On Noise Modeling for Power Line Communications

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***Abstract*—This paper reviews existing noise models including both background and impulsive noise for the in-home PLC scenario, highlighting similarities and differences. With reference to the impulsive noise, it is shown that a simple model, in the frequency band up to 100 MHz, can be derived by considering the noise generated at the source and taking into account the effect of the channel. Capacity considerations are then made, comparing erasure decoding strategies or full decoding strategies.**

I. Introduction

Electric power grids offer a convenient and cheap communication media due to its universal existence in buildings and residences, the ubiquity of outlets, and the simplicity of the power plug. The idea of using the electric power distribution grid for communication purposes is not new at all. For many decades power supply companies have been using their networks for data transmission, even if the main purpose has been management, control and supervision of power plants and distribution facility operations, tasks that call for rather low data rates in the kbit/s range. A next level is the exploitation of the power supply grid for in-home networking purposes, i.e., fast Internet access, voice over IP and home entertainment, tasks requiring data rates in excess of 100 Mbit/s.

The development of PLC systems turns out to be a severe challenge for the communications engineer, having to deal with harsh channels. In fact, the power line network differs considerably from conventional media such as twisted pair and coaxial, especially in terms of topology, structure, and physical properties. For instance, the in-home channel response characteristics that need to be contended with, are time-varying and frequency dependent with attenuation of up to 60 dB [1]. Moreover, the noise in the power line scenario cannot be described by an additive white Gaussian noise (AWGN) model, in contrast to many other communication channels.

In this paper, we focus on noise modeling in the frequency range up to 100 MHz in the home environment. We describe the background noise model, and several existing impulsive noise models. Essentially, impulsive noise is modeled with an arrival process with impulses having a certain amplitude distribution, a certain inter-arrival time distribution, and a certain duration distribution. The amplitude is often statistically modeled with the two terms Gaussian approach [2], with the Middleton's Class A distribution [3], or with a distribution obtained by fitting experimental data [4]. While the two-terms Gaussian model and the Middelton's model do not exhibit significant differences, the model in [4] is particularly distinct. We dig into this difference, and show that by modeling the noise at the source and by taking into account the effect of the channel, the amplitude statistics become more similar. Consequently, we propose a simple colored impulsive noise model.

This paper is organized as follows. Section II summarizes the properties of the power line channel in terms of channel transfer function, and noise classification. Section III presents the most representative models used with emphasis to the impulsive noise. In Section IV, we propose a simple "convergent" impulsive noise model. In Section V, we draw some capacity considerations. Finally, a conclusive discussion is presented.

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II. Properties of Power Line Channels

The general model of a power line channel is depicted in Fig.1

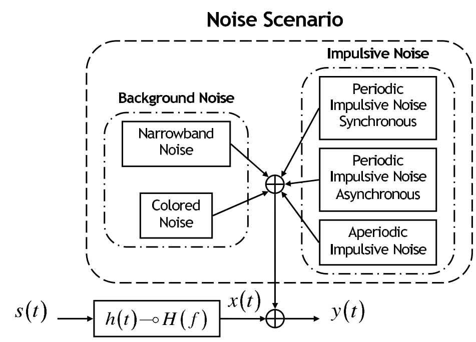


Fig. 1. Noise scenario on power line communications.

The channel is represented by a channel filter with impulse response $h(t)$, and different additive noise components as detailed in the following.

A. Channel Transfer Function

The frequency-selective power line channel can be modeled using a multipath signal propagation approach, according to [5], where the bandpass frequency response with N_p paths can be synthesized as

$$H(f) = B \sum_{p=1}^{N_p} g_p \cdot e^{-j(2\pi d_p/v)f} \cdot e^{-(\alpha_0+\alpha_1 f^K)d_p}, \quad 0 \le f_1 \le f \le f_2 \tag{1}$$

where B is a normalization term, $|g_p| \le 1$ is the transmission/reflection factor for path p, d_p is the length of the path, $v = c/\sqrt{\varepsilon_r}$ with c speed of light and ε_r dielectric constant. The parameters α_0, α_1, K are chosen to adapt the model to a specific network.

A statistical model of the channel can be derived, as proposed in [6], by considering the parameters in (1) as random variables. The main idea lies into the generation of channel realizations from the realizations of the random parameters, as proposed in [7]. The path lengths are assumed to be drawn from a Poisson arrival process with intensity Λ in m^{-1}. The reflection factors g_p are assumed to be real, independent, and uniformly or log-normally distributed. Finally, the parameters α_0, α_1 and K are appropriately chosen to a fixed value.

