Proceedings of IEEE International Symposium on Power Line Communications 2010
Rio de Janeiro, Brasil, March 28-31, 2010

On Power Allocation in Adaptive Cyclic Prefix OFDM

Salvatore D'Alessandro[1], Andrea M. Tonello[1], Lutz Lampe[2]
[1]DIEGM Universitá di Udine, Udine, Italy
[2] Dept. ECE, University of British Columbia, Vancouver, BC, Canada
Email: {salvatore.dalessandro, tonello}@uniud.it; lampe@ece.ubc.ca

***Abstract*—We consider the problem of the joint allocation of power and cyclic prefix (CP) duration to maximize the system capacity in orthogonal frequency division multiplexing (OFDM). A closed form solution to the problem does not exist and the exhaustive search of the optimal values is practically unfeasible. Therefore, we propose a sub-optimal iterative power allocation algorithm jointly with the CP adaptation to see whether capacity improvements can be obtained w.r.t. the conventional choice of a uniform power distribution and a predetermined long CP value. Numerical results show that via power and CP adaptation significant improvements can be obtained.**

I. Introduction

OFDM is one of the most popular modulation techniques in both wireless [1] and wireline communication systems [2], [3]. As it is well known, if OFDM deploys a CP longer than the channel duration, the received signal will be neither affected by inter-symbol interference (ISI) nor by inter-carrier interference (ICI) [4]. In such a case, assuming additive Gaussian noise, the system input-output mutual information with a total power constraint is maximized by Gaussian input signals whose power is computed with the water-filling algorithm [5], [6]. However, the system capacity can be increased with the usage of a CP shorter than the channel duration since the CP introduces a loss in transmission rate by a factor $M/(M+\mu)$, where M and μ respectively denote the number of sub-channels and the CP length in samples. This has been shown in a number of previous papers that consider both PLC and wireless application scenarios (cfr., e.g., [7]–[13]).

When the CP is shorter than the channel duration, the OFDM system can be modeled as a Gaussian interference channel. In our previous work [11] we have considered single tap sub-channel equalization and uniform power distribution across the sub-channels. Then, we have evaluated the capacity improvements via adaptation of the CP to the channel realization. Furthermore, in [12] we have proposed two bit-loading algorithms combined with CP length allocation. Although uniform power allocation at a certain level is simple and allows to fulfill a power spectral density (PSD) mask requirement for coexistence, it is interesting from a research perspective to investigate whether further capacity improvements are attainable with a non-uniform power allocation.

The work of this paper has been partially supported by the European Communitys Seventh Framework Programme FP7/2007-2013 under grant agreement n 213311, project OMEGA - Home Gigabit Networks.

As stated above, for a certain CP, the system can be viewed as a Gaussian interference channel with M total interferers. Both cross-talk ICI and ISI are present in the system. To the authors' knowledge the capacity of an interference channel either with a total power constraint or a PSD constraint is not known [6], [14]. If we assume Gaussian distributed inputs, and we assume joint coding and decoding, the capacity of the system with a total power constraint is achieved with an input covariance matrix that is obtained from the water-filling algorithm derived in [15] for a Gaussian vector channel with crosstalks. However, if we do not assume joint coding and decoding over the set of M sub-channels there is no closed form solution to the power allocation problem (note that in this case the input covariance is diagonal). In this case the interference exhibited by each sub-channel is treated as noise by the sub-channel decoder and the mutual in-out information is not convex as a function of the vector of input powers. Therefore, the optimal input power distribution has to be obtained via an exhaustive search, which is clearly unfeasible.

We point out that a large literature exists about similar, but not identical, problems in digital subscriber lines (DSL). In DSL, up to 100 subscriber lines can be bundled together to form a DSL binder [16]. Each subscriber deploys discrete multi-tone (DMT) modulation over a frequency spectrum common to all the subscribers, thus causing crosstalk interference. A class of algorithms that deals with the crosstalk issues are known as spectrum management techniques. In [17] the authors consider the centralized spectrum management problem, and they find an achievable rate region for the N_U users weighted sum rate. The weighted sum-rate optimization problem is split into M sub-problems, where each sub-problem considers the weighted sum rate in a given sub-channel across the N_U users. Thus, considering the use of L quantized power levels, each sub-problem is solved by doing an exhaustive search for the N_U power levels to be allocated to a certain sub-channel. The algorithm reduces the computational complexity from an order of $O(MN_U L^{MN_U})$ to $O(MN_U L^{N_U} 33^{N_U})$. Unfortunately, this algorithm is not applicable in our context because we deal with M interfering sub-channels and thus the complexity still remains exponential in M.

