CYCLIC BLOCK FMT MODULATION FOR COMMUNICATIONS IN TIME-VARIANT FREQUENCY SELECTIVE FADING CHANNELS

Andrea M. Tonello[(1,2)] *and Mauro Girotto*[(1)]

[(1)]University of Udine, Wireless and Power Line Communications Lab
[(2)]WiTiKee s.r.l., Udine, Italy
e-mail: {tonello, mauro.girotto}@uniud.it
web: www.diegm.uniud.it/tonello/wiplilab, www.witikee.com

ABSTRACT

We present a novel multi-carrier modulation scheme for communications over wireless fading channels. The scheme is based on the filter bank modulation concept, it offers sub-channel frequency confinement and it enjoys an efficient frequency domain implementation. Differently from more conventional filter bank approaches, this scheme uses circular convolutions instead of linear convolutions in the filtering operations. We refer to this scheme as Cyclic Block Filtered Multitone (CB-FMT) modulation. We study the performance of CB-FMT when transmission is over frequency selective time-variant fading channels both in terms of signal-to-interference power ratio (SIR) and in terms of achievable rate. A simple adaptive frequency domain equalizer is proposed. The comparison with the conventional OFDM scheme shows that CB-FMT can enjoy superior SIR and achievable rate performance yet requiring similar complexity.

Index Terms— multi-carrier modulation, time-variant channels, frequency selective fading, equalization.

1. INTRODUCTION

Nowadays, the demand for broadband communications services is growing. Most of broadband systems are currently deploying multi-carrier modulation in the form of orthogonal frequency division multiplexing (OFDM) [1]. Other forms of multi-carrier modulation are under investigation. The aim is to provide higher spectral efficiency and robustness than OFDM, especially in channels with high frequency selectivity, high time selectivity (due to mobility, Doppler effects, carrier frequency offsets and phase noise) and with asynchronous multiple users [2]. The basic concept behind multi-carrier modulation is to split the high data rate information signal into a series of parallel low data rate signals [3]. This technique reduces the equalizer complexity. Filtered Multitone (FMT) modulation, originally proposed for digital subscriber lines [4], is a form of multi-carrier modulation. Compared to OFDM, FMT uses sub-channel filtering to provide a higher sub-channel frequency confinement. In general, to achieve

a high sub-channel pulse length, long pulses have to be deployed. The complexity grows with the frequency selectivity, despite efficient DFT polyphase realizations have been devised [5], [6].

In this paper, we propose a novel modulation scheme based on the filter bank concept. In this scheme, the filter bank linear convolutions are replaced with circular convolutions. Furthermore, we group the sub-channel symbols into blocks. We refer to it as Cyclic Block Filtered Multitone Modulation (CB-FMT). CB-FMT allows simplifying the prototype pulse design and reducing the complexity w.r.t. conventional FMT. This novel modulation scheme has been proposed in [7] and [8] for powerline communication and time-invariant wireless channels, respectively. In this paper, we consider the transmission on time-variant frequency selective fading channels. A simple adaptive frequency domain equalizer is proposed. Then, we study the performance of CB-FMT in terms of signal-to-interference power ratio (SIR) and of achievable rate. A comparison with OFDM is also made.

This paper is organized as follows. In Section 2, we recall the basics of conventional FMT and OFDM. In Section 3, we introduce the CB-FMT scheme and we describe an efficient frequency domain implementation. In Section 4, we study the equalization for CB-FMT in both time-invariant and time-variant channels. In Section 5, we report SIR and achievable rate performance. Finally, the conclusions follow.

2. MULTI CARRIER MODULATION

In multi-carrier modulation, the available bandwidth is divided into K sub-bands. The original high data rate signal is split into K low data rate sequences, denoted with $a^{(k)}(N\ell)$. each sequence is transmitted on a sub-channel with normalized symbol period N. An equalizer is deployed at the receiver to mitigate the inter-symbol interference (ISI) and inter-channel interference (ICI) that may be present due to the communication medium.

