On the Effect of Time and Frequency Offsets in the Uplink of an Asynchronous Multi-user DMT OFDMA System

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Abstract – The effect of time and frequency offsets in the uplink of an asynchronous multi-user system for wireless communications deploying discrete multi-tone modulation and demodulation is studied through analysis. We give analytical expressions for the multiple access interference generated in such a scenario, we study its statistics, and we emphasize its dependency on the tone assignment algorithm used to multiplex the users. Based on our analysis, we also show that an effective way of counteracting these impairments is to insert time and frequency guard intervals, so that a proper trade off with spectral efficiency can be met to optimize system performance.

Introduction

Multiple access schemes based on multi-carrier (MC) modulation [1] offer an interesting solution to the problem of designing systems with high spectral efficiency and high performance for future wireless communications. The key concept of MC modulation is to split the original symbol stream into several parallel streams that are transmitted simultaneously by modulating several carriers [1]. A particular form of MC modulation, known as discrete multi-tone (DMT) or orthogonal frequency division multiplexing (OFDM), allows an easy implementation by using the discrete Fourier transform (DFT). High spectral efficiency is achieved in OFDM since a large number of sub-carriers with overlapping spectra is used. Other advantages of OFDM are the robustness against frequency selective fading, the immunity to inter-symbol interference (ISI), and simplified equalization when a guard time is inserted into the transmitted OFDM symbol [1].

Several approaches are possible in order to combine OFDM with the media access protocol. For instance, OFDM can be combined with TDMA, FDMA, and CDMA. A frequency division approach, named OFDMA, is simply realized by assigning distinct sub-carriers to distinct users [2].

However, the schemes based on OFDM are very sensitive to the time and frequency offsets. In the presence of such offsets, inter-symbol and inter-carrier interference arise and limit the performance of the system. For this reason most of the work on OFDM schemes has been done for the downlink where perfect time and frequency synchronization among users can be obtained [1],[2]. In the uplink, the signals transmitted by users at different distances from the base station are received with different time delays. Furthermore, a frequency offset is present among users whenever their local oscillators are misadjusted and/or movements introduce a frequency Doppler shift.

In this paper we consider an OFDMA based up-link system. We show through analysis that the time and frequency offsets existing in this link jointly induce multiple access interference (MAI), and therefore a penalty in terms of signal to noise plus interference ratio (SINR) on each sub-carrier. Recently, the time offset issue in a similar scenario has been investigated in [3], where however, the frequency offset effect is not considered, and a particular tone assignment algorithm is assumed. An approach to counteract these sources of impairment is to adjust the transmitter starting epoch and the local oscillator frequency at the mobile by a control loop from the base station [4]. However, this strategy might result complicated and not always reliable. Based on our analysis, we show that the MAI power level can be decreased by synchronizing the mobiles to the downlink frame and by inserting appropriate time and frequency guard intervals. Since the MAI is a function of the tone assignment algorithm, the appropriate choice of such an algorithm yields lower interference power levels.

This paper is organized as follows. In section 1 we describe the system model. In section 2 we focus on the receiver structure and we analyze its output. In section 3 we study the statistics of the MAI. Then in section 4, we give and discuss results for two significant tone assignment strategies. Finally, the conclusions follow.

1 System model

We consider an asynchronous multi-user communication system with N_u users, that deploys discrete multi-tone (DMT) modulation in conjunction with frequency division multiple access, frequently referred to as orthogonal frequency division modulation/multiple-access with rectangular pulses, OFDMA. Assigning to the *u-th* user K_u carriers (tones) out of the N available in the system multiplexes the active users.

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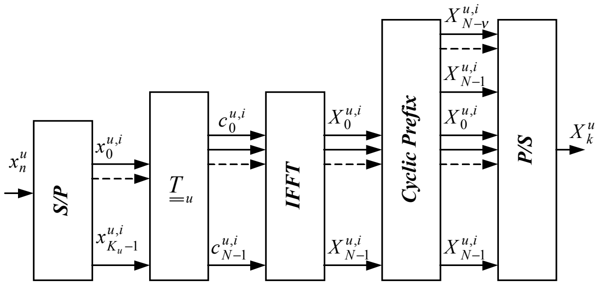

Figure 1: *u-th* user baseband DMT-OFDM transmitter.