In [16], [18] the interference channel is viewed as a noncooperative game where the interference experienced by each player is viewed as noise (thus single user detection is assumed). In such a case it has been shown that a Nash equilibrium can be reached and it corresponds to a set of competitively optimal power levels. Furthermore, it has been shown that under certain conditions, the equilibrium can be achieved using an iterative water-filling (IWF) algorithm. The conditions for the convergence of this algorithm have been derived in [16] for the two-user case, and in [18] for a general multi-user case. It is worth noting that the IWF presented in [16], [18] renders convex the sum rate optimization problem by splitting it into N_U convex sub-problems. Once this has been done, water-filling is applied to each user until convergence is reached. Also this algorithm is not directly applicable to our context because it uses a constraint on the total user power. Therefore, if we treat our M sub-channels as interfering users, the power constraint becomes a PSD constraint. The application of the algorithm in [16], [18] would yield the trivial solution of a constant power distribution at the PSD level.

Inspired by the above results, we consider the problem of the joint allocation of power and CP duration to maximize the system capacity. As stated, a closed form solution to the problem does not exist and the exhaustive search of the optimal values is practically unfeasible. Therefore, we propose a sub-optimal iterative power allocation algorithm jointly with the CP adaptation to see whether capacity improvements can be obtained w.r.t. the conventional choice of a uniform power distribution and a predetermined long CP value that fully equalizes the channel.

The paper is organized as follows. In Section II we derive the system model. Then, in Section III, we formulate the joint power and CP optimization problem. In Section IV, we propose iterative power allocation algorithms to solve the previous problem, and in Section V, we present the numerical results. Finally, in Section VI, we derive the conclusions.

II. System Model

We consider a single user OFDM system. We denote with $a^{(k)}(\ell)$, $k = \{0, \ldots, M-1\}$, $\ell \in \mathbb{Z}$ the M data symbols that are transmitted at time instant ℓ and that have unit-power. Each data block is transformed using an M-point inverse discrete Fourier transform (IDFT). Then, we insert a guard interval in the form of a cyclic prefix that equals the last μ IDFT output coefficients. Therefore, the discrete-time OFDM signal can be written as

$$x(n+\ell N) = \sum_{k\in\mathbb{K}_{on}} \sqrt{P_a^{(k)}(\mu)}\, a^{(k)}(\ell) e^{j2\pi\frac{k(n-\mu)}{M}}, \quad n = 0, \cdots, N-1, \tag{1}$$

where $\mathbb{K}_{on} \subseteq \{0, \cdots, M-1\}$ is the set of used sub-channels, $N = M + \mu$, and $P_a^{(k)}(\mu)$ denotes the transmitted power on sub-channel k. As it will be clear in the following, the sub-channel power is a function of the CP length μ.

The signal is transmitted over a channel with equivalent discrete time channel impulse response given by

$$g_{ch}(n) = \sum_{p=0}^{L_{ch}-1} \alpha_p \delta(n-p), \tag{2}$$

where α_p denote the complex channel coefficients, and $\delta(n) = 1$ if $n = 0$ and zero otherwise.

The signal received in the k-th sub-channel can be written as

$$z^{(k)}(\ell) = \sqrt{P_a^{(k)}(\mu)}\, a^{(k)}(\ell) H^{(k)}(\mu) + I^{(k)}(\ell,\mu) + \eta^{(k)}(\ell). \tag{3}$$

Where, with $H^{(k)}(\mu)$ we denote the amplitude of the data of interest, whereas, with $I^{(k)}(\ell,\mu)$ and $\eta^{(k)}(\ell)$ we respectively denote the interference (ISI plus ICI) and the noise term in sub-channel k. As it is well known, if the CP is longer than the channel impulse response, the system will be orthogonal such that the received symbol will not be affected by interference [7].