2.1. FMT Scheme

In a Filtered Multitone Modulation scheme, each sub-channel signal is obtained by filtering the low-data rate sequence with a prototype pulse $g(n)$. Before filtering, the $a^{(k)}(N\ell)$ sequences are interpolated by a factor N. Each sub-channel signal is translated in the frequency domain by a complex exponential multiplication. The resulting sub-channel signals are summed to obtain the signal $x(n)$. This signal is transmitted over the communication medium. $x(n)$ can be expressed as

$$x(n) = \sum_{k=0}^{K-1} \sum_{\ell \in \mathbb{Z}} a^{(k)}(N\ell) g(n - N\ell) W_K^{-nk}, \qquad (1)$$

where $W_K^n = e^{-i2\pi n/K}$ is the complex exponential. The discrete time received signal can be expressed as

$$y(n) = x * g_{ch}(n) + \eta(n), \qquad (2)$$

where $*$, $g_{ch}(n)$ and $\eta(n)$ are the linear convolution operator, the discrete time equivalent channel impulse response and the background white Gaussian noise, respectively. The receiver is based on an analysis filter bank. The signal in (2) is multiplied by a complex exponential. This operation translates each sub-channel signal in base-band. These signals are filtered with $h(n) = g^*(-n)$ and finally sampled by N. The output of the k-th sub-channel can be expressed as

$$z^{(k)}(Nn) = \sum_{\ell \in \mathbb{Z}} y(\ell) W_K^{\ell k} h(Nn - \ell). \qquad (3)$$

Finally, sub-channel equalization takes place to recover the data streams.

2.2. OFDM Scheme

OFDM is obtained when $N = K$ and the prototype pulse $g(n) = h(-n)$ is a rectangular window, i.e., $g(n)$ is equal to 1 for $n \in \{0, \ldots, N-1\}$ and 0 otherwise. Under these assumptions, the transmitted signal in (1) can be efficiently implemented with a K-points Inverse-Discrete Fourier Transform (IDFT)

$$x(n) = \frac{1}{K} \sum_{k=0}^{K-1} a^{(k)}(N\ell) W_K^{-nk}, \quad N\ell \le n < (\ell+1)N. \quad (4)$$

Before the transmission, the signal in (4) is extended with a cyclic prefix (CP) of μ samples. This operation allows transforming the linear convolution in (2) into a circular convolution. The OFDM receiver discards the CP and, then, it applies a Discrete Fourier Transform (DFT). If the channel is time-invariant and the CP is longer than the maximum time dispersion introduced by the channel, a 1-tap MMSE equalizer can be used. In this case, there is no ISI and ICI.

3. CYCLIC BLOCK FMT MODULATION

The idea behind the Cyclic Block Filtered Multitone Modulation (CB-FMT) scheme is to join the implementation efficiency of OFDM with the sub-channel frequency selectivity of FMT. The linear convolution in (1) is replaced with a circular convolution. To perform the circular convolution, we group the low-data rate sequences $a^{(k)}(N\ell)$ in blocks of L symbols. Then, we consider a prototype pulse $g(n)$ of $M_2 = LN$ samples (eventually zero padded). The transmitted signal can then be expressed as follows

$$x(n) = \sum_{k=0}^{K-1} \sum_{\ell=0}^{L-1} a^{(k)}(N\ell) g((n - N\ell))_{M_2} W_K^{-nk}, \quad (5)$$
$$n \in \{0, \cdots, M_2 - 1\},$$

where $g((n - N\ell))_{M_2}$ is the periodic repetition of the prototype pulse $g(n)$ translated by $N\ell$, i.e., $g((n + aM_2))_{M_2} = g(n), a \in \mathbb{Z}$.

At the receiver side, we replace the linear convolution with the circular convolution. Therefore, the k-th sub-channel output can be written as

$$z^{(k)}(Nn) = \sum_{\ell=0}^{M_2-1} y(\ell) W_K^{\ell k} h((Nn - \ell))_{M_2}, \quad (6)$$
$$n \in \{0, \cdots, L-1\},$$

where $h((Nn - \ell))_{M_2}$ denotes the cyclic shift of $h(n)$.

The prototype pulse design, orthogonality considerations, the frequency domain implementation and the complexity analysis of the CB-FMT scheme are reported in [8].

