Proc. of IFIP-IEEE Wireless Days 2008, Dubai, November 23-37, 2008.

Synchronization for an Efficient Multiuser Filtered Multitone Receiver

Andrea M. Tonello and Francesco Pecile

DIEGM - Università di Udine - Via delle Scienze 208 - 33100 Udine - Italy
e-mail: tonello@uniud.it, francesco.pecile@uniud.it

***Abstract*—We consider the synchronization problem in an uplink multiuser filtered multitone (FMT) modulated system. It differs from OFDMA in the deployment of sub-channel shaping filters. User multiplexing is still accomplished by partitioning the tones among the active users. Users are asynchronous such that the received signals experience independent time offsets, carrier frequency offsets, and multipath fading. We consider the synchronization problem in a novel multiuser receiver that comprises two efficiently implemented fractionally spaced analysis filter banks. In this receiver time/frequency compensation can be jointly done for all users. Despite its lower complexity we show that this receiver approaches the performance of single user FMT.**

***Index Terms*—FMT modulation, Multiuser systems, OFDM, Synchronization, Wireless uplink.**

I. Introduction

We consider the synchronization problem for filtered multitone (FMT) modulation [1] in a multiple access uplink wireless channel [2]. Multiuser FMT differs from orthogonal frequency division multiple access (OFDMA) because it deploys frequency confined sub-channel pulses instead of rectangular time domain pulses that have a sinc frequency response. Then, users are multiplexed by partitioning the tones in an FDMA mode. Multiuser FMT can be efficiently implemented using an inverse fast Fourier transform (IFFT) followed by low-rate sub-channel filtering at the transmitter side [3]. The sub-channel frequency confinement makes multiuser FMT more robust than OFDMA in an asynchronous uplink channel where distinct user signals experience independent time offsets and carrier frequency offsets.

Although the synchronization problem in OFDM is well understood [4], synchronization in FMT systems, and more in general in multiuser FMT has been not extensively investigated. Synchronization involves the estimation of the users' time and frequency offsets as well the estimation of the channel impulse response. In [5] a blind scheme has been considered for synchronization in single user FMT that exploits the redundancy of the oversampled filter bank. In [6] a non-data-aided timing recovery scheme has been proposed.

The synchronization problem depends on the particular receiver structure adopted. Among the conventional receivers we can cite two receiver structures. The first uses an analysis filter bank that is matched to the sub-channels after time and frequency compensation at the sub-channel level. The second uses a filter bank where compensation is done at the user level. For this kind of receivers we proposed in [7] an iterative approach where the analysis filter bank is iteratively matched to the received signals by estimating the time and frequency offsets at its outputs and feeding the estimated parameters back to its input to perform correction.

A drawback of the above receivers is that they have high complexity. To reduce it, we propose in this paper the use of a novel efficient fractionally spaced analysis filter bank that allows detecting all sub-channels of the asynchronous users with lower complexity compared to the traditional single user receiver that requires one synchronous filter bank per user. Compensation of the time offsets and carrier frequency offsets is jointly performed for all the users. The fractionally spaced outputs are processed by sub-channel fractionally spaced RLS equalizers. We then study the synchronization problem in this receiver and we propose a metric for the estimation of the parameters.

II. Multiuser FMT System Model

We consider a multiuser filter bank modulation architecture where multiplexing is performed via partitioning the sub-channels among the users (Fig. 1). We denote with $g(nT)$ the prototype sub-channel pulse, where T is the sampling period. The sub-channel carrier frequency is denoted with $f_k = k/(MT)$, $k = 0,...,M-1$, where M is the total number of tones. It follows that the complex baseband transmitted signal of user u can be written as

$$x^{(u)}(nT) = \sum_{k=0}^{M-1} \sum_{\ell \in \mathbb{Z}} a^{(u,k)}(\ell T_0) g(nT - \ell T_0) e^{j2\pi f_k nT}, \qquad (1)$$

where $a^{(u,k)}(\ell T_0)$ is the k-th sub-channel data stream of user u that we assume to belong to the M-QAM constellation set and that has rate $1/T_0$ with $T_0 = NT \geq MT$. In FMT, the prototype pulse has frequency confined response with Nyquist bandwidth $1/T_0$. The interpolation factor N is chosen to increase the frequency separation between sub-channels and to make the construction of finite impulse response (FIR) pulses easy, thus to minimize the amount of inter-carrier interference (ICI) and multiple access interference (MAI) at the receiver

The work of this paper has been partially supported by the European Community's Seventh Framework Programme FP7/2007-2013 under grant agreement n° 213311, project OMEGA – Home Gigabit Networks.