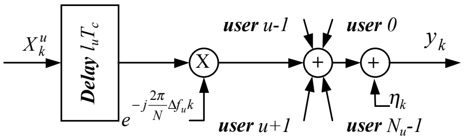

Figure 2: *u-th* user baseband equivalent channel model.

Various tone assignment strategies are possible. In general to minimize the MAI power level distinct tones are assigned to distinct users.

The information bit stream of the *u-th* user is first mapped to complex symbols x_n^u with a spectrally efficient modulation scheme (e.g. M-PSK, M-QAM).

Then, multiplexing and DMT modulation take place as shown in figure 1. After S/P conversion, the *i-th* block of K_u complex symbols $\underline{x}^{u,i} = [x_0^{u,i},\ldots,x_{K_u-1}^{u,i}]^T$ is transformed into a block of N symbols $\underline{c}^{u,i} = \underline{\underline{T}}_u \underline{x}^{u,i}$ by the excitation matrix $\underline{\underline{T}}_u$ of size N by K_u. The excitation matrix is unitary, and is constructed according to a given tone assignment algorithm. Basically, the excitation matrix permutes the elements of $\underline{x}^{u,i}$ into the elements of $\underline{c}^{u,i}$ whose indices correspond to assigned tones, and inserts zeros in correspondence of unassigned tones. A N-point FFT is applied, yielding

$$X_k^{u,i} = \sum_{n=0}^{N-1} c_n^{u,i} e^{j\frac{2\pi}{N}nk} \qquad k = 0,\ldots,N\text{-}1 \tag{1}$$

Hence, a cyclic prefix (guard time) of ν symbols is inserted. The cyclic prefix is typically used to counteract the effect of multi-path fading, and to simplify the equalization task. As it will be shown in what follows, the guard time has also the beneficial effect of reducing the MAI interference level in the presence of asynchronous users. Finally, after P/S conversion the *u-th* user transmits the OFDM modulated symbol stream at rate $F_c=1/T_c = (N+\nu)/T$. After RF conversion, and channel propagation, the signals of all users are superimposed and received at the base station.

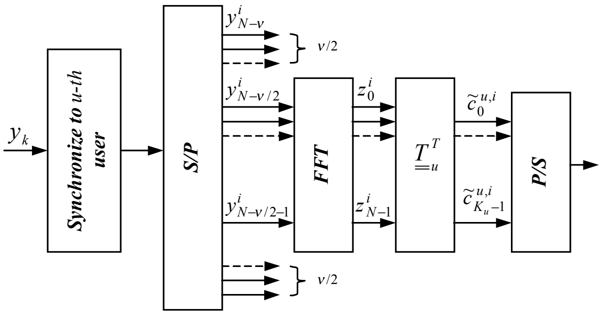

Figure 3: Baseband DMT-OFDM receiver synchronized to user *u*.

In a wireless up-link scenario the users are asynchronous and are received with a time and frequency offset relatively to, say, user *0*. The time offsets originate from different transmission starting epochs and/or different propagation delays. The frequency offsets are due to a mismatch among the local oscillators and/or carrier frequency Doppler shifts arising from movements. At the base station the overall signal is RF down converted, and sampled at rate F_c. Assuming ideal sampling, the overall time and frequency offset effect, in the presence of additive white Gaussian noise, can be represented with the discrete time equivalent channel model in figure 2. In figure 2, $l_u T_c$ represents the relative time delay of the *u-th* user with respect to the *0-th* user, while $\Delta f_u = (f_0 - f_u) N T_c$ is the constant normalized frequency offset between the *u-th* transmitter oscillator at frequency f_u and the local oscillator at frequency f_0.

2 The output from a FFT based receiver

The base station receiver goal is to reconstruct the transmitted information symbol stream of all users. To achieve such a goal we consider a receiver structure obtained by using N_u units identical to the one shown in figure 3 for the *u-th* user. Thus, suppose that we want to demodulate the *0-th* user. First, the receiver acquires time and frequency synchronization with user *0* (that is equivalent to setting $l_0=0$ and $\Delta f_0=0$). Then, it extracts blocks of $N+\nu$ samples, and disregards the first ν/2 and the last ν/2 samples. This operation corresponds to setting a window of N samples starting from the middle of the cyclic prefix. Finally, a N-point FFT is applied. From the FFT output we extract the block of K_u samples belonging to the *0-th* user (with the matrix $\underline{\underline{T}}_u^T$), from which a decision device reconstructs the transmitted information bit stream. The *k-th* sample of the *i-th* OFDM block can be written as:

$$y_k^i = \sum_{u=0}^{N_u-1} e^{-j\frac{2\pi}{N}\Delta f_u k} X_k^{u,i,l_u} + \eta_k \qquad \begin{matrix} i = -\infty,\ldots,\infty \\ k = 0,\ldots,N-1 \end{matrix} \tag{2}$$

where η_k is the thermal noise contribution, while X_k^{u,i,l_u} is the *k-th* element of the *N*-point window shifted by l_u as defined in (3).