III. Problem Formulation

When the CP is shorter than the channel duration, ISI and ICI components arise. Thus, as (3) shows, the OFDM system transmits data over a vector interference channel, i.e., M parallel sub-channels affected by ISI and crosstalks. The system capacity with a total power constraint, Gaussian additive noise, Gaussian input signals, and assuming optimal joint detection can be computed adapting the formulation in [15]. We are, however, interested in computing the system capacity under the assumption of independent Gaussian distributed input signals and using single tap sub-channel equalization, i.e., we treat as noise the interference. Therefore, the capacity is obtained by maximizing the input-output mutual information over the CP length and power distribution. To this end, we can formulate the following optimization problem

$$\begin{aligned} \max_{\mu, \mathbf{P}_a(\mu)} \quad & C(\mu), \\ \text{s.t.} \quad & \sum_{k\in\mathbb{K}_{on}} P_a^{(k)}(\mu) = \overline{P}, \\ & P_a^{(k)}(\mu) \geq 0, \ k \in \mathbb{K}_{on}, \\ & 0 \leq \mu < L_{ch}. \end{aligned} \tag{4}$$

In (4), $C(\mu)$ represents the system capacity. It is defined as:

$$C(\mu) = \frac{1}{(M+\mu)T} \sum_{k\in\mathbb{K}_{ON}} \log_2\left(1 + SINR^{(k)}(\mu)\right), \tag{5}$$

where

$$SINR^{(k)}(\mu) = \frac{P_a^{(k)}(\mu)|H^{(k)}(\mu)|^2}{P_\eta^{(k)} + P_{\mathrm{I}}^{(k)}(\mu)}. \tag{6}$$

Furthermore, T represents the sampling period, and $\mathbf{P}_a(\mu) = \left\{P_a^{(k)}(\mu), \text{with } k \in \mathbb{K}_{on}\right\}$ is the vector of the transmitted powers. In (6), $P_{\mathrm{I}}^{(k)}(\mu)$ and $P_\eta^{(k)}$ respectively denote the interference and the noise power terms on sub-channel k. Details on their computation can be found in [11]. An SNR gap can also be included to take into account the use of modulation and coding schemes [5].

Since the equivalent interference plus noise term, i.e., $P_{eq}^{(k)}(\mu) = P_{\mathrm{I}}^{(k)}(\mu) + P_\eta^{(k)}$, relies on both the vector of the transmitted powers and the CP length, in general, problem (4) is neither convex in μ nor in $\mathbf{P}_a(\mu)$.

A solution to (4) could be found doing and exhaustive search on both μ and $\mathbf{P}_a(\mu)$. However, the exhaustive search in $\mathbf{P}_a(\mu)$ is practically unfeasible. This is true even supposing to quantize the sub-channel power in a finite number L of levels. In fact, for each μ the exhaustive search would have a complexity $O(L^M)$.

IV. Sub-optimal Power Allocation

If we treat the interference as noise not affected by the input power distribution, conventional power allocation solutions for Gaussian parallel channels can be applied. This is because for a certain value of μ, problem (4) becomes convex and its solution corresponds to the water-filling power distribution [6]. Besides water-filling (referred to as true water filling), we can also apply the constant power water-filling proposed by Chow [22], and uniform power allocation at a certain level as specified by a PSD constraint.

These three algorithms are clearly sub-optimal in our context but an improvement can be found if we apply water filling in an iterative fashion. Before describing in detail the proposed iterative water filling algorithm, we briefly recall the basic power allocation strategies.

A. True Water-Filling

Let us fix μ, then the Lagrangian associated to the primal problem (4) is

$$\begin{aligned} L(\mathbf{P}_a(\mu),\lambda,\nu) = \\ -\frac{1}{NT}\sum_{k\in\mathbb{K}_{ON}} \log_2\left(1+\frac{P_a^{(k)}(\mu)|H^{(k)}(\mu)|^2}{P_{eq}^{(k)}(\mu)}\right) \\ +\lambda\left(\sum_{k\in\mathbb{K}_{on}} P_a^{(k)}(\mu)-\overline{P}\right)-\sum_{k\in\mathbb{K}_{on}}\nu_k P_a^{(k)}(\mu). \end{aligned} \quad (7)$$

Consequently, we can define the dual problem as

$$g(\lambda,\nu)=\inf_{\mathbf{P_a}(\mu)} L(\mathbf{P_a}(\mu),\lambda,\nu). \quad (8)$$

Now, observing that under the hypothesis of an interference plus noise term independent from the transmitted power, the primal (4) is convex, it is easy to show that the primal and the dual optimal solutions, namely $\mathbf{P}_a^*(\mu)$ and (λ^*,ν^*) can be obtained satisfacting the Karush-Kuhn-Tucker conditions [21], i.e.,

$$\begin{aligned} P_a^{(k)}(\mu)\geq 0,\ k\in\mathbb{K}_{on}, \\ \sum_{k\in\mathbb{K}_{on}} P_a^{(k)}(\mu)-\overline{P}=0, \\ \nu_k\geq 0,\ k\in\mathbb{K}_{on}, \\ -\nu_k P_a^{(k)}(\mu)=0,\ k\in\mathbb{K}_{on}, \\ \lambda-\nu_k-\frac{|H^{(k)}(\mu)|^2}{NT\ln(2)\left(P_{eq}^{(k)}(\mu)+P_a^{(k)}(\mu)|H^{(k)}(\mu)|^2\right)}=0. \end{aligned} \quad (9)$$