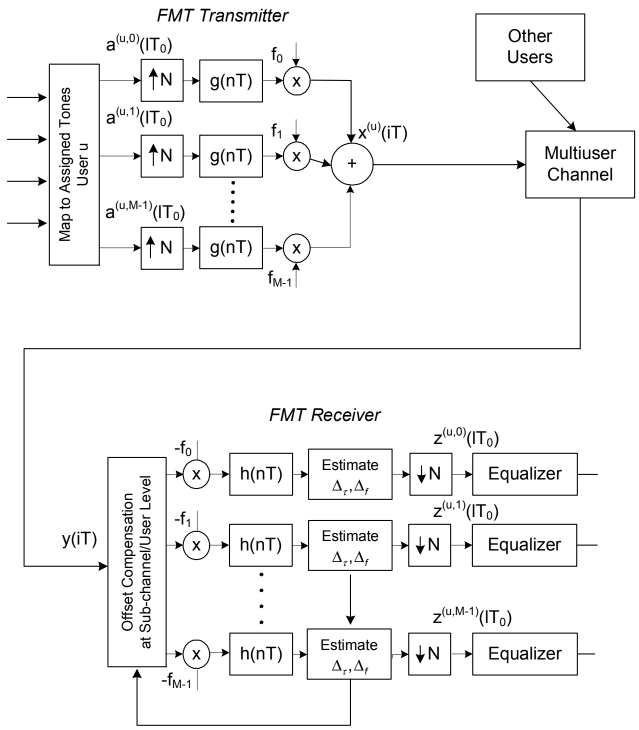

Fig. 1. Multiuser FMT system model with transmitter and receiver of user *u*.

side. Distinct FMT sub-channels can be assigned to distinct users. In this case, the symbols are set to zero for the unassigned FMT sub-channels:

$$a^{(u,k)}(\ell T_0) = 0 \quad \text{for} \quad k \notin K_u, \tag{2}$$

where K_u denotes the set of M_u sub-channel indices that are assigned to user u.

At the receiver, the complex discrete time received signal can be written as

$$y(\tau_i) = \sum_{u=1}^{N_U} \sum_{n \in \mathbb{Z}} x^{(u)}(nT) g_{CH}^{(u)}(\tau_i - nT - \Delta_\tau^{(u)}) e^{j\left(2\pi \Delta_f^{(u)} \tau_i + \phi^{(u)}\right)} + \eta(\tau_i), \tag{3}$$

where $i \in \mathbb{Z}$, N_U is the number of users, $\Delta_\tau^{(u)}$ is the time offset of user u that is due to asynchronous transmission and/or propagation delays, $\Delta_f^{(u)}$ and $\phi^{(u)}$ are the carrier frequency and phase offset, $g_{CH}^{(u)}(t)$ is the fading channel impulse response of user u, and $\eta(\tau_i)$ is the additive white Gaussian noise with zero mean contribution. We assume the time/frequency offsets to be identical for all sub-channels that are assigned to a given user.

III. Receiver Structures

The base station has to detect all users' signals that are affected, according to the model in (3) by carrier frequency offsets, and propagation delays, as well as by different dispersive channel impulse responses. In this section we first describe two conventional receiver structures and then a novel multiuser receiver, that allows lowering the complexity, is devised.

A. Sub-channel Synchronized Receiver and User Synchronized Receiver

In the *sub-channel synchronized receiver* (SCS-RX), synchronization is done at sub-channel level. That is, we deploy an analysis filter bank where each sub-channel filter is matched to the transmit pulse and compensates the frequency offset by an amount $\hat{\Delta}_f^{(u)}$, and adjusts the time phase by an amount $\hat{\Delta}_\tau^{(u,k)}$ that may differ across the sub-channels. The outputs are then sampled at rate $1/T_0$ after which data detection is accomplished with linear sub-channel equalization. It should be noted that while the sub-channel frequency offsets are all identical for a given user, the optimal sub-channel sampling phase can vary across the sub-channels. This is because the propagation channel frequency selectivity translates into different sub-channel equivalent impulse responses [6]. For this reason the SCS-RX not only compensates the time offset of a given user, but it also uses an optimal time phase for each sub-channel. The SCS-RX has a significant drawback since it cannot be implemented using an efficient DFT polyphase filter bank as described in [1], [3].

