An Efficient Implementation of a Wavelet Based Filtered Multitone Modulation Scheme

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Abstract—In this paper we propose an efficient implementation of a novel wavelet based filtered multitone modulation scheme for transmission over broadband channels. Both wireline and wireless applications are considered. The attractive feature of this scheme is its low implementation complexity. Its feasibility has been demonstrated by a prototype hardware implementation using FPGA technology.

Keywords—Filtered multitone modulation (FMT), Multicarrier Modulation, OFDM, Hardware prototype, FPGA implementation.

I. INTRODUCTION

In this paper we describe an efficient implementation of a wavelet based filtered multitone modulation (FMT) scheme. The scheme can find application both in wireless scenarios, and in wireline scenarios, e.g., for very high speed digital subscriber line (VDSL) transmission. Most of the analysis and performance results in this constribution are related to the former scenario. However, we also report some details about a hardware prototype implementation that we developed for VDSL applications. The scheme that we describe falls in the category of multicarrier modulated systems that are typically proposed to simplify the equalization task when transmission is over wideband frequency selective channels [6]. Orthogonal frequency division multiplexing (OFDM) is probably among the most popular multicarrier modulation techniques. Herein, we refer to it as discrete multitone (DMT) modulation. It is essentially based on multicarrier transmission with sub-channel pulses $g(nT)$ that have a rectangular impulse response. Its digital implementation comprises an M-point inverse fast Fourier transform (IFFT), where M equals the number of sub-channels, followed by the insertion of a cyclic prefix. If the channel time dispersion is within the length of the cyclic prefix, simple one-tap equalization can be used. More general multicarrier schemes deploy sub-channel filters with time-frequency concentrated response. Under certain conditions they can be implemented by using an IFFT followed by low-rate sub-channel filtering [1]. These schemes are referred to as filtered multitone modulated systems (FMT) [1] and have been originally proposed for application in VDSL. If the sub-channels have disjoint frequency response, i.e., do not overlap in frequency, it is possible to avoid the inter-carrier interference (ICI), and get low inter-symbol interference (ISI) that can be corrected
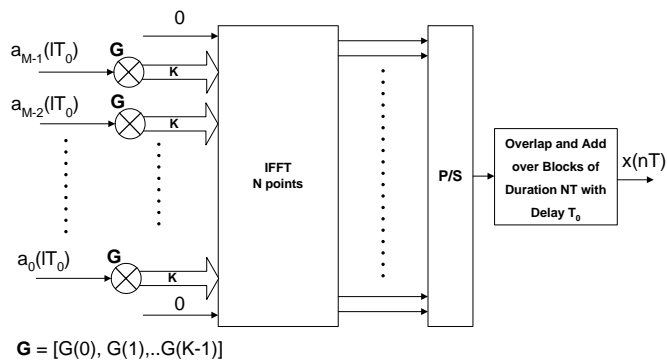
$\mathbf{G} = [G(0), G(1),..G(K-1)]$

Fig. 1. Proposed frequency domain synthesis of the Wavelet FMT (WFMT) transmitter with an N-point core IFFT, and with $T_0 = (N / L)T$.

with sub-channel equalization [1], [3], [4]. FMT modulation has the potentiality of achieving better spectral efficiency than OFDM yet requiring sufficiently simple equalization. Further, the sub-channel spectral containment of FMT based systems makes them an interesting solution for application in asynchronous multiuser systems, i.e., uplink wireless communications where the users received signals are subject to distinct carrier frequency offsets, and time misalignments [3].

Although, FMT modulation can in principle provide better performance, lower out-of-band side lobs, better immunity to narrow band interference than DMT, its practical implementation is typically considered more complex than DMT. The polyphase architectures that are described in [1] are an efficient implementation but not necessarily a low (in the absolute sense) complexity one. In fact, the transmitter (synthesis filter bank) requires to run filtering with a bank of low rate filters at the output of the IFFT. The sub-channel pulses are obtained by the polyphase decomposition of a prototype pulse. Further, in a non-critically sampled FMT implementation, i.e., when the sub-channel data rate is lower than the sub-carrier spacing, the low-rate sub-channel pulses are periodic time-variant [1]. Similarly, the receiver front-end (analysis filter bank) comprises a bank of matched filters followed by a fast Fourier transform (FFT).