This channel model will be used in the following to derive the impulsive noise model, at the receiver, as the superposition of noise components that are generated in some point in the network and that propagate through the channel before being observed at the receiver.

B. Noise Scenario

The noise scenario is rather complicated due to the presence of different sources of disturbance that, according to [8], can be separated in the following classes (also depicted in Fig.1):

1) Background noise
   a) *Colored noise*. It is caused by common building and residential electronic equipment and it has been found to have a power spectral density (PSD) decreasing with increasing frequency.
   b) *Narrowband interference*. It is generally sourced from external broadcast radio bands, i.e., AM, FM and amateur radio.
2) Impulsive noise
   a) *Periodic impulsive noise synchronous with the mains frequency*. It is commonly originated by silicon controlled rectifiers (SCR) in power supplies and it is synchronous with the frequency of 50/100 Hz in Europe (60/120 Hz in the US).
   b) *Periodic impulsive noise asynchronous with the mains frequency*. It is mostly caused by switching power supplies, which are found in various household appliances, and it shows repetition rates between 50 kHz and 200 kHz [9].
   c) *Aperiodic impulsive noise*. It has a sporadic nature, mainly due to transients caused by connection and disconnection of electrical devices.

Even though the background noise has a time-variant nature, it can be considered stationary since it varies very slowly over periods of seconds and minutes or sometimes even hours. On the other hand, the impulsive noise cannot be considered stationary. During the occurrence of such impulses the noise power is perceptibly higher and may cause a significant increase in error rate.

Therfore, the overall noise process can be written as

$$\eta(t) = n_b(t) + n_{imp}(t) \tag{2}$$

where the first component denotes the background noise, while the second component denotes the impulsive contribution.

III. Noise Modeling

In this section the background and the impulsive noise are discussed in more detail.

A. Background Noise

Colored background noise is caused by the superimposition of numerous noise sources, e.g, computers, dimmers or hair dryers, which can create disturbances in the frequency range 0-100 MHz herein considered [10], [11]. Usually it is characterized by a fairly low power spectral density, which, however, significantly increases towards lower frequencies. Although not considered here, narrow band interferences localized in particular frequencies can also be added. They are generated from radio communication devices.

A commonly accepted model is the simple three-parameter model presented in [4], where the noise is considered Gaussian with the power spectral density (PSD)

$$R_{n_b}(f) = a + b\,|f|^c \quad \left[\frac{dBm}{Hz}\right] \tag{3}$$

where a, b and c are parameters derived from measurements and f is the frequency in MHz.

By setting the following parameters $[a, b, c] = [-145, 53.23, -0.337]$ we obtain a worst case scenario, while with the parameters $[a, b, c] = [-140, 38.75, -0.72]$ we obtain a best case scenario. The resulting PSDs are depicted in Fig.2, where the considered frequency band is between 1 MHz and 100 MHz.

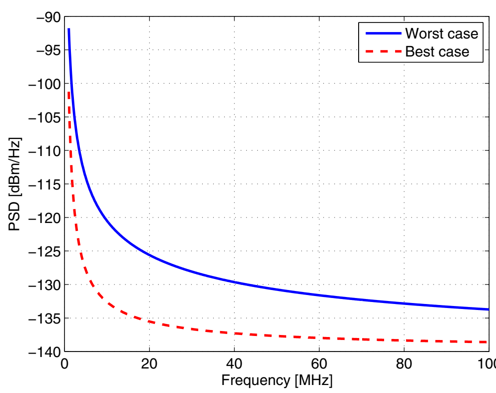


Fig. 2. Bad and good background noise PSD.

B. Impulsive Noise

The impulsive noise can be modeled with an arrival process specified by the statistics of the impulse amplitude A_{imp}, inter-arrival time t_{iat}, and duration t_{wid}. Most of the literature focuses on the statistics of the amplitude, as the Middleton's classification of electromagnetic interference [3] and afterwards the two-term Gaussian mixture [2] model. The temporal characteristics have been modeled via Markov chains in [8], or simply via a Bernoulli process [1]. Some peculiar statistics have been derived via curve fitting tools from experimental measurements in [4].

In the following we discuss such statistics, highlighting similarities and differences.