$$\ldots, X_{N-1}^{u,i-1}, X_{N-\nu}^{u,i}, \ldots, \left[X_{N-\frac{\nu}{2}}^{u,i}, \ldots, X_{N-1}^{u,i}, X_{0}^{u,i}, \ldots, X_{N-\frac{\nu}{2}-1}^{u,i} \right], \ldots, X_{N-1}^{u,i}, \ldots \qquad \underset{-|l_u|}{\leftarrow} \quad \underset{+|l_u|}{\rightarrow} \tag{3}$$

The *n-th* sub-carrier FFT output is given by:

$$z_n^i = \frac{1}{N}\sum_{k=0}^{N-1} y_k^i e^{-j\frac{2\pi}{N}nk} = c_n^{0,i} + \sum_{u=1}^{N_u-1} \widetilde{c}_n^{\,u,i} + w_n \tag{4}$$

where

$$\widetilde{c}_n^{\,u,i} = \frac{1}{N}\sum_{k=0}^{N-1} X_k^{u,i,l_u} e^{-j\frac{2\pi}{N}(n+\Delta f_u)k} \tag{5}$$

According to (4) and (5) the *n-th* sub-carrier output is the sum of the symbol transmitted by the *0-th* user $c_n^{0,i}$, a noisy term w_n, and a multiple access interference (MAI) term $\sum_{u=1}^{N_u-1} \widetilde{c}_n^{\,u,i}$. Note that (5) is the interference contribution on the *n-th* sub-carrier during the *i-th* OFDM block due to the *u-th* user. It can be shown that the MAI is zero only under the following hypothesis:

- The *n-th* tone is univocally assigned to user *0*.
- All the other users do not experience any frequency offset relatively to user *0*.
- All the other users have a time delay $|l_u| \le \nu/2$.

For all the other cases the MAI differs from zero and constitutes a source of impairment. The statistics of the MAI are analyzed in the next section.

3 Statistics of the MAI

The MAI defined in (5) can be considered, in first approximation, Gaussian distributed. Under the hypothesis of equally likely i.i.d transmitted symbols, the mean is zero. The evaluation of the power is somewhat cumbersome, and for brevity we report only the results.

The power of the MAI due to the *u-th* user on the *n-th* sub-carrier given its time and frequency offsets is given by

$$M_u(n \mid l_u, \Delta f_u) = E[|\widetilde{c}_n^{\,u,i}|^2 \mid l_u, \Delta f_u] = \tag{6}$$

$$\begin{cases} for\ |l_u| \le \nu/2 \\ P_u \displaystyle\sum_{k_u \in \Gamma_u} \frac{\sin^2 \pi(n+\Delta f_u)}{N^2 \sin^2 \frac{\pi}{N}(n+\Delta f_u - k_u)} \\ for\ |l_u| > \nu/2 \\ P_u \displaystyle\sum_{k_u \in \Gamma_u} \frac{1-\cos\pi(n+\Delta f_u - k_u)\cos\frac{\pi}{N}(2|l_u|-\nu-N)(n+\Delta f_u-k_u)}{N^2 \sin^2 \frac{\pi}{N}(n+\Delta f_u - k_u)} \end{cases}$$

where $P_u = E[|c_n^{u,i}|^2]$ (i.e. the *u-th* user has equal power sub-carriers). In (6) $\Gamma_u \in \{0, \ldots, N-1\}$ is the set of K_u tones assigned to user *u*.