In this paper we propose and describe an efficient FMT scheme that is based on the deployment of wavelets. We refer to it as wavelet FMT (WFMT), Fig. 1. It should be noted that the design of the prototype pulse is of great importance both

from a performance perspective and an implementation complexity point of view. The goal is to design a prototype pulse that has concentrated impulse and frequency response. The contribution is twofold. Firstly, we describe the theoretical aspects behind the scheme. Secondly, we report some details on the hardware implementation that has been done at DATA-JCE Ltd using FPGA technology.

The scheme comprises, at the transmitter side, the following stages: frequency domain synthesis of the sub-channel wavelet and data modulation. At the receiver side it comprises the analysis of the sub-channel wavelets, followed by simplified sub-channel equalization, and detection.

The prototype wavelet is designed to fulfil with the requirement of being time and frequency concentrated. We show that the wavelet can be synthesized via an IFFT with a small number K of non-zero coefficients.

The WFMT transmitter uses an N-point core IFFT and transmits M data streams a_i each at rate $1/T_0 = L/(NT)$. As shown in Fig. 1, the data streams modulate the frequency shifted wavelet coefficients. Some of the outermost frequencies can be set to zero for spectral containment reasons. The coefficients are passed through the IFFT, P/S converted and finally an overlap and add operation takes place to obtain the synthesized multicarrier signal. At the receiver the WFMT signal is processed by an analysis filter-bank that implements demodulation of the sub-channels wavelets. The analysis of the sub-channel wavelets is provided by the analysis of its frequency components by an FFT. To compensate the distortions introduced by a frequency selective channel we propose to use a simple equalization scheme.

II. PROTOTYPE WAVELET

Our multicarrier system deploys a prototype wavelet $g(nT)$ where T is the transmission period (chip period). The prototype wavelet is designed to fulfil the requirements of being real, even, to have finite support NT, and to be orthogonal to its translations of multiples of $T_0 = NT/L$, for integers N and L, i.e., $\kappa_g(nT_0) = g * g_-(nT_0) = \delta(n)$. From these constraints, it follows that the prototype wavelet can be obtained by a weighted sum of cosine components. We assume that the wavelet synthesis comprises only K cosine components. It can be shown that the value K is relatively small, and is a function of the level of inter-symbol interference (ISI) that is considered acceptable in the system design. In Fig. 2 we show the level of inter-symbol interference (ISI) as a function of K for wavelets that were used for the realization of an efficient FMT algorithm. The minimal number of cosine components required for the prototype wavelet synthesis is 9. The practical FMT system that we developed uses a prototype wavelet constructed from K=11 components, and it is shown in Fig. 3. Such a low number of frequency components give us the possibility to use an inverse discrete Fourier Transform (IDFT) (or its
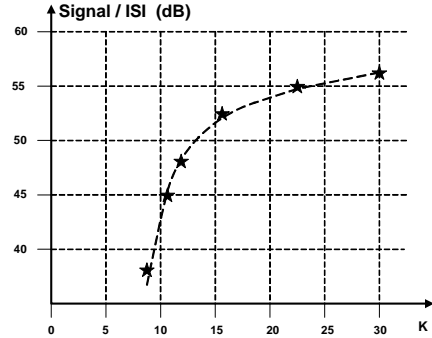

Fig. 2. Signal-to-interference power ratio as a function of the number K of wavelet frequency components.
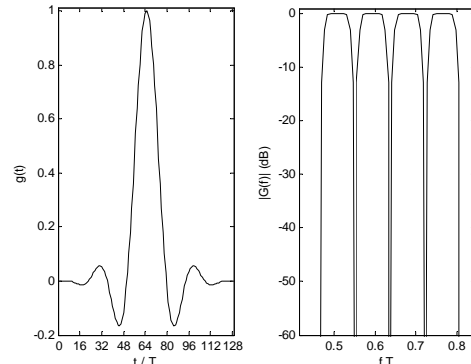

Fig. 3. Wavelet impulse response and frequency response for four adjacent sub-channels. Wavelet is generated using a 128 point core FFT with K=11 non-zero frequency components.

efficient version, the IFFT) for the prototype wavelet synthesis. We denote with $G(k) = G(kF)$ the N-point DFT of $g(nT)$ with $F = 1/(NT)$. Note that with the above design the wavelet has bandwidth practically equal to KF. In other words the wavelet has K non-zero frequency components. Further, in our design we choose $K = L + 3$.

