Capacity Considerations on the Uplink of a Multi-user DMT OFDMA System Impaired by Time Misalignments and Frequency Offsets

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Abstract: The uplink of an asynchronous DMT/OFDM based communication system is studied and capacity implications are derived. Multiple users share a Gaussian channel through DMT OFDMA modulation and multiplexing. We consider a bank of single user detectors, at the output of which multiple access interference arises whenever time misalignments and frequency offsets exist among users. Consequently, we determine an inner bound to both the capacity of a given user link, and the region of achievable information rates for joint reliable communications (i.e., capacity region). Such capacity inner bounds are random variables and function of several system parameters. The associated complementary cumulative distribution functions are defined, and are evaluated for several system scenarios characterized by different tone assignment schemes and strategies of power allotment to sub-carriers. As a result, a pragmatic approach for enlarging the capacity region is devised. The approach is based on the appropriate insertion of a guard time, the partition of tones to users, and the choice of the power profile to be assigned to the sets of tones.

1. Introduction

Several advantages of multi-carrier modulation and in particular of DMT-OFDM (discrete multi-tone/orthogonal frequency division multiplexing) for wireless communications are well recognized. These include robustness against frequency selective fading, frequency diversity, simple implementation with FFT (fast Fourier transform) based joint modulation and multiplexing [1], [2]. However, a major limit is its sensitivity to time misalignments and frequency offsets that can arise in an asynchronous multi-user uplink scenario. Some investigation of the effect of the time asynchronism can be found in [3]. In [4] and [5] a general analysis of the effects of time and frequency asynchronism in both Gaussian and multi-path fading channels is presented. In particular, in [4], the detrimental effect due to multiple access interference is measured in terms of signal energy over noise plus interference ratio, while in [5] symbol error rate performance is also investigated.

In this paper we address the problem of defining the *achievable information rates* for reliable communications (i.e. with arbitrarily small probability of error) of a set of time and frequency asynchronous users that share a Gaussian channel with an orthogonal frequency division multiplexing scheme. In particular, demodulation is accomplished with a bank of parallel single user detectors, each perfectly time and frequency synchronized to a given user [4], [5].

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Considering N_U users, the determination of the *capacity region*, which is the closure of the set of all achievable rate N_U-tuples, of such a scenario, is in general a formidable task [6],[7]. We determine an *inner bound*, i.e. a subset of the capacity region that can be achieved when deploying the proposed receiver. A trivial *outer bound* is found by considering the perfectly time and frequency synchronous system.

We emphasize that in this scenario the capacity region is determined as a function of several system parameters. Once we have constrained the total number of sub-carriers, and the total power per user, the capacity region still depends on the number of users, the number of sub-carriers per user, the sub-carrier allocation scheme to users, the power allotment to sub-carriers, the deployment of time and frequency guards [4]. Finally, there is a dependency on the time and frequency offsets of each user, which are random variables.

To proceed into the analysis we take the approach of treating the maximum rate for reliable communications of a given user link (link capacity) as a random variable. We fix a maximum time and frequency offset within the system, and then we compute the complementary cumulative distribution functions (ccdf) of the link capacity. These curves show the probability that a user link has a larger capacity than the corresponding abscissa, over all system realizations. The dependency on the tone allocation and power allotment scheme is illustrated by numerically computing, through Monte Carlo simulations, the ccdf for several assignment schemes in a fully loaded system. Among these, a promising dynamic tone allocation scheme yields increased capacity, compared to the conventional block and interleaved tone allocations.

We further consider the joint ccdf of a set of users, from which it possible to determine, for instance, the N_U-tuples of achievable rates with a given probability.

Finally, from the comparison of the capacities achieved with the aforementioned schemes, system design guidelines are devised.

This paper is organized as follows. In Section 1 we revise the OFDM based communication system and the proposed detector [4]. In Section 2 the resulting multiple user interference channel model is described. Sections 3 and 4 discuss the system capacity problem. The evaluation of the ccdfs for several system scenarios is carried out in Section 5. Finally, the conclusions follow.

