

# Iterative Synchronization for Multiuser Filtered Multitone Systems

Andrea M. Tonello and Francesco Pecile

DIEGM - Università di Udine - Via delle Scienze 208 - 33100 Udine - Italy  
 phone: +39 0432 558 288 - fax: +39 0432 558 251 - e-mail: tonello@uniud.it

**Abstract**—We address the synchronization problem in a filtered multitone (FMT) modulated system. An FMT based system differs from the popular OFDM scheme in the deployment of sub-channel shaping filters. We assume a multiuser uplink scenario where sub-channels are partitioned among the active users. Users are asynchronous such that the received signals experience independent time offsets, carrier frequency offsets, and multipath fading. We propose to recover symbol/frame timing and carrier frequency for each active user by exploiting the sub-channel separability with an iterative procedure that uses training sequences. Two synchronization metrics are described. Detection uses simple linear sub-channel equalizers with RLS training.

**Keywords**—Filtered multitone modulation, Multiuser systems, OFDM, Synchronization, Wireless uplink.

## I. INTRODUCTION

In this paper we consider the synchronization, i.e., acquisition of the carrier frequency and the frame/symbol timing, in a filtered multitone (FMT) modulated system. Although the synchronization problem in OFDM is well understood [1], in an FMT system it presents several challenges. Orthogonal frequency division multiplexing (OFDM) is probably among the most popular multicarrier modulation techniques. It is essentially based on multicarrier transmission with sub-channel pulses that have a rectangular impulse response. More general multicarrier schemes deploy sub-channel filters with time-frequency concentrated response. Under certain conditions they can be implemented by using an inverse fast Fourier transform (IFFT) followed by low-rate sub-channel filtering. These schemes are referred to as filtered multitone modulated systems (FMT) [2].

Synchronization algorithms for a single user FMT system where presented in [3]-[4]. In [3] a blind scheme has been considered. In [4] both time domain and frequency domain algorithms with training have been investigated. In this paper we consider the multiuser uplink scenario. Multiuser architectures are possible both with OFDM and FMT. They are simply obtained by partitioning the available tones (sub-channels) among the active users [5]. In the uplink, multiuser OFDM is severely affected by time misalignments and carrier frequency offsets that can be large in a cellular, high mobility environment. This is due to the fact that in conventional OFDM, sub-channels exhibit *sinc* like frequency response, therefore their orthogonality can be easily lost in the absence of precise synchronization [6]-[7]. In an asynchronous multiuser environment, increased robustness and better performance can be obtained with filtered multitone (FMT) modulation

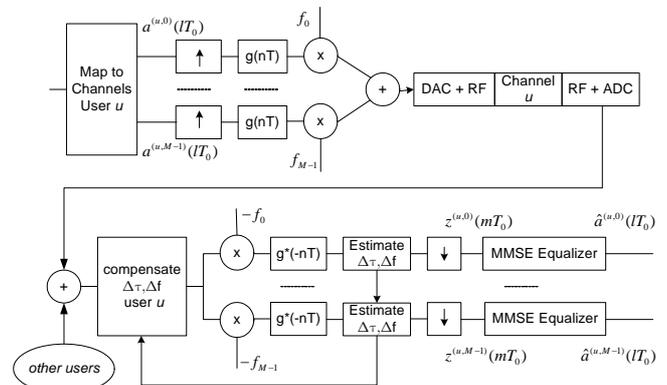


Fig. 1. Multiuser FMT system model with transmitter and receiver of user  $u$ .

architectures where the sub-channels are shaped with appropriate time-frequency concentrated pulses.

In our multiuser FMT system detection is single user based. It works by running a filter bank that is matched to the sub-channels that are assigned to the desired user after compensation of the time and frequency offsets. We propose an iterative synchronization approach that exploits the sub-channel separability to estimate the users' time/frequency offset. We consider two synchronization metrics one of which has been already presented in [8]. It should be noted that to allow for an efficient implementation of the receiver, during the detection stage, accurate time/frequency offset compensation has to be done up-front, i.e., before running the receiver filter bank. This is because the efficient receiver implementation that is based on low-rate sub-channel matched filtering followed by a fast Fourier transform (FFT) [2], requires to compensate the time/frequency offset deploying a common estimate for all the sub-channels that are assigned to the user under consideration.