3.1. Frequency Domain Implementation

In this section, we report the necessary basis of CB-FMT frequency domain implementation, shown in Fig. 1. The M_2-points DFT of the transmitted signal in (5) reads as

$$X(i) = \sum_{k=0}^{K-1} A^{(k)}(i - Qk) G(i - Qk), \qquad (7)$$
$$i \in \{0, \ldots, M_2 - 1\}$$

where Q is a constant s.t. $M_2 = LN = KQ$, $A^{(k)}(i)$ and $G(i)$ are the L-points IDFT of the block of data symbols transmitted on the k-th sub-channel, and the M_2-points DFT of the prototype pulse $g(n)$. When the sub-channel prototype pulses are frequency confined, i.e., $G(i) \neq 0$ only for $i \in \{0, \ldots, Q-1\}$, the equation (7) can be simplified as

$$X(i) = A^{(k)}(i - Qk) G(i - Qk), \qquad (8)$$
$$i \in \{Qk, \cdots, Q(k+1) - 1\}, k \in \{0, \cdots, K-1\}.$$

Under this assumption there is no inter-channel interference (ICI) between different sub-channels.
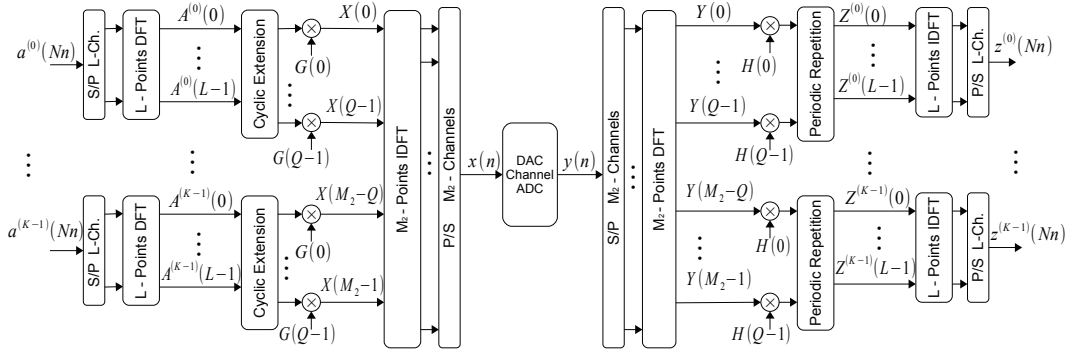
Fig. 1. Frequency domain implementation of the CB-FMT transceiver.

Frequency domain (FD) processing takes place also at the receiver. Thus, we compute a M_2-points DFT of the received signal $y(n)$ and we obtain $Y(p)$. Then, (6) becomes

$$z^{(k)}(Nn) = \sum_{p=0}^{L-1} Z^{(k)}(p - Qk)W_L^{-(p-Qk)n}, \qquad (9)$$

where $Z^{(k)}(p - Qk)$ is expressed as follow

$$Z^{(k)}(p - Qk) = \sum_{q=0}^{N-1} Y(p + Lq)H(p + Lq - Qk). \quad (10)$$

In (10), $H(p)$ is the M_2-points DFT of the prototype pulse $h(n)$. To avoid inter-symbol interference (ISI), we choose $g(n)$ as a Nyquist pulse, i.e., $g*g_-^*(Nm) = \sum_m g(n)g^*(Nm+n)$ differs from zero only for $m = 0$. Finally, we choose $h(n) = g^*(-n)$.

4. EQUALIZATION

The analysis done up to now has shown that CB-FMT is perfectly orthogonal under ideal conditions. Now, we focus on transmission over a time-variant frequency selective channel with response $g_{ch}(l, n)$. The channel is assumed to be a causal time-variant FIR filter with P coefficients. Thus, the discrete time received signal can be expressed as

$$y(n) = \sum_{l=0}^{P-1} g_{ch}(l, n)x(n - l) + \eta(n), \qquad (11)$$

$$g_{ch}(l, n) = \sum_{l=0}^{P-1} \alpha_l(n)\delta(n - l), \qquad (12)$$

where $\delta(n) = 1$ for $n = 0$ and zero otherwise, while $\eta(n)$ is the Gaussian background noise. Clearly, the l-th channel coefficient $\alpha_l(n) = \alpha_l$ is constant when the channel is static.

As shown in Fig. 1, the receiver computes a M_2-points DFT of the received signal. This first receiver stage is similar to that used in OFDM. This suggests to apply a cyclic prefix

on the transmitted signal to transform the linear convolution in (11) in a circular convolution. We insert a cyclic prefix such that $x(n) = x(n + \mu)$ for $n \in \{0, \ldots, \mu - 1\}$, where μ in the CP length in samples. The convolution becomes cyclic if $\mu \geq P - 1$. Under this assumption, after some algebraic manipulation, we can express the M_2-points DFT of $y(n)$ as

$$Y(q) = \sum_{p=0}^{M_2-1} X(p)G_{ch}(p, q - p) + N_\eta(q), \qquad (13)$$

where $X(p)$ is the signal in (8), $G_{ch}(p, q)$ is two-dimensional DFT of the channel response, defined as

$$G_{ch}(p, q) = \sum_{l=0}^{M_2-1} \sum_{n=0}^{M_2-1} g_{ch}(l, n)W_{M_2}^{lp+nq}, \qquad (14)$$

and $N_\eta(q)$ is the M_2-points DFT of the noise samples.