1) Amplitude statistics: In the two terms Gaussian model, the overall noise is written as $\eta(t) = \varepsilon(t)n_1(t) + (1 - \varepsilon(t))n_2(t)$, where $\varepsilon(t)$ is a Bernoulli random process, and $n_1(t)$, $n_2(t)$ are two independent Gaussian process with zero mean and power respectively equal to σ_b^2 and $K\sigma_b^2$ which is K times larger than the background one. It follows that the noise at any given time instant, is a random value whose probability density function (pdf) has the form

$$p_\eta(v) = (1 - P)\mathcal{N}\left(0, \sigma_b^2\right) + P\mathcal{N}\left(0, K\sigma_b^2\right) \quad (4)$$

where the first term $\mathcal{N}\left(0, \sigma_b^2\right)$ denotes a Gaussian pdf with zero mean and variance σ_b^2, while P is the probability that the impulsive noise occurs.

It should be noted that the background noise variance is obtained as follows from the PSD model in (3)

$$\sigma_b^2 = 2R_{IN}\int_{f_1}^{f_2} 10^{\left[R_{n_b}(f)-30\right]/10}df \quad \left[V^2\right] \quad (5)$$

where R_{IN} is the input impedance of the receiver, usually equal to 50 Ω, and f_1=1 MHz, while f_2=100 MHz.

In Fig.3, the blue solid line depicts the amplitude pdf, assuming $\sigma_b^2 = 2.6456 \cdot 10^{-5}\ V^2$, $K = 100$, $P = 0.01$, and the worst case background noise PSD, by way of an example.

In the Middleton's model, the noise is categorized into three different types (A, B and C). Among them, class A is known to represent the power line channel noise scenario. According to it, the background plus impulsive noise amplitude has a pdf equal to

$$p_\eta(v) = \sum_{k=0}^{\infty} \frac{e^{-A}A^k}{k!} \cdot \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp\left(-\frac{v^2}{2\sigma_k^2}\right) \quad (6)$$

with

$$\sigma_k^2 = \left(1 + \frac{1}{\Gamma}\right)\left(\frac{(k/A) + \Gamma}{1 + \Gamma}\right)\sigma_b^2 \quad (7)$$

It can be considered as a weighted sum of Gaussian pdfs where the parameter A is called impulsive index while the parameter Γ is called background-to-impulsive noise ratio. A good approximation of this distribution, is simply obtained by cutting off the cumulative sum after the third term according to [2]. In such a case, it is very similar to the one obtained in the two terms Gaussian model.

The amplitude pdf is depicted in Fig.3 assuming $\sigma_b^2 = 2.6456 \cdot 10^{-5}\ V^2$, $A = \Gamma = 0.01$ and truncating the cumulative sum at the third term. This is completely comparable to the two-terms Gaussian model having an occurrence rate equal to 1% and an impulsive noise variance 100 times larger than the background noise variance.

We now consider the model derived from fitting measured data in [12]. In particular, it has been obtained by the observation of noise generated by an electric drill and a light dimmer. The measurements have been done classifying a voltage sample as impulsive noise only when the sample exceeds a threshold, chosen so that background noise is unlikely to cross this threshold. Furthermore, a positive threshold has been set, so that only positive values of amplitude have been recorded. To overcome this limitation, we propose to introduce a random sign flip, so that with a simple modification of what was obtained in [4], the amplitude distribution is a double-sided Beta like. Thus, its pdf is

$$p_{A_{imp}}(v) = \frac{1}{2}f\left(\frac{|v| - 8}{9}\right) \quad (8)$$

where $f(u)$ denotes a Beta distribution with parameters $a = 3$ and $b = 2$, i.e.,

$$f(u) = \frac{\Gamma(5)}{\Gamma(3)}u^2(1 - u) \quad (9)$$

and $\Gamma(x)$ is the gamma function. The parameters have been herein set according to the data available in [4] where it has been observed that the impulsive noise samples are within the range $8 - 17$ mV.

The resulting pdf is depicted in Fig.3, which highlights the significant difference with the other distributions.

2) Inter-arrival time and duration: With the underlying Bernoulli process in the two-term Gaussian model the inter-arrival time, in a discrete-time model, follows a geometric distribution which can be viewed as the discrete counterpart of the exponential distribution. This suggests to model the inter-arrival time of the continuous time arrival process with an exponential distribution

$$p_{t_{iat}}(v) = \lambda e^{-\lambda v} \quad (10)$$

where v is expressed in $[s]$ and the mean λ can be related to the occurrence probability P for the two-term Gaussian mixture, as follows

$$\lambda = -\ln(1 - P) \quad (11)$$

Differently, in the experimental model in [4], the inter-arrival time is modeled with a Gamma distribution with parameters $a = 4.2$ and $b = 1$ as follows

$$p_{t_{iat}}(v) = \frac{1}{b^a\Gamma(a)}v^{a-1}e^{-\frac{v}{b}} \quad (12)$$

where v is expressed in $[ms]$, while the mean is $ab = 4.2\ ms$.