To simplify the complexity of the SCS-RX we can compensate the time offset and the frequency offsets with a common value $\hat{\Delta}_\tau^{(u)}$, and $\hat{\Delta}_f^{(u)}$, for all the sub-channels of a given user. We refer to this receiver as *user synchronized receiver* (US-RX). Now, one efficient analysis filter bank per user can be deployed. Then sub-channel equalization with symbol spaced equalizers is performed. Although the US-RX can be implemented in an efficient way it still requires one filter bank per user. Further, it suffers from a performance penalty compared to the SCS-RX.

B. Fractionally Spaced Multiuser Receiver

To reduce further the complexity and increase the performance we propose a novel architecture that uses only two fractionally spaced analysis filter banks to detect all M signals that are partitioned among the N_U users, as depicted in Fig. 2. We refer to this receiver as *fractionally spaced receiver* (FS-RX). The filter bank outputs are processed with fractionally spaced linear sub-channel equalizers [8], whose coefficients are obtained according to the MMSE criterion. The fractionally spaced equalizer allows using a common time phase for all the users. Fine synchronization is not required. Only synchronization at symbol level is required, and this can be done at the output of the bank of filters. It should be noted that with ideal band limited pulses neither ICI nor MAI is present. However, the use of an inexact sampling phase (as a result of imperfect synchronization) may yield increased sub-channel ISI which has to be handled with equalization. Further, a problem to be solved is the joint compensation of the carrier frequency offsets that differ among the users. This

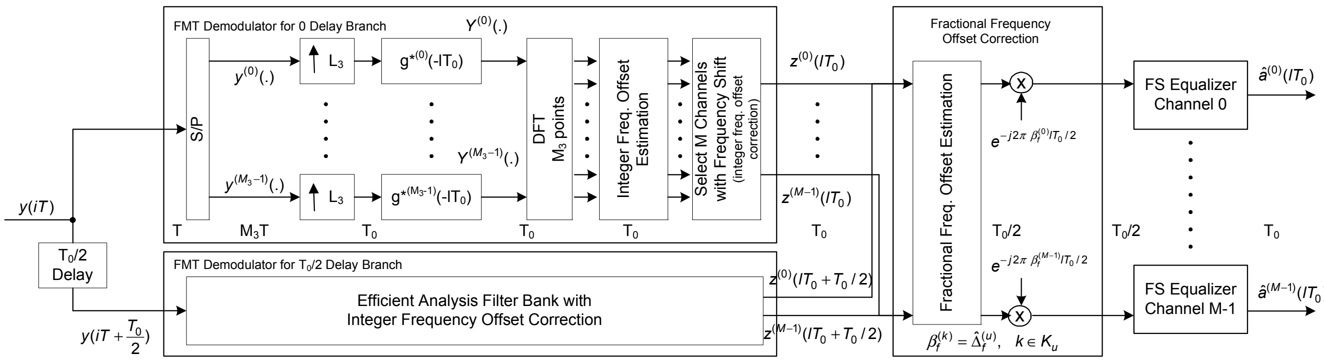

Fig. 2. Fractionally spaced multiuser receiver efficient implementation.

is accomplished by supposing to correct part of the frequency offset before the filter bank (pre-compensation) and part after it (post-compensation).

We define $M_3 = QM_2 = K_3 M = L_3 N$, where Q is a positive integer, and $M_2 = l.c.m(M,N)$ is the least common multiple between M and N. Then, we define the frequency offset as the sum of an integer multiple of $1/M_3T$, and a fractional part $\tilde{\Delta}_f^{(u)}$, i.e., $\Delta_f^{(u)} = q^{(u)}/(M_3T) + \tilde{\Delta}_f^{(u)}$, where we choose $q^{(u)}$ according to the following rule

$$q^{(u)} = \underset{-\lfloor K_3/2\rfloor \le q < \lfloor K_3/2 \rfloor}{\arg\min}\left(\left|\Delta_f^{(u)} - \frac{q}{M_3T}\right|\right), \qquad (4)$$

that corresponds to minimize the fractional frequency offset at the output of the receiver filter bank. In (4) we have assumed $\left|q^{(u)}\right| < \lfloor K_3/2 \rfloor$ such that adjacent FMT sub-channels do not completely overlap as a result of the frequency offset.

