp. 311-335.

[3] V. Tarokh, N. Seshadri, R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction", *IEEE Trans. Info. Theory , March 1998*, pp. 744-765.

[4] V. Tarokh, H. Jafarkhani, R. Calderbank, "Space-time block coding for wireless communications: performance results", *IEEE JSAC, March 1999*, pp. 451-460.

[5] E. Zehavi, "8-PSK trellis codes for a Rayleigh channel", *IEEE Trans. on Comm., May 1992,* pp. 873-884.

[6] G. Caire, G. Taricco, E. Biglieri, "Bit-interleaved coded modulation"*, IEEE Trans. Info. Theory, May 1998,* pp. 927-946.

[7] C. Berrou, A. Glavieux, P. Thitimajshima, "Near Shannon limit error correcting coding and decoding: turbo codes", *Proc. of IEEE ICC 1993*, pp. 1064-1070.

[8] J. Hagenauer, E. Offer, L. Papke, "Iterative decoding of binary block and convolutional codes", *IEEE Trans. Info. Theory, March 1996*, pp. 429-445.

[9] X. Li, J.A. Ritcey, 'Bit-interleaved coded modulation with iterative decoding using soft feedback', *IEE Elect. Letters, May 1998*, pp. 942-943.

[10] S. ten Brink, J. Speidel, R.H. Yan, "Iterative demapping and decoding for multilevel modulation", *Proc. of IEEE Globecom 98, Sydney, November 1998,* pp. 579-584 .

[11] A. Tonello, "Iterative MAP detection of coded M-DPSK signals in fading channels with application to IS-136 TDMA", *Proc. of IEEE VTC-'99-Fall, Boston, September 1999,* pp. 1615-1619.

[12] A. Tonello, "Space-time bit-interleaved coded modulation over frequency selective fading channels with iterative decoding", *to appear on Proc. of IEEE Globecom 2000, S. Francisco, Nov. 2000.*