## II. MULTIUSER FMT SYSTEM MODEL

The complex baseband transmitted signal  $x^{(u)}(nT)$  of user  $u$  is obtained by a filter bank modulator (Fig. 1) with prototype pulse  $g(nT)$ , e.g., a root-raised-cosine pulse, and sub-channel carrier frequency  $f_k = k/(MT)$ ,  $k = 0, \dots, M-1$ , with  $T$  being the transmission period

$$x^{(u)}(nT) = \sum_{k=0}^{M-1} \sum_{l \in \mathbb{Z}} a^{(u,k)}(lT_0) g(nT - lT_0) e^{j2\pi f_k nT} \quad (1)$$

where  $a^{(u,k)}(lT_0)$  is the  $k$ -th sub-channel data stream of user  $u$  that we assume to belong to the M-PSK/M-QAM constellation

set and that has rate  $1/T_0$  with  $T_0 = NT \geq MT$ . The interpolation factor  $N$  is chosen to increase the frequency separation between sub-channels, thus to minimize the amount of inter-carrier interference (ICI) and multiple access interference (MAI) at the receiver side. Distinct FMT sub-channels can be assigned to distinct users. In this case, the symbols are set to zero for the unassigned FMT sub-channels:

$$a^{(u,k)}(IT_0) = 0 \quad \text{for } k \notin K_u \quad (2)$$

where  $K_u$  denotes the set of  $M_u$  sub-channel indices that are assigned to user  $u$ . At the receiver, after RF demodulation, and analog-to-digital conversion, the discrete time received signal can be written as

$$y(\tau_i) = \sum_{u=1}^{N_U} \sum_{n \in \mathbb{Z}} x^{(u)}(nT) g_{CH}^{(u)}(\tau_i - nT - \Delta_\tau^{(u)}) e^{j(2\pi \Delta_f^{(u)} \tau_i + \phi^{(u)})} + \eta(\tau_i) \quad (3)$$

where  $\tau_i = iT + \Delta_0$ ,  $i \in \mathbb{Z}$  and  $\Delta_0$  is a sampling phase.  $N_U$  is the number of users,  $\Delta_\tau^{(u)}$  is the time offset of user  $u$ ,  $\Delta_f^{(u)}$  and  $\phi^{(u)}$  are the carrier frequency and phase offset,  $g_{CH}^{(u)}(t)$  is the fading channel impulse response of user  $u$ , and  $\eta(\tau_i)$  is the additive white Gaussian noise with zero mean contribution. Note that we assume the time/frequency offsets to be identical for all sub-channels that are assigned to a given user.

Here we consider a single user based FMT receiver where we first acquire time/frequency synchronization with each active user [8]. Then, for each user, we compensate the time/frequency offsets, we run FMT demodulation via a bank of filters that is matched to the transmitter bank, and we sample the outputs at rate  $1/T_0$ . Note that once compensation of the offsets is performed the FMT front-end can be efficiently implemented as described in [2], i.e., polyphase filtering and FFT. The acquisition of the time/frequency offsets is performed as described in the next section.

### III. ITERATIVE SYNCHRONIZATION

In the synchronization stage we estimate the time/frequency offsets of each user (Fig.1). This is accomplished with a training based approach, i.e., each user transmits a frame that comprises a known training data portion  $a_{TR}^{(u,k)}(IT_0)$ ,  $k \in K_u$ ,  $l = 0, \dots, N_{TR} - 1$ . The training sequences, one per sub-channel, are assumed to be random and to have good autocorrelation. Then, the estimation of the time/frequency offsets is done for each user with an iterative procedure where we first filter the received signal with a bank of filters that is matched to the transmit filter bank. Second, the time offset and the frequency offset of the user are estimated. Third, we re-run the filter bank by now pre-compensating with the estimated time/frequency offsets. The procedure can be repeated iteratively.

Therefore, at the first iteration (when we do not have any a priori knowledge of the time/frequency offset of the desired user) we run the following filter bank for the channels of user  $u$

$$z_{it=1}^{(u,k)}(nT) = \sum_{i \in \mathbb{Z}} y(iT) g^*(iT - nT) e^{-j2\pi f_k iT} \quad (4)$$

The outputs are used to compute a correlation metric that uses the known training symbols. In particular, we consider two approaches for the correlation metric. In a first method, the correlation is computed along the temporal direction, i.e., as a

function of  $nT$ , for each sub-channel of index  $k$ . The metric allows to determine an estimate of the time offset and of the frequency offset independently for each sub-channel. From these estimates we determine an average value  $\widehat{\Delta}_{\tau,it}^{(u)}$  and  $\widehat{\Delta}_{f,it}^{(u)}$ .