Where, the first and the second conditions represent the constraints on the transmitted power, and the third imposes the lagrangian multipliers associated with the inequality to be non negative. The fourth condition is the so called slackness condition, it implies that if $P_a^{(k)}(\mu)>0$ then $\nu_k=0$. Viceversa, if $P_a^{(k)}(\mu)=0$ then $\nu_k>0$. The last condition has been obtained computing the gradient of (7) w.r.t. $P_a^{(k)}(\mu)$ and setting it equal to zero.

It is easy to show that the solution to (9) is given by

$$P_a^{(k)}(\mu)=\begin{cases}\overline{\lambda}-\frac{P_{eq}^{(k)}(\mu)}{|H^{(k)}(\mu)|^2} & \text{if } \overline{\lambda}\geq P_{eq}^{(k)}(\mu)/|H^{(k)}(\mu)|^2,\\ 0 & \text{if } \overline{\lambda}< P_{eq}^{(k)}(\mu)/|H^{(k)}(\mu)|^2,\end{cases} \quad (10)$$

with

$$\overline{\lambda}=1/\left(\lambda NT ln(2)\right), \quad (11)$$

and

$$\lambda=\frac{|\mathbb{K}_{on}|}{NT ln(2)}\left(\overline{P}+\sum_{k\in\mathbb{K}_{on}}\frac{P_{eq}^{(k)}(\mu)}{|H^{(k)}(\mu)|^2}\right)^{-1}. \quad (12)$$

In (12), $|\mathbb{K}_{on}|$ indicates the cardinality of $\mathbb{K}_{on}$.

Equation (10) is the water-filling power distribution. This is because if we think to the inverse of the SNRs across sub-channels ($P_{eq}^{(k)}(\mu)/|H^{(k)}(\mu)|^2$) as a bowl, then we can think to pour such a bowl with water (represented by the powers $P_a^{(k)}(\mu)$) to a certain constant level $\overline{\lambda}$.

We refer to the solution (10) as true water-filling. This is to distinguish (10) from the following constant power water-filling algorithm.

B. Constant Power Water-Filling

When the SNR is high, the sum-log in (5) weakly depends on the optimal power distribution. This observation has allowed Chow [22] to empirically show that a uniform power distribution has negligible loss w.r.t. true water-filling where uniform power distribution means to uniformly distribute the overall power (see second line of (4)) to the sub-channels where true water-filling allocates positive power, i.e.,

$$P_a^{(k)}(\mu)=\begin{cases}\overline{P}/|\mathbb{K}_{on}| & \text{if } \overline{\lambda}\geq P_{eq}^{(k)}(\mu)/|H^{(k)}(\mu)|^2,\\ 0 & \text{if } \overline{\lambda}< P_{eq}^{(k)}(\mu)/|H^{(k)}(\mu)|^2,\end{cases} \quad (13)$$

with $\mathbb{K}_{on}=\left\{k:\overline{\lambda}\geq P_{eq}^{(k)}(\mu)/|H^{(k)}(\mu)|^2\right\}$, and $\overline{\lambda}$ given by (11).

C. Power Allocation with Constraint on the PSD

Another more simple algorithm consists in uniformly distributing the total power over the set of active sub-channels $\mathbb{K}_{on}$. For instance, with a PSD constraint we can allocate to each active sub-channel a power equal to the PSD level. State-of-the-art broadband PLC (BPLC) systems transmit at a constant PSD level equal to -50 dBm/Hz over the 0-37.5 MHz band [3], [19], [20] and some tones are switched off according to a fixed notching mask. This power allocation allows BPLC devices to be compliant with the EN 55022 standard [23]. It is interesting to note that with a PSD constraint the uniform and constant power distribution at the PSD level is optimal only in the absence of interference, which can be easily proved by observing that in the absence of interference capacity is maximized by wasting all available power on each sub-channel.

D. Iterative Water-Filling

The iterative power allocation algorithm that we propose is described by the pseudo-code below.