Now, the FS-RX pre-compensates the integer part of the frequency offset by the estimated value $\hat{q}^{(u)}/(M_3T)$ before sub-channel filtering, while it post-compensates the fractional part $\tilde{\Delta}_f^{(u)}$ at the sub-channel output where we assume to sample the outputs at rate $2/T_0$. Therefore, the two sub-channel outputs of index $k \in K_u$ belonging to user u are

$$\begin{aligned} z^{(k,\hat{q}^{(u)})}(mT_0+\delta) &= \sum_{i\in\mathbb{Z}} y(iT) g^*(iT - mT_0 - \delta) e^{-j2\pi\left(f_k + \frac{\hat{q}^{(u)}}{M_3T}\right)iT} \\ &= e^{j\left(2\pi\left(\tilde{\Delta}_f^{(u)}+\varepsilon_q^{(u)}\right)mT_0 + \varphi^{(u)}\right)}\Big[a^{(u,k)}(mT_0) g_{EQ}^{(u,k)}(\delta - \Delta_\tau^{(u)}) \\ &\quad + \sum_{\ell \ne m} a^{(u,k)}(\ell T_0) g_{EQ}^{(u,k)}(mT_0 - \ell T_0 + \delta - \Delta_\tau^{(u)})\Big] \\ &\quad + \mathrm{I}^{(u,k)}(mT_0+\delta), \end{aligned} \qquad (5)$$

where $\varphi^{(u)} = 2\pi\left(\left(\tilde{\Delta}_f^{(u)} + \varepsilon_q^{(u)}\right)\delta - f_k \Delta_\tau^{(u)}\right) + \phi^{(u)}$ and $\varepsilon_q^{(u)} = q^{(u)} - \hat{q}^{(u)}$ is the residual error in the correction of the integer part of the frequency offset. The sampling phase δ is set to zero, and to $T_0/2$. The interference term $\mathrm{I}^{(u,k)}(mT_0+\delta)$ comprises noise and ICI and MAI that are small due to the sub-channel spectral containment. The sub-channel equivalent response reads

$$\begin{aligned} g_{EQ}^{(u,k)}(mT_0+\delta-\Delta_\tau^{(u)}) &= \sum_{i\in\mathbb{Z}} g_{CH}^{(u)}(iT) e^{-j2\pi f_k iT} \\ &\times \left[\sum_{n\in\mathbb{Z}} g(nT - iT + mT_0 + \delta - \Delta_\tau^{(u)}) g^*(nT) e^{j2\pi\left(\tilde{\Delta}_f^{(u)}+\varepsilon_q^{(u)}\right)nT}\right]. \end{aligned} \qquad (6)$$

The factor $e^{j2\pi\left(\tilde{\Delta}_f^{(u)}+\varepsilon_q^{(u)}\right)mT_0}$ in (5) introduces a time variant rotation of the constellation, but it can be fully compensated at the sub-channel filter output. The factor $e^{j2\pi\left(\tilde{\Delta}_f^{(u)}+\varepsilon_q^{(u)}\right)nT}$ in the inner sum in (6) cannot be compensated, and it yields a frequency mismatch between the transmitted sub-channel, and the analysis sub-channel filter which is the smallest for $\varepsilon_q^{(u)} = 0$ when the estimation of the integer part of the frequency offset is perfect. Therefore, the pre-compensation of only the integer part of the frequency offset translates in both a sub-channel SNR loss and increased ISI.

The joint correction of the integer part of the frequency offset for all the users can be realized in an efficient receiver implementation. Similarly to the derivation in [3], if we apply the polyphase decomposition to (5) with period M_3T, under the hypothesis of pre-compensating the frequency offset by $\hat{q}^{(u)}/(M_3T)$, we obtain

$$z^{(k,\hat{q}^{(u)})}(mT_0+\delta) = \sum_{i=0}^{M_3-1} Y^{(i)}(mT_0+\delta) e^{-j\frac{2\pi(K_3k+\hat{q}^{(u)})}{M_3}i}, \qquad (7)$$

with

$$Y^{(i)}(mT_0+\delta) = \sum_{\ell\in\mathbb{Z}} y^{(i)}(\ell L_3 T_0 + \delta) g^{(i)*}(\ell L_3 T_0 - mT_0), \qquad (8)$$

$$y^{(i)}(\ell L_3 T_0 + \delta) = y(iT + \ell L_3 T_0 + \delta),\ i = 0,\ldots,M_3-1. \qquad (9)$$