In a second method, we compute a common value for the time/frequency offsets (for all sub-channels of the desired user) by implementing a metric that jointly processes the outputs (4) along both the temporal and the sub-channel direction, i.e., as a function of  $nT$  and  $k$ . This second method turns out to require shorter training sequences, thus it saves redundancy.

Once the estimates above are computed, we re-run the receiver filter bank. However, now the filter bank can exploit the knowledge of the estimated time/frequency offsets. Thus, for this new iteration we compute

$$z_{it+1}^{(u,k)}(nT) = \sum_{i \in \mathbb{Z}} y(iT + \widehat{\Delta}_{\tau,it}^{(u)}) g^*(iT - nT) e^{-j2\pi (f_k + \widehat{\Delta}_{f,it}^{(u)}) iT} \quad (5)$$

Now, using the outputs in (5) we can re-compute the synchronization metrics in an iterative fashion.

#### A. Sub-Channel Synchronization Metric

In a first method (see also [8] and Appendix I), the outputs  $z_{it}^{(u,k)}(nT)$  are used to compute the following correlation metric that uses the known training symbols

$$P_{it}^{(u,k)}(n) = \sum_{l=0}^{N_{TR}-K-1} \left[ Z_{it}^{(u,k)}(IT_0; nT) Z_{it}^{(u,k)*}(IT_0 + KT_0; nT) \right] \quad (6)$$

$$Z_{it}^{(u,k)}(IT_0; nT) = z_{it}^{(u,k)}(IT_0 + nT) \frac{a_{TR}^{(u,k)}(IT_0)^*}{|a_{TR}^{(u,k)}(IT_0)|^2} \quad (7)$$

Metric (6) is used to locate the training sequence and to estimate the time offset of sub-channel  $k$  of user  $u$  as follows

$$\widehat{\Delta}_{\tau,it}^{(u,k)} = T \arg \max_n \left\{ |P_{it}^{(u,k)}(n)|^2 \right\}, \quad (8)$$

while it is used to estimate the frequency offset as follows

$$\widehat{\Delta}_{f,it}^{(u,k)} = \frac{1}{2\pi KT_0} \arg \left\{ P_{it}^{(u,k)}(n_{\max}) \right\} \quad n_{\max} = \widehat{\Delta}_{\tau,it}^{(u,k)} / T. \quad (9)$$

At this point, we compute the average values (across the assigned sub-channels)

$$\widehat{\Delta}_{\tau,it}^{(u)} = \frac{1}{M_u} \sum_{k \in K_u} \widehat{\Delta}_{\tau,it}^{(u,k)} \quad \widehat{\Delta}_{f,it}^{(u)} = \frac{1}{M_u} \sum_{k \in K_u} \widehat{\Delta}_{f,it}^{(u,k)}. \quad (10)$$

The frequency offset estimation holds for  $|\Delta f| < 1/(2KT_0)$ . Further, the value  $K \geq 1$  is chosen to minimize the variance of the estimator.

#### B. Joint Synchronization Metric

In a second method (see also Appendix II) the outputs  $z_{it}^{(u,k)}(nT)$  are used to compute the following correlation metric

$$P_{it}^{(u)}(n) = \sum_{k \in K_u} \left[ Z_{it}^{(u,k)}(0; nT) Z_{it}^{(u,k)*}(KT_0; nT) \right] \quad (11)$$

where  $Z_{it}^{(u,k)}(IT_0; nT)$  is defined in (7). Metric (11) is used to locate the training sequence and to estimate the time offset of user  $u$  as follows

$$\widehat{\Delta}_{\tau,it}^{(u)} = T \arg \max_n \left\{ |P_{it}^{(u)}(n)|^2 \right\}, \quad (12)$$

while it is used to estimate the frequency offset as follows

$$\widehat{\Delta}_{f,it}^{(u)} = \frac{1}{2\pi KT_0} \arg \left\{ P_{it}^{(u)}(n_{\max}) \right\} \quad n_{\max} = \widehat{\Delta}_{r,it}^{(u)} / T. \quad (13)$$

The frequency offset estimation holds for  $|\Delta f| < 1/(2KT_0)$ .