1) Set the number of maximum iterations N_{it}.
   Set $iteration = 0$.
   Initialize $\mathbb{K}_{on}$ equal to the set of active sub-channels given by the power spectral density mask constraint.
   Uniformly distribute the power across the active sub-channels at a power P_{SD} given by the PSD constraint, i.e., $P_a^{(k)}(\mu, iteration) = P_{SD}$ with $k \in \mathbb{K}_{on}$.
   Compute the $SINR^{(k)}(\mu, iteration)$ with (6).
   Compute the capacity $C(\mu, iteration)$ with (5).
2) $for\ iteration = 1, ..., N_{it}$
   a) Run true water-filling (10) or constant power water-filling (13). It gives back $P_a^{(k)}(\mu, iteration)$ with $k \in \mathbb{K}_{on}$. Update $\mathbb{K}_{on}$ according to the set of active sub-channels computed with one of the water-filling algorithms.
      Compute the $SINR^{(k)}(\mu, iteration)$ with (6).
      Compute the capacity $C(\mu, iteration)$ with (5).
   b) $if\ C(\mu, iteration) < C(\mu, iteration - 1)$
      Set $it_{opt} = iteration - 1$.
      Set $iteration = N_{it}$.
      end

   end
3) Set $\mathbf{P}_a^*(\mu) = \mathbf{P}_a(\mu, it_{opt})$.
   Set $C^*(\mu) = C(\mu, it_{opt})$.

It is worth noting that the algorithm stops if the maximum number of iterations or a local maximum is reached. However, numerical results show that usually the algorithm converges in less than ten steps. This result agrees with the one obtained in [16] where IWF is used to deal with the crosstalk interference typical of DSL multiple access channel.

Now we come back to the initial problem (4). In order to jointly compute the optimal CP and the power distribution, we can run the IWF for different values of CP length, and we can choose the optimal CP length as the value that maximizes capacity, i.e.,

$$\mu_{opt} = \underset{\mu \in \{0,...,L_{ch}-1\}}{\operatorname{argmax}} \{C^*(\mu)\}. \tag{14}$$

In the next section we compare the performance of the proposed algorithm with that obtained adapting the CP length but uniformly allocating the power according to a certain PSD constraint. In this case the optimal CP is obtained as

$$\mu_{opt,PSD} = \underset{\mu \in \{0,...,L_{ch}-1\}}{\operatorname{argmax}} \{C(\mu, 0)\}. \tag{15}$$

In [11] we used metric (15) to show that the CP length that maximizes capacity is shorter than the channel duration. Furthermore, we showed that the adaptation of the CP to the channel realization can significantly improve the system capacity w.r.t. the conventional choice of using a CP as long as the channel duration.

To simplify the complexity given by the exhaustive search done over the values of μ in (14) or in (15), we could use one of the sub-optimal metrics presented in [11], [13]. Therefore, we could first compute the sub-optimal CP length and then run IWF.

V. Numerical Results

A. Channel Model

We consider the statistical PLC channel model presented in [24], which is representative of indoor PLC channels. It synthesizes the channel frequency response according to a multipath model (cf. e.g. [25]) with a finite number of components. The frequency response can be written as

$$G_{\text{ch}}(f) = A \sum_{i=1}^{N_p} g_i \mathrm{e}^{-(\gamma_0 + \gamma_1 f^q) d_i} \mathrm{e}^{-\mathrm{j}2\pi f (d_i / v_p)}\ ,\ 0 \le f \le 1/T, \tag{16}$$

where the number of components N_p is drawn from a Poisson process with average path rate per unit length $\Lambda = 0.2$ $path/m$. The maximal network length is fixed to $L_{\max} = 800$ m. Thus, the interarrival delays d_i follow an exponential distribution with expected value Λ^{-1}. The reflection factors g_i are considered to be uniformly distributed in $[-1, 1]$. The other parameters in (16) have been chosen to fit channel responses from measurement results as follows: $q = 1$, $\gamma_0 = 0.3 \cdot 10^{-2}$, $\gamma_1 = 4 \cdot 10^{-10}$, $v_p = 2 \cdot 10^8$ m/s. The channel impulse response (2) can be obtained in closed form by computing the inverse Fourier transform of (16) when $q = 1$ as shown in [24]. The maximum channel length is $L_{ch}T = 5.57$ μs (209 samples with $T = 1/37.5$ μs), which is very similar to the CP length used in HPAV [3]. The coefficient A is adjusted in such way that the average path loss (PL) at zero frequency equals $\{30, 50, 70\}$ dB.

B. System Parameters

We assume a sampling frequency of 37.5 MHz and transmit in the 2-28 MHz band as it is done by state-of-the-art broadband PLC systems. The number of OFDM sub-channels M is 384, a quarter of the sub-channels used in HPAV [3].