Equations (7)-(8) suggest the scheme in Fig. 2 where each of the two analysis filter banks comprises the following steps:

- Serial-to-parallel conversion, interpolation by a factor L_3, filtering with the polyphase pulses $g^{(i)*}(-mT_0) = g^*(iT - mT_0)$, $i = 0,\ldots,M_3-1$;
- Computation of an M_3-point DFT, and sampling the DFT outputs with index $K_3k + \hat{q}^{(u)}$ for $k \in K_u$. This is to obtain the sub-channel signals of user u, and to partly compensate

the frequency shift introduced by the integer part of the carrier frequency offset;

- Combining the signals in (7) for $\delta=\{0,T_0/2\}$ to obtain a set of M sample streams at rate $T_0/2$;
- Compensation of the fractional frequency offset with multiplication by $e^{-j\pi\tilde{\Delta}_f^{(u)}mT_0}$;
- Finally, a fractionally spaced sub-channel equalizer processes the signals.

It should be noted that the correction of the integer part of the frequency offset is done by choosing the appropriate output tone of the M_3-point DFT (shifted tone). If we increase Q, we reduce the amount of the residual frequency offset $\tilde{\Delta}_f^{(u)}$, at the expense of complexity since the size of the DFT increases. In the FS-RX the parameters to be estimated are $\tilde{\Delta}_f^{(u)}$ and the integer part of the frequency offset.

IV. Synchronization for the Multiuser Receiver

The multiuser analysis filter bank requires the estimation of the integer and fractional frequency offset of each user as well as symbol time synchronization. We use a training approach to estimate the time/frequency offsets. Each user transmits a frame of data that comprises a known training data portion $a_{TR}^{(u,k)}(\ell T_0)$, $k\in K_u$, $\ell=0,...,N_{TR}-1$, i.e., a training sequence per sub-channel. The training sequence is also used to train the MMSE sub-channel equalizer using an RLS algorithm.

We propose a synchronization metric that implements, at the filter bank output, an appropriately defined correlation with the training data. The correlation is done in time (along the time dimension for a given sub-channel) and in frequency (across the sub-channels). The metric is defined as

$$P^{(q)}(n)=\sum_{k\in K_u}\sum_{m=0}^{N_{TR}-K-1} Z^{(k,q)}(mT_0;nT_0/2)^{*}Z^{(k,q)}(mT_0+KT_0;nT_0/2) \quad (10)$$

$$Z^{(k,q)}(mT_0;nT_0/2)=z^{(k,q)}\left(mT_0+nT_0/2\right)\frac{a_{TR}^{(k,q)}(mT_0)^{*}}{|a_{TR}^{(k,q)}(mT_0)|^2}, \quad (11)$$

where $z^{(k,q)}\left(mT_0+nT_0/2\right)$, $k\in K_u$, $|q|<\lfloor K_3/2\rfloor$, is efficiently obtained as in (7). The lag K in (10) is chosen to take into account the presence of the sub-channel ISI, and it has to be such that KT_0 is larger than the sub-channel time dispersion.

Metric (10) is used to jointly estimate the integer part of the frequency offset and the time phase as follows

$$\left(n_{\max}^{(u)},\hat{q}^{(u)}\right)=\underset{n,\,|q|<\lfloor K_3/2\rfloor}{\arg\max}\left\{\left|P^{(q)}(n)\right|^2\right\},\quad u=1,...,N_U. \quad (12)$$

According to (12) we search the peak of the correlation (10) for each of the K_3 possible values of the integer frequency offset. The highest of these peaks, yields both the estimate of $\hat{q}^{(u)}$, and symbol timing $\hat{\Delta}_\tau^{(u)}=n_{\max}^{(u)}T_0/2$.

We emphasize that the estimate of the integer part of the frequency offset $\hat{q}^{(u)}/M_3T$, is used in the efficient implementation (7) by picking for user u the M_3-point DFT outputs of index $K_3k+\hat{q}^{(u)}$ for $k\in K_u$.