#### IV. PERFORMANCE RESULTS

In this section we report several performance results for a multiuser system that deploys  $M = 64$  sub-channels and a truncated root-raised-cosine prototype pulse with roll-off  $\alpha_1 = 0.2$ . The interpolation factor is  $N = 80$ . It should be noted that if we assume a transmission bandwidth  $1/T = 10$  MHz, the sub-channel Nyquist bandwidth is 125 kHz and the frequency guards are equal to  $0.04/(MT) = 6.25$  kHz. The channel is assumed to be Rayleigh faded with an exponential power delay profile with taps that have average power  $\Omega_p \sim e^{-pT/(\gamma T_0)}$  with  $p \in \mathbb{Z}^+$ ,  $\gamma = \{0.025, 0.1\}$ , and truncation at -20 dB. The data symbols and the training sequences belong to the 4-PSK signal set.

In Fig.2 we report BER as a function of the signal-to-noise ratio assuming a fully loaded system with 4 users that are allocated in an interleaved fashion to 16 sub-channels. The users are asynchronous with independent uniformly distributed time/frequency offsets respectively in  $[0, 2T_0]$  and  $[-\Delta_{f,\max}, \Delta_{f,\max}]$  with maximum carrier frequency offset  $\Delta_{f,\max} = 0.02/(MT)$ . The ideal curve assumes perfect synchronization and channel estimation, while the other curves assume practical synchronization (with 2 iterations) and channel estimation. For the ideal case we use a sub-channel MMSE equalizer with 1 or 3 taps. For the practical case we use RLS training of the equalizer over the known synchronization sequences. Note that the 3 tap equalizer yields a small gain over the single tap equalizer because in this scenario the sub-channels have quasi-flat frequency response. We also report the ideal performance of the single user case in flat fading which basically corresponds to the performance of 4-PSK in flat fading. If we compare the curves associated to the asynchronous multiuser case, we do not see any degradation. This is because the system keeps its orthogonality when the carrier frequency offsets are smaller than  $0.02/(MT)$ . The practical curves with both synchronization metrics and 2 iterations are within about 2 dB from the ideal ones. The sub-channel synchronization metric uses training sequences of length 15 random symbols per sub-channel, while the joint synchronization metric uses only 2 training symbols per sub-channel. Further, we have fixed  $K=3$ .

In Fig.3-4 we report BER as a function of the carrier frequency offset. Without compensation of the frequency offsets the performance dramatically decreases. With practical synchronization the BER curve has a flat behavior, i.e., we are capable of estimating and compensating the synchronization parameters very effectively for a large range of frequency offsets. The joint metric performs almost identically to the sub-channel metric. However, the former requires much less redundancy. Two synchronization iterations yield some advantage only for large frequency offsets when the receiver filter bank is significantly mismatched.

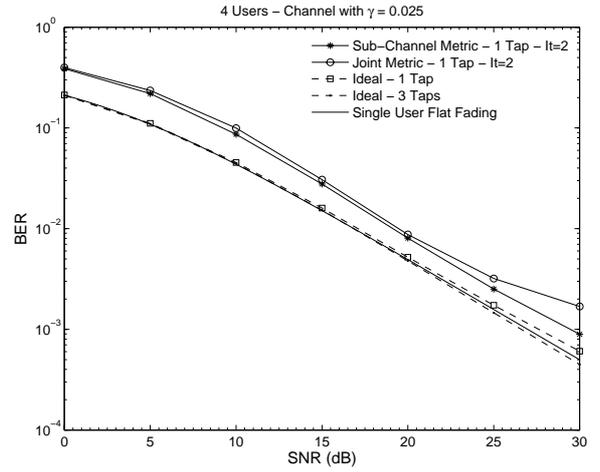


Fig. 2. Average BER with 4 asynchronous users as a function of the SNR. Ideal and practical performance with sub-channel and joint synchronization metric.