When the power constraint is imposed on the transmitted signal PSD, we consider a PSD mask of -50 dBm/Hz in 2-28 MHz. Whereas, when the constraint corresponds to the total transmitted power, we set $\overline{P}$ equal to the integral of the PSD over the band 2-28 MHz.

We assume white Gaussian noise with a PSD of -140 dBm/Hz, which is typical for indoor PLC scenarios. We set the SNR gap to 9 dB. Since the channel introduces a path loss, with the parameters that we have assumed and taking into account the SNR gap, we obtain three channel scenarios that for a constant transmit PSD are characterized by an average SNR (averaged across the sub-channels) equal to $\{46.6, 26.6, 6.6\}$ dB. In the following we use the term SNR to indicate the average SNR.

C. Baseline System

In order to see whether capacity improvements can be obtained with the proposed joint CP and power allocation algorithms, we define a baseline system. It uses a constant CP duration of 5.57 μs for all channel realizations, and it transmits in each sub-channel a signal whose power is defined by the PSD level of -50 dBm/Hz.

D. Simulation Results

We run simulations for 100 channel realizations and for the three SNR cases. Fig. 1 shows the capacity obtained using the baseline system, the capacity obtained with iterative true water-filling and with constant power water-filling. Furthermore, we show the capacity achieved by transmitting with constant sub-channel power at the PSD mask level and optimizing the CP length, namely $C(\mu_{opt,PSD}, 0)$ (see (15)). The SNR equals 6.6 dB. In Figs. 2 and 3 we show the same capacity comparisons respectively for an SNR of 26.6 dB and 46.6 dB.

From Figs. 1–3, we notice that when the SNR is of {6.6, 26.6, 46.6} dB, the capacity gains given by the joint CP and power allocation w.r.t. the baseline system are respectively equal to: {29%,10%,4%} when using iterative true water-filling, {28%,10%,4%} when using constant power IWF, and {22%,10%,4%} when we transmit at the PSD level over all the used sub-channels and we optimize the CP.

It is worth noting that for high SNRs, the use of the IWF algorithms does not give improvements w.r.t. the case of transmitting at the PSD level across all the sub-channels. Furthermore, in agreement with the result obtained by Chow [22], we can affirm that also for IWF, for high SNRs, the

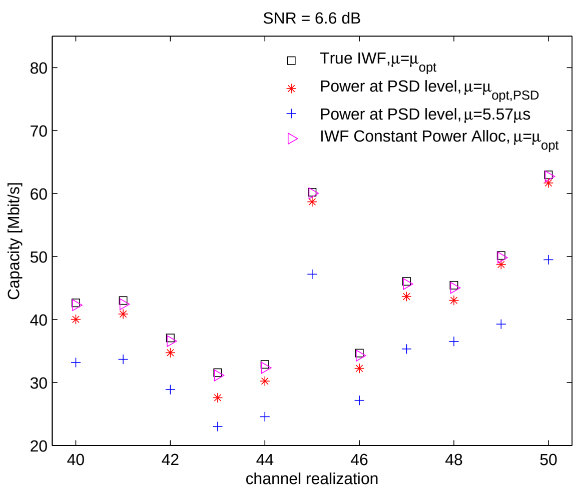

Fig. 1. Capacities according to the different power allocation algorithms presented in Section IV. The SNR is fixed to 6.6 dB. For the sake of readability, only realizations 40 to 50 are shown out of 100.

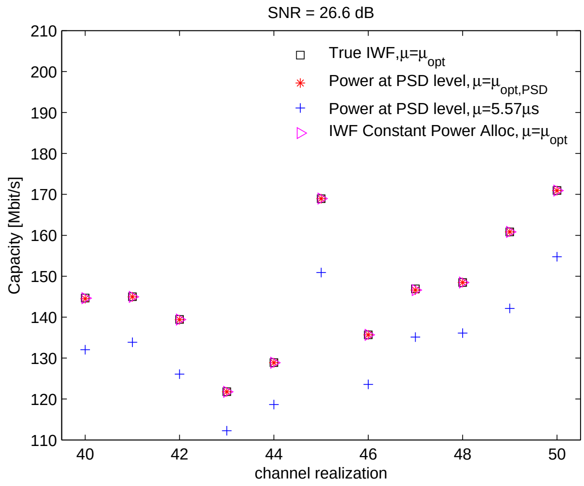

Fig. 2. Capacities according to the different power allocation algorithms presented in Section IV. The SNR is fixed to 26.6 dB. For the sake of readability, only realizations 40 to 50 are shown out of 100.