To avoid interference (ISI and ICI) at the receiver output, an equalization process is required. We use a 1-tap subchannel MMSE equalizer [9], with coefficient expressed for q-th DFT bin as

$$H_{EQ}^*(q) = R_{YY}^{-1}(q)R_{YX}(q), \qquad (15)$$

$$R_{YY}(q) = E[Y(q)Y(q)^*], \qquad (16)$$

$$R_{YX}(q) = E[Y(q)X(q)^*], \qquad (17)$$

where $(\cdot)^*$ and $E[\cdot]$ are the complex complex conjugate operator and the expectation operator, respectively.

4.1. Time-Invariant Equalization

When the channel is time-invariant, the channel impulse response is independent from the time instant n. The two-dimensional DFT can be expressed as

$$G_{ch}(p, q) = \begin{cases} \sum_{l=0}^{P-1} \alpha_l W_{M_2}^{lp} & q = 0 \\ 0 & \text{otherwise} \end{cases} \qquad (18)$$

Substituting (18) in (13), we simply obtain

$$Y(q) = X(q)G_{ch}(q, 0) + N_\eta(q). \qquad (19)$$

Thus, when the channel is static and there is no noise, we can perfectly equalize the received signal $Y(q)$ simply dividing it by $G_{ch}(q,0)$ (Zero Forcing equalization). When the noise is present, better performance is obtained with MMSE equalization. Equations (16) and (17) now become

$$R_{YY}(q) = L|G(q)|^2|G_{ch}(q,0)|^2 + M_\eta, \qquad (20)$$

$$R_{YX}(q) = L|G(q)|^2 G_{ch}(q,0), \qquad (21)$$

where M_η is the power of the background white noise. Substituting eq. (20) and (21) in (15), we obtain the equalizer coefficients as

$$H_{EQ}(q+kQ) = \frac{G_{ch}^*(q+kQ,0)}{|G_{ch}(q+kQ,0)|^2 + \frac{M_\eta}{|G(q)|^2}}, \qquad (22)$$

$$q \in \{0,\ldots,Q-1\}, k \in \{0,\ldots,K-1\}.$$

4.2. Adaptive Equalization

When the channel is time-variant the simple expression in (19) is not valid. Thus, to derive the MMSE equalizer, the general expression in (13) has to be considered. We still propose an 1-tap MMSE equalizer. Thus, we need to compute the correlation coefficients in (16) and in (17). After some algebraic manipulations, we obtain

$$R_{YY}(q) = R_{YY}^{(1)}(q) + R_{YY}^{(2)}(q), \qquad (23)$$

$$R_{YY}^{(1)}(q) = L \sum_{p=0}^{M_2-1} |G_1(p)|^2 |G_{ch}(p,q-p)|^2 + M_\eta, \qquad (24)$$

$$R_{YY}^{(2)}(q) = 2L\,\mathrm{Re}\left\{ \sum_{s=0}^{K-1} \sum_{u=0}^{Q-L-1} G(u)G^*(u+L)\times \right.$$
$$\left. \times G_{ch}^{(s)}(u,q-u)\left(G_{ch}^{(s)}(u+L,q-u-L)\right)^* \right\}, \qquad (25)$$

$$R_{YX}(q) = R_{YX}^{(1)}(q) + R_{YX}^{(2)}(q), \qquad (26)$$

$$R_{YX}^{(1)}(q) = L|G_1(q)|^2 G_{ch}(q,0), \qquad (27)$$

$$R_{YX}^{(2)}(q) = \begin{cases} LG(q_1+L)G^*(q_1)G_{ch}(q,L) \\ \qquad\qquad 0 \le q_1 < Q-L \\ LG(q_1-L)G^*(q_1)G_{ch}(q,-L) \\ \qquad\qquad L \le q_1 < Q-1 \\ 0 \qquad\qquad \text{otherwise} \\ \quad (q_1 = q \bmod Q) \end{cases}, \qquad (28)$$

where $G_1(q) = G(q \bmod Q)$ and $G_{ch}^{(s)}(u,q-u) = G_{ch}^{(s)}(u+sQ,q-u-sQ)$. The correlation coefficients in (23) and (26) comprise the sum of two terms. The first term, i.e., that in (24) and in (27), is similar to that in (20) and in (21) obtained for the static channel case. To better understand the second term, i.e., that in (25) and in (28), we may rewrite (8) as

$$X^{(k)}(i) = X(i+kQ) = A^{(k)}(i)G(i), \qquad (29)$$
$$k \in \{0,\ldots,K-1\}, i \in \{0,\ldots,Q-1\}.$$