III. TRANSMITTER

The complex baseband transmitted signal is obtained by a filter bank modulator with prototype wavelet $g(nT)$ and sub-channel carrier frequency

$$f_k = k_0 F + kKF \quad k = 0,...,M-1 \quad MK \le N \quad (1)$$

for an integer $0 \le k_0 < K$ that is chosen to set to zero the outermost frequencies. Therefore, it can be written as

$$x(nT) = \sum_{k=0}^{M-1} \sum_{l=-\infty}^{\infty} a^{(k)}(lT_0) g(nT - lT_0) e^{j2\pi f_k nT} \quad (2)$$

where $a^{(k)}(lT_0)$ is the k-th sub-channel data stream that we assume to belong to the M-QAM constellation set.

A possible conventional efficient implementation that is based on polyphase filtering is described in [1]. However, by exploiting the fact that the wavelet can be synthesized with a small number K of frequency components, we can use an alternative efficient implementation as shown in Fig. 1. To derive it we can manipulate (2) to obtain

$$x(nT) = \sum_{l=-\infty}^{\infty} x_l(nT) \qquad (3)$$

$$x_l(nT) = \sum_{k=0}^{M-1} a^{(k)}(lT_0)e^{j\frac{2\pi}{L}(Kk+k_0)l}g(nT-lT_0)e^{j\frac{2\pi}{N}(Kk+k_0)(n-\frac{N}{L}l)} \qquad (4)$$

for $n = lN/L,...,lN/L+N-1$, and zero otherwise. The finite time support signal (4) can be obtained by a N-point IDFT as

$$x_l(nT) = \frac{1}{N}\sum_{k=0}^{M-1}\sum_{m=0}^{K-1} a^{(k)}(lT_0)e^{j\frac{2\pi}{L}(Kk+k_0)l}G(m)e^{j\frac{2\pi}{N}n(m+kK+k_0)} \qquad (5)$$

where $G(m)$ is the m-th frequency component of the wavelet.

In conclusion as shown in Fig. 1, the M sub-channel data symbols $a_k(lT_0) = a^{(k)}(lT_0)e^{j2\pi(Kk+k_0)l/L}$ at a given instant lT_0 weight the frequency shifted wavelet components. Some zeros are added at the edges. Then, a N-point IDFT is computed. Finally, P/S conversion and an *overlap and add* operation, at rate $1/T_0$, take place. The overall data rate is equal to $R = ML/(NT)$. Note that in our design the sub-channel expansion factor equals $K/L = K/(K-3)$, and decreases with an increasing number of wavelet frequency components K.

IV. CHANNEL MODEL AND RECEIVED SIGNAL

If we assume a discrete time base band channel model with tap amplitudes α_p, i.e., $h(nT) = \sum_{p=0}^{N_P}\alpha_p\delta(nT-pT)$, then the received signal in the absence of noise reads

$$y(nT) = \sum_{k=0}^{M-1}\sum_{l=-\infty}^{\infty} a^{(k)}(lT_0)e^{j\frac{2\pi}{L}(Kk+k_0)l}g_{EQ}^{(k)}(nT-lT_0) \qquad (6)$$

where the k-th sub-channel equivalent impulse response is

$$g_{EQ}^{(k)}(nT) = \sum_{p=0}^{N_P}\alpha_p g(nT-pT)e^{j\frac{2\pi}{N}(Kk+k_0)(n-p)} \qquad (7)$$

In wireless the channel taps are assumed to be independent, zero mean, complex Gaussian (Rayleigh fading).