Under the hypothesis of zero frequency offset, (6) can be rewritten as

$$M_u(n|l_u, \Delta f_u = 0) = \begin{cases} for\ |l_u| \le \nu/2 \\ 0 \\ for\ |l_u| > \nu/2 \\ P_u \displaystyle\sum_{k_u \in \Gamma_u} \frac{2\sin^2 \frac{\pi}{N}(|l_u| - \frac{\nu}{2})(n-k_u)}{N^2 \sin^2 \frac{\pi}{N}(n-k_u)} \end{cases} \tag{7}$$

If we further assume users with disjoint set of tones (i.e. $\Gamma_{u'} \cap \Gamma_{u''} = \emptyset$), we can define the overall signal-to-interference power ratio on the *n-t*h sub-carrier belonging to the *0-th* user, as

$$SIR_{u=0}(n \mid \underline{l}, \underline{\Delta f}) = P_0 \left(\sum_{u=1}^{N_u-1} M_u(n \mid l_u, \Delta f_u) \right)^{-1} \tag{8}$$

Similarly, the signal-to-noise-plus-interference power ratio is defined as

$$SINR_{u=0}(n \mid \underline{l}, \underline{\Delta f}) = P_0 \left(\sigma^2 + \sum_{u=1}^{N_u-1} M_u(n \mid l_u, \Delta f_u) \right)^{-1} \tag{9}$$

where we have assumed $E[|w_n|^2] = \sigma^2$. Finally, averaging over the set of tones assigned to user *0*, we get the average SIR and SINR seen by the *0-th* user

$$\overline{SIR}_{u=0}(\underline{l}, \underline{\Delta f}) = \frac{P_0}{K_0} \sum_{n \in \Gamma_0} \left(\sum_{u=1}^{N_u-1} M_u(n \mid l_u, \Delta f_u) \right)^{-1} \tag{10}$$

$$\overline{SINR}_{u=0}(\underline{l}, \underline{\Delta f}) = \frac{P_0}{K_0} \sum_{n \in \Gamma_0} \left(\sigma^2 + \sum_{u=1}^{N_u-1} M_u(n|l_u, \Delta f_u) \right)^{-1} \tag{11}$$

4 Analysis of the MAI for some particular cases

It should be noted that (6)-(11) are a function of the particular tone assignment criteria. We herein consider two particular cases. In the first case we regularly interleave the tones of distinct users across the overall set of *N* tones. We refer to this scheme as *interleaved tone assignment* (figure 4). In the second case we assign disjoint blocks of K_u contiguous tones to each user. We refer to this second scheme as *block tone assignment* (figure 5). Assuming K_u=*K*, N_u=*N/K* (i.e. users with equal number of tones), the set of tone indices assigned to user *u∈{0,...,N_u-1}* in the above schemes can be expressed respectively as

$$k_{u,i} = u + \frac{N}{K}i \quad (12) \qquad k_{u,i} = uK + i \quad (13) \qquad i = 0, \ldots, K-1$$

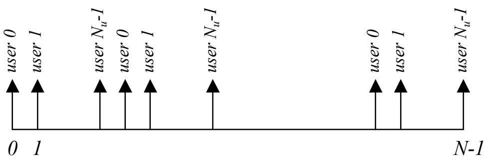

Figure 4: Interleaved tone assignment.

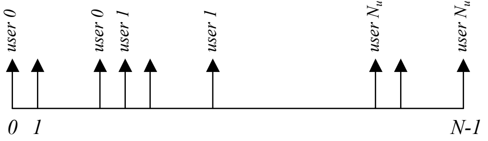

Figure 5: Block tone assignment.

In order to quantify the MAI power or equivalently the SIR, we consider a system characterized by $N=1024$ and $K=64$. We are interested on evaluating the power of the MAI generated by one active user on all the sub-carriers but the ones assigned to it. Basically this user acts as an interferer on all the sub-carriers that do not belong to its set of tones. We further consider both cases when this single interferer uses the set of tones defined in (12) or (13) (with $u=1$).

In general, when this interferer has both a time and a frequency offset relatively to the user that we are demodulating, equation (6) applies.

First, in figure 6 we plot the SIR (with $P_u=1$) on the *n-th* sub-carrier as a function of the frequency offset when the time offset is smaller than half the guard time. Dashed curves are for the interleaved tone assignment, while solid curves are for the block tone assignment.