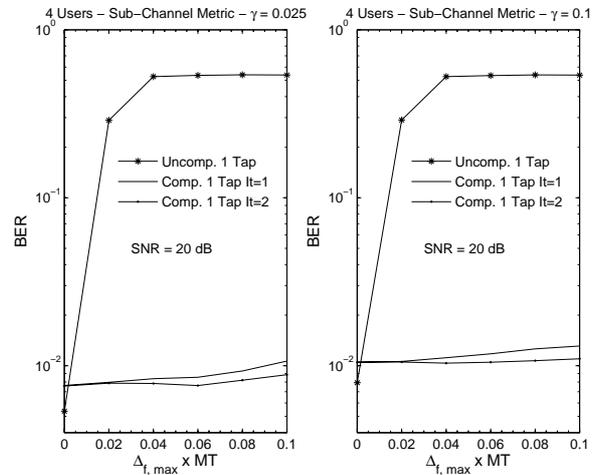


Fig. 3. Average BER with 4 asynchronous users as a function of the maximum frequency offset. 1 tap RLS equalizer, and 1-2 synchronization iterations with sub-channel synchronization metric.

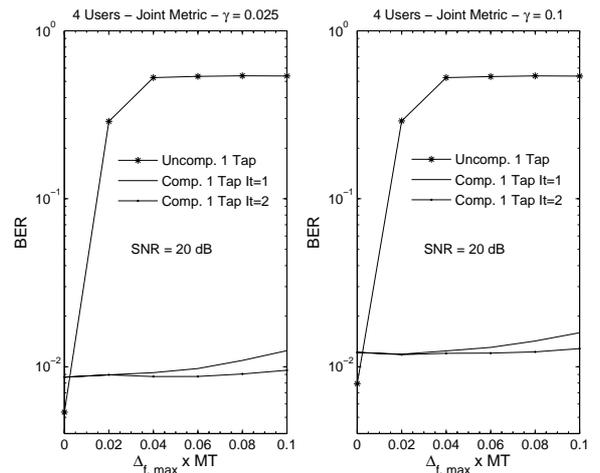


Fig. 4. Average BER with 4 asynchronous users as a function of the maximum frequency offset. 1 tap RLS equalizer, and 1-2 synchronization iterations with joint synchronization metric.

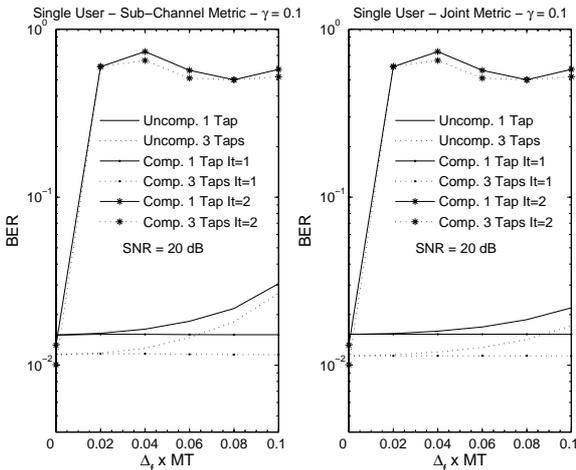


Fig. 5. Average BER with single user. 1 and 3 taps RLS equalizer, and 1-2 synchronization iterations with sub-channel and joint synchronization metric.

The results have shown that more than one synchronization iteration yields some advantage when the carrier frequency offsets are large or the number of sub-channels that are assigned to a given user is high. To this respect, in Fig.5 we report results for a single user system with  $M = 32$  sub-channels, roll-off  $\alpha_1 = 0.2$ , and interpolation factor  $N = 40$ . All sub-channels are assigned to the user. Now note that the performance gain that is provided with 2 iterations is significant both with the sub-channel and the joint metrics.

## V. CONCLUSIONS

We have presented an iterative synchronization approach with two synchronization metrics for a multiuser FMT system. The overall receiver performs well and demonstrates the effectiveness of the algorithms and their robustness to a wide range of time/frequency offsets and channel frequency selectivity. Multiuser FMT is a robust air-interface that provides better performance than multiuser OFDM yet allowing an efficient FFT based implementation. A performance comparison with multiuser OFDM and multicarrier CDMA (MC-CDMA) is reported in [9].