The exponential with $\lambda = 10.1\ ms$ (according to (11) with $P = 0.01$) and the Gamma pdfs are plotted in Fig.4, which shows that some similarities exist.

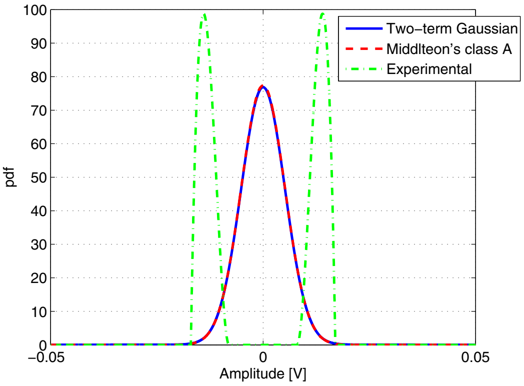


Fig. 3. Amplitude distribution comparison.

Finally, a simple way to model the impulse duration is to consider it a constant equal to a certain value, e.g., 100 μs according to [1], [13]. Otherwise, following [4], for the experimental case it can be modeled with the two-terms Gaussian model, and precisely as

$$p_{t_{wid}}(v) = P_1 \mathcal{N}\left(m_1, \sigma_1^2\right) + P_2 \mathcal{N}\left(m_2, \sigma_2^2\right) \quad [\mu s] \tag{13}$$

where $P_1 = 0.0736$, $m_1 = 4.9$, $\sigma_1 = 0.2$ and $P_2 = 0.0318$, $m_2 = 4.2$, $\sigma_2 = 0.25$. The mean value of a random variable distributed according to (13), using this set of parameters, is 4.7 μs.

IV. Impulsive Noise Analysis and Proposed Model

While the inter-arrival time distribution in the considered models is very similar, the amplitude distribution of the experimental model is significantly different from the others as Fig.3 shows. This can be explained by the fact that the latter model applies at the source that generates the noise rather than at the receiver.

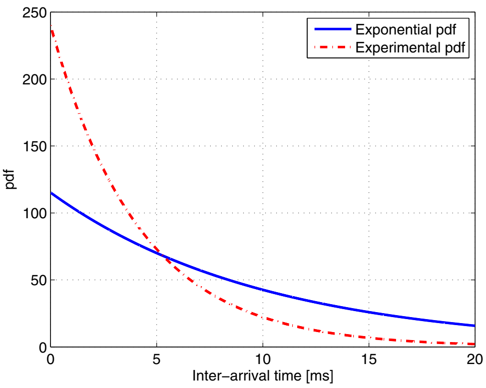
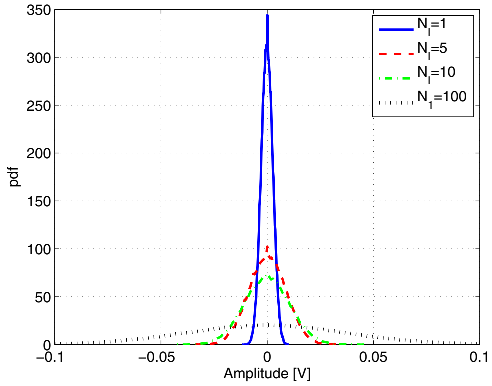


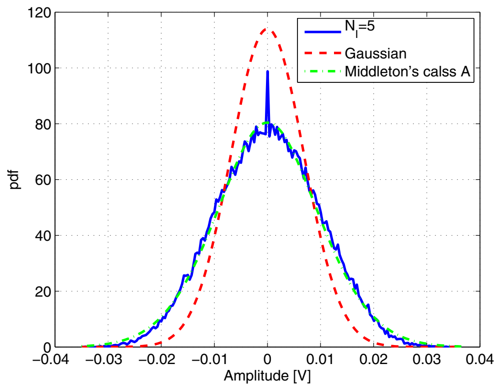
Fig. 4. Inter-arrival distribution comparison.

Starting from the statistical channel impulse response, described above, we set $f_1 = 0$ and $f_2 = 100$ MHz. Assuming an indoor application where the number of paths is typically high, we fix for the underlying Poisson process an intensity $\Lambda = 1/15\ m^{-1}$. The maximum considered path length is $L_{max} = 300\ m$. Finally, we choose $K = 1$, $\alpha_0 = 10^{-5}$ m^{-1} and $\alpha_1 = 10^{-9}\ s/m$, according to [6].