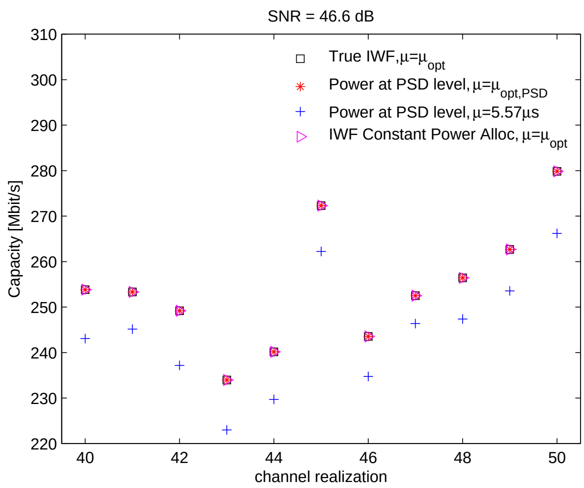

Fig. 3. Capacities according to the different power allocation algorithms presented in Section IV. The SNR is fixed to 46.6 dB. For the sake of readability, only realizations 40 to 50 are shown out of 100.

constant power water-filling has a negligible loss w.r.t. true water-filling.

Fig. 4 shows the optimal CP (14) and (15) cumulative distribution functions (CDFs) for the three SNR values. Regarding (14), the optimal CP-CDF is shown for both the IWF algorithms, i.e., for the true water-filling, and for the constant power water-filling.

As we can see, only for a low SNR, the optimal CP duration depends on the used power allocation algorithm. Furthermore, the higher the SNR the longer the optimal CP is. This is because, for high SNRs, the interference term dominates the SINR denominator. Thus, a long CP maximizes the capacity (5). Consequently, for high SNRs with a long CP, the interference term is small, and therefore, the use of IWF

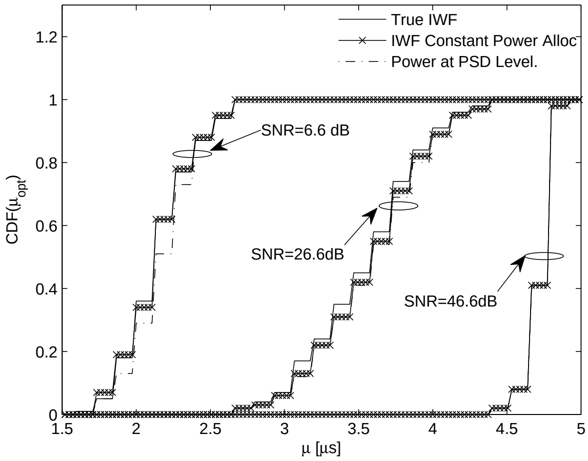

Fig. 4. Optimal CP CDF for the three proposed power allocation algorithms. The SNR equals $\{6.6,26.6,46.6\}$ dB.

is unnecessary.

Summarizing:

- Only for low SNRs, the joint adaptation of power and CP length to the channel realization further improves capacity w.r.t. a uniform power distribution.
- Constant power IWF gives negligible loss w.r.t. true IWF.
- The optimal CP duration is not appreciably dependent on the used power allocation algorithm.

VI. Conclusions

In this paper we have investigated the problem of power allocation and CP length adaptation for the OFDM transmission system. We have shown that, the use of power allocation algorithms based on IWF together with the adaption of the CP can improve the OFDM system capacity. This is especially true for low SNR regions. We have further shown that the optimal CP length does not strongly depends on the power distribution, and that the constant power IWF gives results that are similar to true IWF.

References

[1] IEEE Std.802.11, "Wireless LAN Medium Access Control and Physical Layer Specification," Jun. 2007.

[2] J.A. Bingham, *ADSL, VDSL, and Multicarrier Modulation*, Wiley, 2000.

[3] Home Plug AV White Paper, Version HPAVWP-050818, 2005.

[4] S. B. Weinstein, P. M. Ebert, "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform," *IEEE Trans. Commun. Technology*, vol. 9, no. 5, Oct. 1971.

[5] I. Kalet, "The multitone channel," *IEEE Trans. Commun.*, vol. 37, no. 2, pp. 119-124, Feb. 1989.

[6] T. M. Cover, J. A. Thomas, *Elements of Information Theory*, Wiley, 2006.