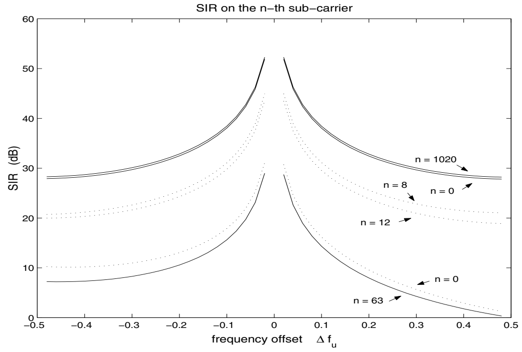

Figure 6: SIR by a single user on the *n-th* sub-carrier as a function of the normalized frequency offset, with time offset smaller than the guard interval. Solid lines: block tone assignment scheme. Dashed lines: interleaved tone assignment. *N*=1024, *K*=64.

Note that when $|l_u| \le \nu/2$ and we consider (12), equation (6) can be evaluated in closed form, yielding

$$M_u(n \mid l_u, \Delta f_u) = \frac{P_u}{N_u{}^2} \frac{\sin^2(\pi(n - k_{u,0} + \Delta f_u))}{\sin^2(\frac{\pi}{N_u}(n - k_{u,0} + \Delta f_u))} \quad (14)$$

The worst case in terms of SIR occurs on the sub-carrier n that is adjacent to a sub-carrier belonging to the interfering set of tones (as figure 6 also confirms).

Considering the block tone assignment, we report in figure 7 the MAI as a function of the sub-carrier for several values of the normalized frequency offset.

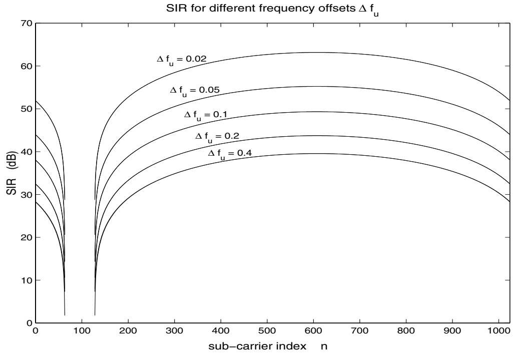

Figure 7: SIR by a single user for a given normalized frequency offset as a function of the sub-carrier, with time offset smaller than the guard interval. Block tone assignment scheme, *N*=1024, *K*=64.

Now, assuming no frequency offset, equation (7) applies. In figure 8 we plot the SIR evaluated on different sub-carriers as a function of the time offset. Again the worst case in terms of SIR is seen to occur on an adjacent sub-carrier *n*.

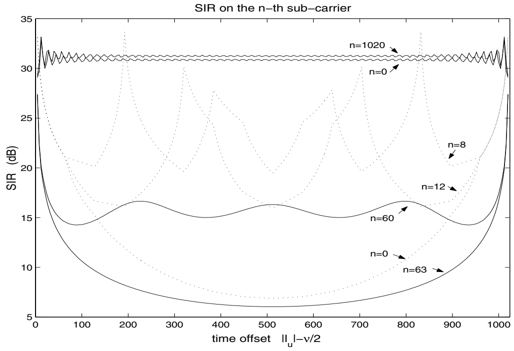

Figure 8: SIR by a single user on the *n-th* sub-carrier as a function of the time offset exceeding the guard interval when there is no frequency offset. Solid lines: block tone assignment scheme. Dashed lines: interleaved tone assignment. *N*=1024, *K*=64.

It is interesting to observe that for the interleaved tone assignment case, as long as $|l_u| - \nu/2 \le K_u$, the MAI power in the absence of frequency offset is linearly increasing with the time delay and does not depend on the sub-carrier index *n*, as expressed by

$$M_u(n|l_u, \Delta f_u = 0) = 2 P_u K_u (|l_u| - \nu/2)/N^2 \quad (15)$$

Finally, we examine the joint effect, on an adjacent sub-carrier (worst case), of non-zero frequency offset and time offset larger than the guard time. Looking at figure 9 and figure 10, both assignment strategies exhibit strong penalties on the SIR, with a slight advantage for the interleaved case. On the other hand if the block strategy is deployed the SIR can be greatly bettered by turning off just the first tone in each block (figure 11).

From the above analysis, it follows that while we can suppress the detrimental effects of the time offset by an

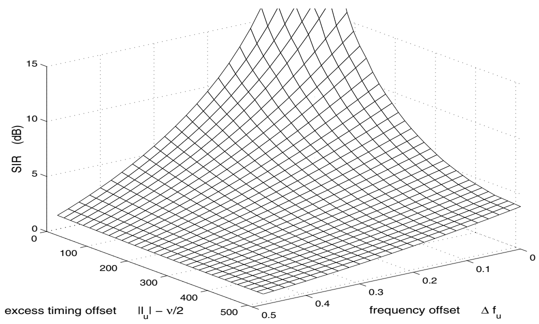

Figure 9: SIR by a single user on an adjacent sub-carrier (worst case) as a function of the time offset exceeding the guard interval and the normalized frequency offset. Interleaved tone assignment scheme, *N*=1024, *K*=64.