When $Q > L$ and $L \le i < Q-1$ we have $X^{(k)}(i+L) = A^{(k)}(i+L)G(i+L) = A^{(k)}(i)G(i+L)$. Thus, the coefficients $X^{(k)}(i)$ and $X^{(k)}(i+L)$ are correlated, i.e., $E\left[X^{(k)}(i)\left(X^{(k)}(i+L)\right)^*\right] = E\left[|A^{(k)}(i)|^2\right]G(i)G^*(i+L) \ne 0$. The equalization coefficients take this correlation into account. In the particular condition $Q = L$, this correlation is zero. Thus, the terms (25) and (28) are always null.

5. NUMERICAL RESULTS

To evaluate the performance of CB-FMT in terms of signal-to-interference ratio (SIR) and achievable rate, we assume a time variant channel with coefficients modeled with the Clarke's isotropic scattering model [10]. The coefficients are assumed to be stationary complex Gaussian processes, with zero mean and correlation

$$E[\alpha_l(m)^* \alpha_{l'}(m+n)] = \Omega_{l,l'} J_0(2\pi f_D n), \qquad (30)$$

where $\Omega_{l,l'} = E[\alpha_l(m)^* \alpha_{l'}(m)]$, f_D is the maximum Doppler and $J_0(\cdot)$ is the zero order Bessel function of the first kind. We assume $\Omega_{l,l'} = \Omega_0 e^{-l/\gamma}\delta_{l,l'}$, where $\delta_{l,l'}$, γ and Ω_0 are the Kronecker delta, the normalized delay spread and the power normalization constant, respectively. The channel is truncated at -10 dB for a given γ.

5.1. CB-FMT and OFDM Design Parameters

CB-FMT is compared with OFDM assuming an identical bandwidth $1/T = 20$ MHz. We set the cyclic prefix equal to $0.4\,\mu$s for both systems. Finally, the we set the number of OFDM sub-channels equal to 240. To have comparable complexity [8], we choose for CB-FMT $M_2 = KQ = LN = 240$. As a prototype pulse, we choose the root raised cosine pulse with a roll-off equal to $\beta = 0.25$. Under this assumption, the $Q/L = N/K = \beta + 1$ ratio is equal to $5/4$. The relationship between the parameters is the following: $Q = M_2/K, N = 5/4K, L = M_2/N$. Thus, K is the only degree of freedom. All the parameters must be integer. Thus, $K \in \{4,8,12,16,24,48\}$. Specifically, the final set of chosen parameters is $K = 8, N = 10, L = 24, Q = 30$. For this value of K, the SIR in CB-FMT for a Doppler of $f_D T = 10^{-4}$ is maximized.

5.2. Performance on time-variant channels

Fig. 2.A shows the SIR as a function of the normalized Doppler frequency for $\gamma = 2$. Fig. 2.B shows the SIR as a function of the normalized delay spread γ for $f_D T = 10^{-4}$. The performance of both CB-FMT and of OFDM with the 1-tap equalizer described in Section 4 is reported. In particular, the curve labeled with "time-invariant equalizer" has been obtained with an equalizer whose coefficients are computed according to Section 4.1, i.e., as if the channel were static.
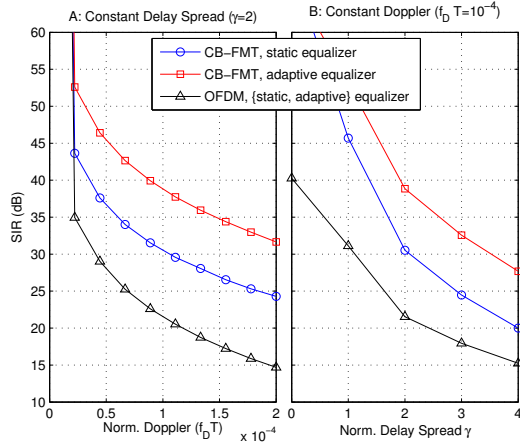
Fig. 2. Average SIR as a function of the normalized Doppler frequency and the normalized delay spread.