Now, in Fig.5(a) we show the pdf of the impulsive noise amplitude, assuming to have obtained it by the superposition of N_I noise processes that are filtered by N_I distinct channel responses. The distribution of each noise amplitude at the source is according to (8). The considered values of N_I are equal to 1, 5, 10, 100.



(a) Filtered noise processes pdfs.



(b) Monte Carlo simulation results.

Fig. 5. Experimental amplitude pdf at receiver side.

We now try to fit the obtained distribution either with a Middleton's distribution or a Gaussian distribution minimizing the mean squared error between them. Performing a Monte Carlo simulation to estimate the best fitting parameters for the resulting noise model with $N_I = 5$, yields $A = 5.8$ and $\Gamma = 0.1$ for Middleton's distribution, while $\sigma^2 = 4.7 \cdot 10^{-5}$ V^2 for the Gaussian distribution. The results in Fig.5(b) show that the Middleton's distribution perfectly fits the impulsive noise distribution observed at the receiver.

To derive a model for the impulsive noise power spectral density, we assume it to be white at the source, with zero mean and power

$$\sigma^2_{n_{imp,S}} = \int_D v^2 p_{A_{imp}}(v)\, dv \tag{14}$$

where D is the support of the amplitude distribution. Therefore, the PSD at the source is

$$R_{n_{imp,S}}(f) = R_{n_{imp,S}} = \frac{\sigma^2_{n_{imp,S}}}{f_2 - f_1}. \qquad (15)$$

It follows that the PSD of the impulsive noise observed at the receiver, assuming N_I independent sources of noise that propagate through channels with frequency response $H_k(f)$, is

$$\begin{aligned} R_{n_{imp}}(f) &= R_{n_{imp,S}} \sum_{k=1}^{N_I} |H_k(f)|^2 \\ &\to N_I R_{n_{imp,S}} E\left[|H(f)|^2\right] \end{aligned} \qquad (16)$$

where we show that the arithmetic sum goes to N_I times the average path loss of the channel for a large number of terms. Furthermore, the average path-loss of the assumed statistical channel model can be computed in closed form as follows

$$R_{n_{imp}}(f) = R_{n_{imp,S}} \frac{N_I B \Lambda}{1 - e^{-\Lambda L_{\max}}} \frac{1 - e^{-2(\alpha_0 + \alpha_1 f)L_{\max}}}{2(\alpha_0 + \alpha_1 f)} \qquad (17)$$

with the normalization factor $B = 48.8 \cdot 10^{-3}$ so that the channel path loss represents a typical profile encountered in practice. The resulting PSD is depicted in Fig.6 with $N_I = 5$.

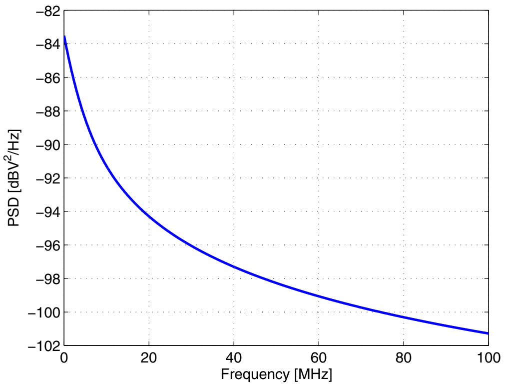


Fig. 6. Impulsive noise PSD profile at the receiver side for $N_I = 5$.

In conclusion, all the presented models essentially converge towards a single compliant impulsive noise model whose amplitude has a Middleton's distribution and inter-arrival time that is exponential distributed as summarized in Table I.

TABLE I
PROPOSED IMPULSIVE NOISE MODEL

	pdf	Parameters
Inter-arrival time	exponential	$\lambda = 10.1\ ms$
Amplitude	Middleton's class A	$A = 5.8$, $\Gamma = 0.1$

V. CHANNEL CAPACITY

In this section, we infer the capacity of the channel in the presence of impulsive noise. In particular, we consider two coding/decoding schemes. In the first one, we assume to implement an erasure decoding strategy, i.e., we disregard the samples affected by impulsive noise. In the second one, we assume full decoding. If P denotes the probability that a sample is affected by impulsive noise, the capacity associated to the two decoding strategies can be written as

$$C_{\text{erasure}} = (1 - P) \cdot C_b \qquad (18)$$

$$C_{\text{non-erasure}} = (1 - P) \cdot C_b + P \cdot C_i \qquad (19)$$

where C_b and C_i denote the capacity of a channel affected affected by colored Gaussian background noise and impulsive noise only, respectively.