Now, (10) can be written as follows using (5)

$$P^{(\hat{q}^{(u)})}(n)=e^{j2\pi\left(\tilde{\Delta}_f^{(u)}+\varepsilon_q^{(u)}\right)KT_0}\sum_{k\in K_u}\left|g_{EQ}^{(u,k)}(nT_0/2-\Delta_\tau^{(u)})\right|^2(N_{TR}-K)+\hat{I}^{(u)}(n), \quad (13)$$

where $g_{EQ}^{(u,k)}(nT_0/2-\Delta_\tau^{(u)})$ is given by (6), which implies that the estimates are chosen to maximize the sum of the energies of the useful data terms. It should be noted that the contribution of ICI and MAI in $\hat{I}^{(u)}(n)$ is small due to the good spectral sub-channel containment. Finally, the fractional frequency offset is estimated as follows

$$\hat{\tilde{\Delta}}_f^{(u)}=\frac{1}{2\pi KT_0}\arg\left\{P^{(\hat{q}^{(u)})}(n_{\max}^{(u)})\right\}. \quad (14)$$

The fractional frequency offset estimation holds for $|\tilde{\Delta}f|<1/(2KT_0)$.

V. Performance Results

Performance is evaluated for an FMT system with 1 and 4 asynchronous users, $M=32$ tones, and interleaved tone allocation. The interpolation factor is $N=40$. The prototype pulse has duration $12T_0$, and it is designed according to the method in [3] to achieve a theoretical bandwidth equal to $1.25/T_0=1/MT$. The carrier frequency offsets are independent and uniformly distributed in $[-\Delta_f^{\max},\ \Delta_f^{\max}]$, while the time offsets are uniformly distributed in $[0,\ T_0]$. The user channels are assumed to be Rayleigh faded with an exponential power delay profile with independent T-spaced taps that have average power $\Omega_p\sim e^{-pT/(0.05T_0)}$ with $p\in\mathbb{Z}^+$ and truncation at -20 dB. The data symbols and the random training sequences of length 20 belong to the 4-PSK signal set. With 20 MHz bandwidth the channel has overall duration equal to 500 ns, and the data rate is 32 Mbit/s.

Average bit-error-rate (BER) is shown in Fig. 3 and Fig. 4. The ideal curves assume perfect synchronization and channel estimation and are plotted for the SCS-RX (used as a reference point for the performance) and the FS-RX. The curves that assume practical synchronization and channel estimation are plotted for the FS-RX in order to show the performance of the synchronization metric proposed in Section IV. RLS training for the sub-channel 3 taps equalizers uses a forgetting factor $\lambda=0.95$. The factor K in the estimator is set equal to 3.

In Fig. 3 the maximum carrier frequency offset normalized to the sub-carrier spacing is $\varepsilon_f=\Delta_f^{\max}MT=0.05$. First we consider the ideal curves. The proposed FS-RX nearly achieves the performance of the SCS-RX. The performance of the FMT system with 4 asynchronous users is close to that of the single user system, which shows the robustness to the

MAI. The curves of the FS-RX with $Q=1$ and $Q=4$ are nearly overlapped.

For the single user case, the practical curves are close to the ideal ones. For the 4 users case the practical curves present a loss that goes from 2 dB to 4 dB with respect to the ideal curves. This discrepancy is mainly due to the estimation of the frequency offsets. Differently from the single user case we have fewer sub-channels per user, so when computing the metric we have a lower redundancy in the frequency dimension.

In Fig. 4 we plot the BER as a function of the maximum carrier frequency offset. Again we first analyze the ideal curves. For $N_U=1$, the SCS-RX has performance that is independent of ε_f since the frequency offset is fully compensated. For $N_U=4$ the SCS-RX exhibits a rapid performance degradation only for $\varepsilon_f>0.04$. That is, the MAI degrades performance only when the carrier frequency offsets exceed ¼ the excess band of the pulse. The proposed FS-RX approaches the performance of the SCS-RX as Q increases. For the practical case, comparing the 1 user and the 4 users case we can notice again the higher gap present in the 4 users case due to the lower performance of the frequency offset estimation.

VI. CONCLUSIONS

In this paper we have proposed a training based synchronization approach for multiuser FMT systems and applied it to a novel fractionally spaced multiuser receiver. It is based on a correlation approach that exploits the separability of sub-channel signals that belong to different uplink users. We have further considered simple RLS adaptive equalization. Numerical results show the effectiveness of the algorithms and their robustness to a wide range of time/frequency offsets and channel frequency selectivity. The proposed solution nearly achieves the performance of the ideal single user receiver.