### APPENDIX I: SUBCHANNEL SYNCHRONIZATION METRIC

The sub-channel synchronization metric has been derived as a result of the following observations. First, if the carrier frequency offsets fulfil the relation  $\Delta_f T_0 \sim 0$ , i.e., they are much smaller than the sub-channel bandwidth, (4) can be written in correspondence of the training sequence as

$$z_{it=1}^{(u,k)}(lT_0 + nT) = e^{j2\pi\Delta_f^{(u)}lT_0} a_{TR}^{(u,k)}(lT_0) h_{EQ}^{(u,k)}(nT) + I^{(u,k)}(lT_0; nT) + \eta^{(u,k)}(lT_0 + nT) \quad (14)$$

for  $l=0, \dots, N_{TR}-1$ ,  $n \in \mathbb{Z}$ , and for a certain sub-channel equivalent impulse response  $h_{EQ}^{(u,k)}(nT)$ . If we further assume the interference term that comprises ISI, ICI, and MAI components, to be small, the metric (6) has the peak in correspondence of the training sequence (assuming it to have

good autocorrelation properties). That is, for  $n = n_{\max}$ , we obtain

$$P_{it=1}^{(u,k)}(n_{\max}) = e^{j2\pi\Delta_f^{(u)}KT_0} \left| h_{EQ}^{(u,k)}(n_{\max}T) \right|^2 (N_{TR} - K) + \hat{\eta}^{(u,k)}(n_{\max}T). \quad (15)$$

Thus, (8)-(9) follow. The term  $K$  is chosen to minimize the variance of the estimator. In particular to take into account the presence of the ISI, it has to be, possibly, such that  $KT_0$  is larger than the sub-channel time dispersion.

Clearly, the above approximations are improved when we possess an a priori knowledge of the carrier frequency offset. That is, when we have obtained a first estimate of the carrier frequency offset, we can re-run the filter bank that compensates for it as described by (5). In other words, we iteratively refine the knowledge of the carrier frequency offset, which allows to center at the appropriate frequency the receive filter bank of the desired user.

### APPENDIX II: JOINT SYNCHRONIZATION METRIC

The joint synchronization metric has been derived observing that (11) in correspondence to the training sequence yields

$$P_{it=1}^{(u)}(n) = e^{j2\pi\Delta_f^{(u)}KT_0} \sum_{k \in K_u} \left| h_{EQ}^{(u,k)}(nT - \Delta_\tau^{(u)}) \right|^2 + I^{(u)}(n) + \eta^{(u)}(n). \quad (16)$$

Therefore, while in (15) the maximum is in correspondence to the peak of the squared magnitude of the sub-channel equivalent response, in (16) the maximum is in correspondence to the sum of the sub-channel equivalent responses.

It should be noted that the joint synchronization metric turns out to be effective when the user is allocated to a sufficient number of sub-channels.

### REFERENCES

- [1] T. Schmidl, D. Cox, "Robust Frequency and Timing Synchronization for OFDM", *IEEE Trans. Comm.*, pp. 1613-1621, December 1997.
- [2] G. Cherubini, E. Eleftheriou, S. Ölçer, "Filtered Multitone Modulation for Very High-Speed Digital Subscriber Lines", *IEEE JSAC*, pp. 1016-1028, June 2002.
- [3] A. Assalini, A. Tonello, "Time-Frequency Synchronization in Filtered Multitone Modulation Based Systems", *Proc. of WPMC 03*, Yokosuka, Japan, pp. 221-225, October 2003.
- [4] A. Tonello, F. Rossi, "Synchronization and Channel Estimation for Filtered Multitone Modulation", *Proc. of WPMC 2004*, Abano Terme, Italy, pp. 590-594, Sept. 2004
- [5] A. Tonello, "Asynchronous Multicarrier Multiple Access: Optimal and Sub-optimal Detection and Decoding", *Bell Labs Tech. Journal*, vol. 7 n. 3, pp. 191-217, 2003.
- [6] S. Kaiser, W. Krzymien, "Performance Effects of the Uplink Asynchronism in a Spread Spectrum Multi-carrier Multiple Access System", *European Trans. on Telec.*, pp. 399-406, July-August 1999.
- [7] A. Tonello, N. Laurenti, and S. Pupolin, "Analysis of the uplink of an asynchronous DMT OFDMA system impaired by time offsets, frequency offsets, and multi-path fading", *Proc. of IEEE VTC 00-Fall*, Boston, USA, pp.1094-1099, September 2000
- [8] A. Tonello, F. Pecile, "Synchronization Algorithms for Multiuser Filtered Multitone (FMT) Systems", *Proc. of IEEE VTC 2005 Spring*, Stockholm, Sweden, May 2005.
- [9] A. Tonello, "Performance Analysis of a Multiple Antenna Concatenated DMT-FMT Scheme in the Uplink", *Proc. of IEEE ISWCS 2005*, Siena, Italy, Sept. 2005.