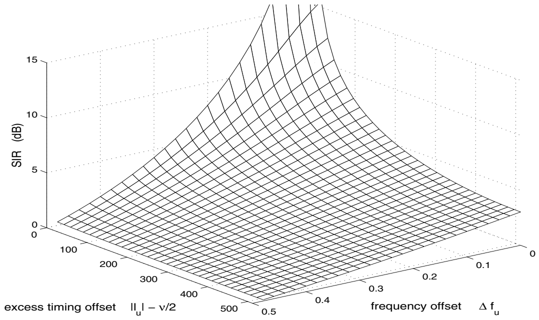

Figure 10: SIR by a single user on an adjacent sub-carrier (worst case) as a function of the time offset exceeding the guard interval and the normalized frequency offset. Block tone assignment scheme, *N*=1024, *K*=64.

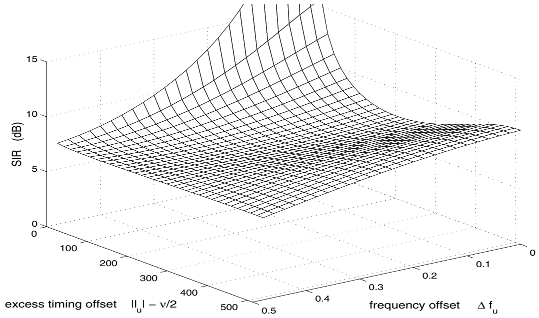

Figure 11: SIR by a single user on an adjacent sub-carrier (worst case) as a function of the time offset exceeding the guard interval and the normalized frequency offset. Block tone assignment scheme with frequency guard, *N*=1024, *K*=64. The interferer does not transmit on the first tone (i.e. tone index 64).

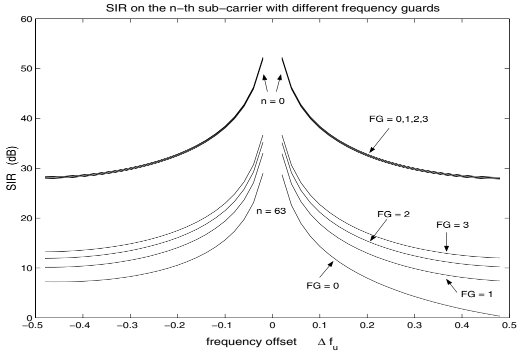

Figure 12: SIR by a single user on the *n-th* sub-carrier as a function of the normalized frequency offset, with time offset smaller than the guard interval. Block tone assignment scheme, *N*=1024, *K*=64, and different number of unused tones at the lower end of each block.

appropriate time-guard, the frequency offset effect can not be entirely suppressed. However, if we deploy a block tone assignment algorithm, the insertion of a frequency-guard between blocks belonging to distinct users can significantly lower the interference. Equivalently we can turn off some of the tones at the beginning of each block. In this way trading off with the spectral efficiency can optimize the system performance. The SIR gain with such an approach is shown, for instance, in figure 12.

Conclusions

In this paper we have investigated the effect of both time and frequency offsets on the uplink of an OFDM based multi-user communication system. We have shown that multiple access interference arises whenever the users are not synchronous. We have derived a general expression for the MAI power, showing its dependency on the tone assignment algorithm. The MAI due to time offsets can be completely removed by inserting a guard time longer than the maximum delay. We have then evaluated the MAI power for an assignment scheme based on interleaving the users' tones among the overall band, and for an assignment scheme based on allocating disjoint blocks of contiguous tones to the users. For both scenarios the worst effect is seen on the sub-carrier that is closest to an interfering tone. However, for the latter case, using a frequency-guard between adjacent blocks of tones can significantly lower the MAI due to the frequency and the excess time offset. Thus, for this scenario the combined use of time-guards, frequency-guards, and spectral efficiency trade off, can help optimizing the system performance.

Finally, we point out that the effect of a frequency selective fading channel can also be incorporated into our analysis, and this is part of a forthcoming paper [5].

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