[7] J. Seoane, S. Wilson, and S. Gelfand, "Analysis of Intertone and Interblock Interference in OFDM when the Length of the Cyclic Prefix is Shorter than the Length of the Impulse Response of the Channel," *Proc. of IEEE Global Telecomm. Conf. (GLOBECOM)*, Phoenix, USA, Nov. 1997, pp. 32–36.

[8] M. Batariere, K. Baum, and T. Krauss, "Cyclic Prefix Length Analysis for 4G Systems," *Proc. of IEEE Vehicular Technology Conference (VTC Fall)*, Los Angeles, CA, USA, 2004, pp. 543–547.

[9] Z. Zhao-yang and L. Li-feng, "A Novel OFDM Transmission Scheme with Length-Adaptive Cyclic Prefix," *Journal of Zhejiang University SCIENCE*, vol. 2004, pp. 1336–1342, 2004.

[10] V. M. Golubović, Z. R. Petrović, "The Impact of the Cyclic Prefix Length in DMT on the PLC Systems Design," *Proc. of IEEE Int. Conf. on Telecomm. in Modern Satellite, Cable and Broadcasting Services (TELSIKS)*, Serbia, 2007, pp. 347–350.

[11] A. M. Tonello, S. D'Alessandro, L. Lampe, "Bit, Tone and Cyclic Prefix Allocation in OFDM with Application to In-Home PLC," *Proc. of IEEE IFIP Wireless Days 2008*, Dubai, United Emirates, Nov. 2008, pp. 1–5.

[12] S. D'Alessandro, A. M. Tonello, L. Lampe, "Bit-Loading Algorithms for OFDM with Adaptive Cyclic Prefix Length in PLC Channels," *Proc. of IEEE Int. Sym. on Power Line Commun. and its Applications (ISPLC) 2009*, Dresden, Germany, March 29 – Apr. 1 2009, pp. 177–181.

[13] S. D'Alessandro, A. M. Tonello, L. Lampe, "Improving WLAN Capacity via OFDMA and Cyclic Prefix Adaptation," *Proc. of IEEE IFIP Wireless Days 2009*, Paris, France, Dec. 2009.

[14] E. Calvo, J. R. Fonollosa, J. Vidal, "On the Totally Asynchronous Iterative Channel with Single-User Receivers," *Proc. of IEEE Int. Sym. on Inf. Theory (ISIT) 2009*, Seoul, Korea, 2009.

[15] W. Yu, W. Rheem, S. Boyd, J. M. Cioffi, "Iterative Water-filling for Gaussian Vector Multiple Access Channels," *IEEE Trans. Inf. Theory*, vol. 50, no.1, pp. 145–152, Jan. 2004.

[16] W. Yu, G. Ginis, J. M. Cioffi, "Distributed Multiuser Power Control for Digital Subscriber Lines," *IEEE J. on Sel. Areas in Commun.*, vol. 20, no. 5, June 2002.

[17] R. Cendrillon et. al., "Optimal Multiuser Spectrum Balancing for Digital Subscriber Lines," *IEEE Trans. Commun.*, vol. 54, no. 5, May 2006.

[18] S. T. Chung, S. J. Kim, J. Lee, J. M. Cioffi, "A Game-Theoretic Approach to Power Allocation in Frequency-Selective Gaussian Interference Channels," *Proc. of IEEE Int. Sym. on Inf. Theory*, Yokohama, Japan, June 2003.

[19] Universal Powerline Association, "UPA Digital Home Specification White Paper," May 2006.

[20] S.Galli, O. Logvinov, "Recent Developments in the Standardization of Power Line Communications within the IEEE," *IEEE Commun. Mag.*, vol. 46, no. 7, pp. 64-71, July 2008.

[21] S. Boyd, L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[22] W. Yu, J. M. Cioffi, "Constant-Power Waterfilling: Performance Bound and Low-Complexity Implementation," *IEEE Trans. Commun.*, vol. 54, no. 1, Jan. 2006.

[23] M. Tlich, R. Razafferson, G. Avril, A. Zeddam, "Outline about the EMC Properties and Throughputs of the PLC Systems up to 100 MHz," *Proc. of IEEE Int. Sym. on Power Line Commun. and its Applications (ISPLC) 2008*, Jeju Island, Korea, Apr. 2008, pp. 259–262.

[24] A. Tonello, "Wideband impulse modulation and receiver algorithms for multiuser power line communications," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, no. Article ID 96747, 2007.

[25] M. Zimmermann, K. Dostert, "A Multipath Model for the Powerline Channel," *IEEE Trans. on Commun.*, vol. 50, pp. 553–559, April 2002.