V. RECEIVER

The receiver front-end consists of a matched filter bank. Filters are matched to the equivalent sub-channel response [4]. The k-th filter output sample reads

$$z^{(k)}(mT_0) = e^{-j\frac{2\pi}{L}(Kk+k_0)m}\sum_n y(nT)g_{EQ}^{(k)*}(nT-mT_0)+\eta^{(k)}(mT_0) \qquad (8)$$

where $\eta^{(k)}(mT_0)$ is the Gaussian noise contribution. Further,

$$z^{(k)}(mT_0) = a^{(k)}(mT_0)\kappa_{EQ}^{(k)}(0)$$
$$+\sum_{l\neq m} a^{(k)}(lT_0)e^{j\frac{2\pi}{L}(Kk+k_0)(l-m)}\kappa_{EQ}^{(k)}(mT_0-lT_0) \qquad (9)$$
$$+ICI^{(k)}(mT_0)+\eta^{(k)}(mT_0)$$

where the first term represents the useful data contribution, the second additive term is the ISI contribution, the third term
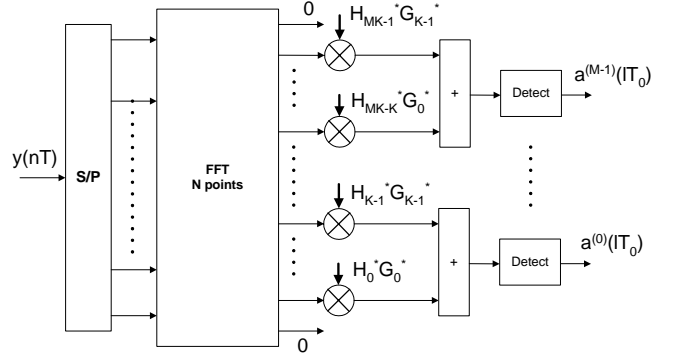

Fig. 4. Simplified matched filter receiver.

is the intercarrier interference (ICI) contribution. Further,

$$\kappa_{EQ}^{(k)}(mT_0) = g_{EQ}^{(k)} * g_{EQ-}^{(k)*}(mT_0) \qquad (10)$$

is the equivalent sub-channel autocorrelation. If we assume frequency concentrated non-overlapping sub-channels the ICI term is zero.

To derive a simple detector we can assume that the prototype wavelet has short duration, such that convolved with the channel yields an equivalent impulse response with time support still equal to NT. Then, the matched filtering operation can take place in the frequency domain, as follows

$$z^{(k)}(mT_0) = e^{-j\frac{2\pi}{L}(Kk+k_0)m}\sum_{n=Kk+k_0}^{K(k+1)+k_0-1} Y_m(n)G_{EQ}^*(n) \qquad (11)$$

$$G_{EQ}(n) \approx G(n-kK-k_0)H(n) \quad n\in k_0+[kK,(k+1)K)$$

$$Y_m(n) = DFT\{y(kT+mNT),\, k=0,...,N-1\} \quad n=0,...,N-1$$

that is, $Y_m(n)$ is the DFT of the m-th received block of N samples, while $G_{EQ}(n)$ is the DFT of the equivalent channel response. This simplified matched filter receiver is shown in Fig. 4. A further simplification can be obtained by choosing, e.g., with a minimum mean square error criterion, a single channel weight $H^{(k)}$ within a single sub-channel k, which corresponds to assume the channel flat within a band KF.

A. Example of Performance over a Wireless Fading Channel

In Fig. 5 we show bit-error-rate (BER) performance of the proposed WFMT scheme. We assume a Rayleigh faded channel with exponential power delay profile (truncated to 32 chips) with root-mean-square delay spread τ_{rms}. Further, we use BPSK signalling, and parameters N=128, K=11, L=8, M=11 with the wavelet in Fig. 3. If for instance the bandwidth is equal to 20 MHz (as in the WLAN standard IEEE 802.11a), the delay spreads considered range from 50 ns up to 200 ns. We further consider channel coding with an interleaved convolutional encoder of rate ½ and constraint length 5. The simple equalization scheme above is deployed under the assumption of knowing the channel.