REFERENCES

[1] G. Cherubini, E. Eleftheriou, S. Ölçer, "Filtered Multitone Modulation for Very High-Speed Digital Subscribe Lines," *IEEE JSAC*, pp. 1016-1028, June 2002.

[2] A. Tonello, "Asynchronous Multicarrier Multiple Access: Optimal and Sub-optimal Detection and Decoding," *Bell Labs Tech. Journal*, vol. 7 n. 3, pp. 191-217, 2003.

[3] A. Tonello, "Time Domain and Frequency Domain Implementations of FMT Modulation Architectures," *Proc. of ICASSP 2006*, Toulouse, vol. 4, pp. 625-628, March 2006.

[4] T. Schmidl, D. Cox, "Robust Frequency and Timing Synchronization for OFDM," *IEEE Trans. Comm.*, pp. 1613-1621, December 1997.

[5] A. Assalini, A. Tonello, "Time-Frequency Synchronization in Filtered Multitone Modulation Based Systems," *Proc. of WPMC '03*, Yokosuka, Japan, pp. 221-225, October 2003.

[6] V. Lottici, M. Luise, C. Saccomando, F. Spalla, "Non-Data-Aided Timing Recovery for Filter-Bank Multicarrier Wireless Communications," *IEEE Trans. on Signal Processing*, vol.54, no.11, pp. 4365-4375, Nov. 2006.

[7] A. Tonello, F. Pecile, "Iterative Synchronization for Multiuser Filtered Multitone Systems," *Proc. of IEEE ISWCS 2005*, Siena, Italy, Sept. 2005.

[8] G. Ungerboeck, "Fractional Tap-Spacing Equalizer and Consequences for Clock Recovery in Data Modems," *IEEE Trans. Comm.*, pp. 856-864, August 1976.

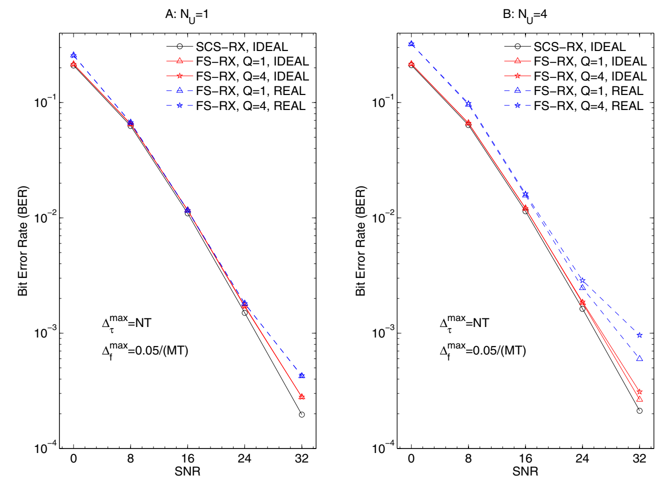

Fig. 3. Average BER as a function of the SNR for 1user and 4 users case. Ideal performance for SCS-RX and FS-RX. Practical performance for the FS-RX.

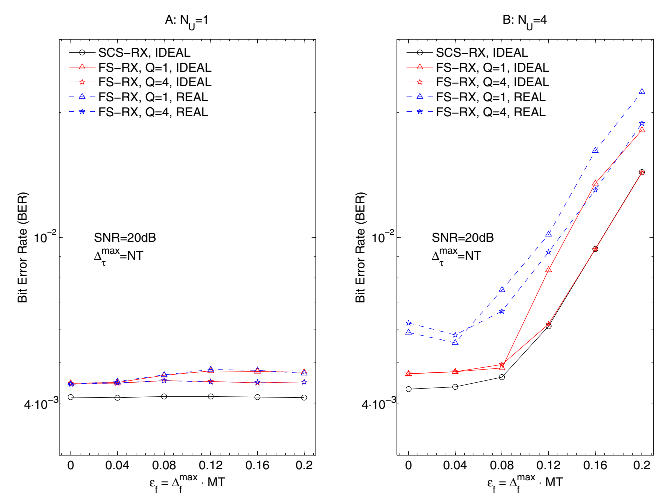

Fig. 4. Average BER as a function of the maximum frequency offset for 1user and 4 users case. Ideal performance for SCS-RX and FS-RX. Practical performance for the FS-RX.