To proceed, we assume the source input signal to be Gaussian distributed with constant power spectral density R_s. In the numerical examples, we assume $R_s = -50\ dBm/Hz$. Then,

$$C_b = \int_{f_1}^{f_2} \log_2\left[1 + \frac{|H(f)|^2 R_s}{R_{n_b}(f)}\right] df \quad [bit/s] \qquad (20)$$

where the background noise PSD is given by (3).

The capacity of the "portion" of the channel affected by impulsive noise, under the assumption of a Gaussian input signal, can be computed by evaluating the mutual input-output information rate as

$$C_i = h(Y) - h(N_{imp}) \quad [bit/s] \qquad (21)$$

where $h(Y)$ is the differential entropy per second of the output signal, while $h(N_{imp})$ is the differential entropy per second of the impulsive noise.

If we model the impulsive noise at frequency f so that it has a certain pdf $\hat{p}_{A_{imp}}(a, f)$ per-unit-frequency, then,

$$C_i = \int_{f_1}^{f_2} \left(\hat{h}(Y, f) - \hat{h}(N_{imp}, f)\right) df \qquad (22)$$

where

$$\hat{h}(N_{imp}, f) = -\int_{-\infty}^{+\infty} \hat{p}_{A_{imp}}(a, f) \log_2\left[\hat{p}_{A_{imp}}(a, f)\right] da \qquad (23)$$

$$\begin{aligned} \hat{h}(Y, f) = -\int_{-\infty}^{+\infty} &\left[\hat{p}_{A_{imp}} * \hat{p}_x(a, f)\right] \cdot \\ &\cdot \log_2\left[\hat{p}_{A_{imp}} * \hat{p}_x(a, f)\right] da \end{aligned} \qquad (24)$$

where $\hat{p}_x(a, f)$ is the pdf of the information signal at the receiver side (see Fig.1) and $*$ denotes convolution.

To proceed, we assume the background noise to have the bad PSD profile in (3) while the impulsive noise to have the PSD in Fig.6. Furthermore, we make the simplified assumption that $\hat{p}_{A_{imp}}(a, f)$ is a Gaussian pdf, or a Middleton's class A pdf. The transmitted signal, assumed Gaussian and white, propagates through the statistical channel with frequency response $H(f)$. Then, the complementary distribution function of the capacity is shown in Fig.7 which has been obtained by drawing a large number of channel response realizations and assuming two different values of impulsive noise occurrence rate P.

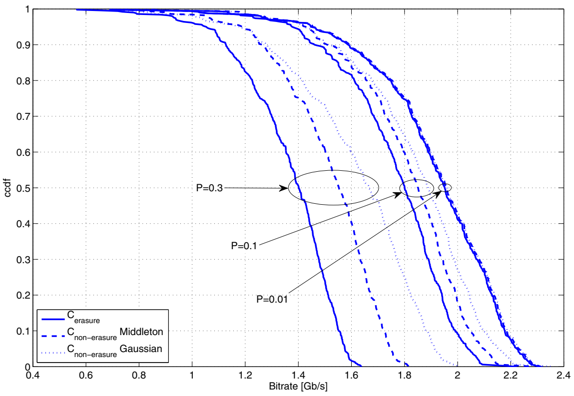


Fig. 7. Channel efficiency with different coding techniques combined with the proposed noise model.

As the figure shows, the higher P, the higher the loss in achievable rate. Assuming the impulsive noise to have a Middleton's distribution yields lower capacities than assuming it Gaussian. This can also be explained by the fact the the Gaussian input distribution is not optimal in the latter case. Erasure decoding does not yield significant capacity losses w.r.t. full decoding if P is large.

CONCLUSION

We have discussed several available models for the background and impulsive noise in PLC wide band channels. We have shown that by modeling the noise at the source and taking into account the effect of the propagation channel, the two-terms Gaussian, the Middeleton's and the Esmalian's et al. models converge to an arrival process model with impulses having interarrival time that is exponentially distributed, and amplitude that has a Middleton's distribution. Furthermore, the impulsive noise process has a certain colored spectrum related to the average path loss of the propagation channel. Finally, some capacity considerations are made. They highlight that erasure decoding is not affected by severe capacity loss compared to full decoding if the impulsive noise has a Middleton's distribution and the probability of occurrence is large.

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