For comparison we report also the BER performance of an OFDM (DMT) system that uses a 128 point FFT, a cyclic

prefix of length 32, and data rate identical to the WFMT system. The same channel encoder is deployed for the OFDM system. Fig. 5 shows that the BER performance of the proposed WFMT system is good even without channel coding. Some error floor appears at high SNRs and high delay spreads. However, performance can be improved with more powerful equalization. Note also that some frequency diversity exploitation is possible also without coding since over a certain SNR region the BER improves as the delay spread increases despite the simple equalization scheme. On the contrary the uncoded OFDM scheme does not provide any diversity gain and further an SNR penalty (although small) is introduced because the conventional OFDM receiver is not the optimal matched filter receiver [4]. With coding there is a deep performance improvement in both the WFMT and the OFDM scheme that is more pronounced for high delay spreads as a result of higher frequency diversity. However, the coded WFMT scheme shows superior performance than the coded OFDM scheme still having identical data rate and comparable complexity.

VI. HARDWARE IMPLEMENTATION

The most significant issue of FMT implementation is its complexity. Several authors give different complexity estimation for FMT based on different architectures of poly-phase filters and equalizers. For the correct estimation of the FMT chip complexity, size, and for verification of simulation results, an FPGA prototype of the proposed WFMT system was developed. The parameters of the prototype have been chosen for VDSL applications but they can be adapted to wireless applications as considered in this paper and are an interesting example to understand what the overall complexity is. The prototype demonstrates that the proposed WFMT scheme can be implemented using a chip that has similar size and cost than a DMT chip. It includes a transmitter and receiver implemented in FPGA VERTEX2 (3000) of Xilinx. Because of the FPGA limitations the system clock of 60 MHz was provided instead of 88 MHz that is needed in the implemented VDSL operation mode B. As a result, the signal bandwidth was limited to 3 MHz instead of 4.4 MHz. A 1024 point core FFT was used to obtain 44 sub-channels, 32 sub-channels for the downstream (138 kHz-3.7 MHz), and 8 for the upstream. The bandwidth of each FMT sub-channel is about 59 kHz. Constellations from 2 to 12 bit/symbol were used.

It is interesting to note that the WFMT chip size does not depend on the signal bandwidth provided that we keep the same core FFT size, and number of sub-channels. However, it is clear that the system clock changes for different operation modes, e.g., in VDSL it ranges from 88 MHz (mode A-B) up to 260 MHz (mode C) [5].

Both the transmitter and receiver tests have shown a close coincidence with the simulation results [5]. The complexity estimate accounts for 1007 KGates, and 162 KB of RAM, including transmitter, receiver, framer, and processor. For
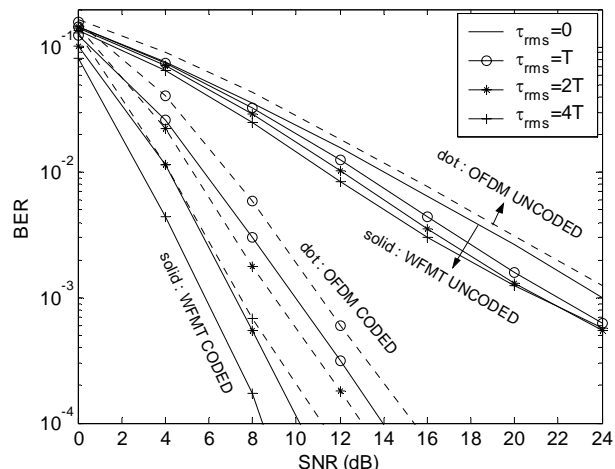

Fig. 5. Average BER in Rayleigh fading with exponential power delay profile with delay spread τ_{rms} for the WFMT scheme (solid curves) and an OFDM scheme (dotted curves) with identical data rate.

0.18 µm technology the required chip size is 28.2 mm^2 which shows that the scheme can be realized in a small silicon area.

VII. CONCLUSIONS

We have presented a wavelet based multitone system and an alternative (compared to the filter bank approaches in [1]) efficient implementation. Simulation performance results show that the scheme is robust to channel frequency selectivity yet requiring a simple detection algorithm. Although, we have not discussed it herein, the FMT schemes, and in particular the WFMT scheme of this paper, shows more robustness to carrier frequency offsets and timing errors than OFDM, and is capable of yielding higher spectral efficiency. An hardware prototype implementation targeted for VDSL applications has been done and it shows that the overall chip complexity is small. Current research activity includes the design and development of an hardware prototype for wireless applications.

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