Fig. 3. Average achievable rate as a function of the normalized Doppler frequency and delay spread.

In the "adaptive equalizer" curves, the coefficients have been computed according to Section 4.2. The SIR in CB-FMT is in the order of 15 dB better than in OFDM.

Fig. 3 shows the average achievable rate as a function of maximum Doppler assuming Gaussian input data signals (which can be computed using the Shannon capacity formula). Despite the fact that CB-FMT and OFDM have different peak data rates, i.e., $R_{cb-fmt} = 0.8 R_{ofdm}$ with the considered parameters, CB-FMT has higher achievable rate when the adaptive equalizer is used especially at high Doppler and/or delay spread. This is because CB-FMT has higher SIR and exploits the channel diversity.

6. CONCLUSIONS

We have presented a novel multi-carrier scheme referred to as Cyclic Block Filtered Multitone Modulation. The system is a filter bank scheme that uses circular convolutions instead of linear convolutions. An efficient frequency domain implementation has been described. We have focused on the equalization problem in a time-variant frequency selective fading channel and we have proposed a simple 1-tap frequency domain MMSE equalizer. Finally, numerical results have shown that CB-FMT is more robust than OFDM in terms of SIR and it provides higher achievable rate especially at high Doppler and delay spread.

7. REFERENCES

[1] S. Weinstein and P. Ebert, "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform," *IEEE Trans. on Communication Technology*, vol. 19, pp. 628 – 634, October 1971.

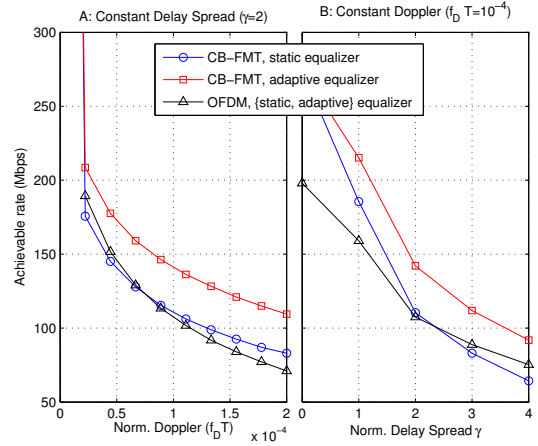[2] A. M. Tonello and F. Pecile, "Analytical Results about the Robustness of FMT Modulation with Several Proto-type Pulses in Time-Frequency Selective Fading Channels," *IEEE Trans. on Wireless Communication*, vol. 7, pp. 1634–1645, May 2008.

[3] J.A.C. Bingham, "Multicarrier Modulation for data Transmission, an Idea whose Time Has Come," *IEEE Commununication Magazine*, vol. 31, pp. 5 – 14, May 1990.

[4] G. Cherubini, E. Eleftheriou, and S. Olcer, "Filtered Multitone Modulation for Very High-Speed Digital Subscribe Lines," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1016–1028, June 2002.

[5] A. M. Tonello, "Time Domain and Frequency Domain Implementations of FMT Modulation Architectures," in *Proc. of IEEE Int. Conf. on Acoustic, Speech, and Signal Processing (ICASSP 2006)*, Toulose, France, May 2006.

[6] N. Moret and A. M. Tonello, "Design of Orthogonal Filtered Multitone Modulation Systems and Comparison among Efficient Realizations," *EURASIP Journal on Advanced Signal Processing*, 2010.

[7] A. M. Tonello and M. Girotto, "Cyclic Block FMT Modulation for Broadband Power Line Communications," in *Proc. of IEEE Int. Symp. on Power Line Communications and Its Applications (ISPLC 2013)*, Johannesburg, South Africa, March 2013, pp. 247–251.

[8] A. M. Tonello, "A Novel Multi-carrier Scheme: Cyclic Block Filtered Multitone Modulation," in *Proc. of IEEE International Conference on Communications (ICC 2013)*, Budapest, Hungary, June 2013, pp. 3856–3860.

[9] Louis L. Scharf, *Statistical Signal Processing*, AddisonWesley, 1991.

[10] G. L. Stuber, *Principles of Mobile Communication*, Kluwer Academic Publishers, 2nd edition, 1996.