2. Asynchronous Multi-user Communication System

We consider a communication system where N_U users deploy discrete multi-tone modulation (DMT) and share a Gaussian channel with an orthogonal frequency division multiple access scheme (OFDMA). The overall bandwidth W is subdivided into N equally spaced sub-carriers (tones) among which K_u are assigned to the u-th user. The transmitter of user u is shown in Figure 1. Its discrete time complex transmitted signal x_n^u, n=-∞,...,+∞, is serially to parallel converted into blocks of length K_u, $\underline{x}^{u,i} = [x_0^{u,i},...,x_{K_u-1}^{u,i}]^T$, which are mapped into blocks of length N, $\underline{c}^{u,i} = \underline{T}_u\, \underline{x}^{u,i}$, by the excitation matrix $\underline{T}_u$ of size N by K_u. The excitation matrix is determined by the tone assignment scheme. Basically, it permutes the elements of $\underline{x}^{u,i}$ into the elements of $\underline{c}^{u,i}$ whose indices correspond to the tones assigned to user u, and inserts zeros in correspondence of the other tones.

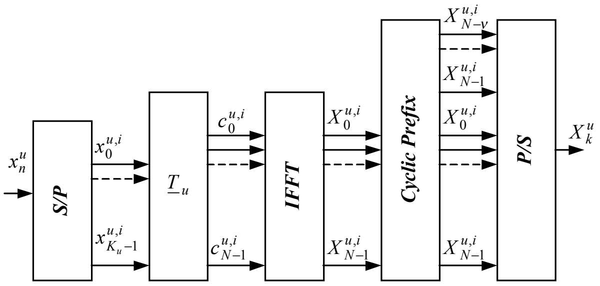

Figure 1. Baseband DMT-OFDMA transmitter of user u.

DMT modulation is implemented through a N-point FFT, yielding

$$X_k^{u,i} = \sum_{n=0}^{N-1} c_n^{u,i} e^{j\frac{2\pi}{N}nk} \tag{1}$$

We consider the insertion of a cyclic prefix (guard time) of ν symbols. As shown in [4], the guard time has the beneficial effect of reducing the MAI interference detrimental effects in the presence of asynchronous users, at the expense of some bandwidth loss.

Finally, the modulated symbol stream is transmitted at rate $W=F_c=1/T_c=(N+\nu)/T$. After RF conversion, and channel propagation, the signals of all users are superimposed and received at the base station.

In a wireless up-link scenario the users are asynchronous and are received with a time and frequency offset relatively to, say, user u'. The time offsets originate from different transmission starting epochs and/or different propagation delays. The frequency offsets are due to a mismatch among the local oscillators and/or carrier frequency Doppler shifts arising from movements. At the base station the overall signal is RF down converted, and sampled at rate F_c. We assume ideal sampling, so that the overall time and frequency offset effect, in the presence of additive white Gaussian noise, can be represented with the discrete time equivalent channel model in Figure 2.

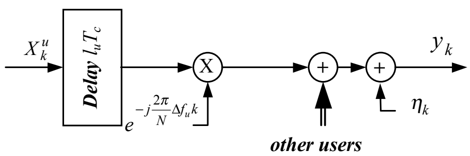

Figure 2. Baseband equivalent channel model for user u.

In Figure 2, $l_u T_c$ represents the relative time delay of the u-th user with respect to the u'-th user, while $\Delta f_u = (f_{u'} - f_u)NT_c$ is the constant normalized frequency offset between the u-th transmitter oscillator at frequency f_u and the local oscillator at frequency $f_{u'}$.

In order to reconstruct the transmitted information symbol stream of all users, we consider a bank of N_U single user detectors identical to the one shown in Figure 3 [4]. Demodulation for the u′-*th* user is accomplished by first acquiring time and frequency synchronization with user u' (that is equivalent to setting $l_{u'}=0$ and

$\Delta f_{u'}=0$). Then, blocks of $N+\nu$ samples are extracted. A window of N samples is set starting from the middle of the cyclic prefix. Finally, a N-point FFT is applied. From the FFT output we extract blocks of $K_{u'}$ decision variables belonging to the *u′-th* user.

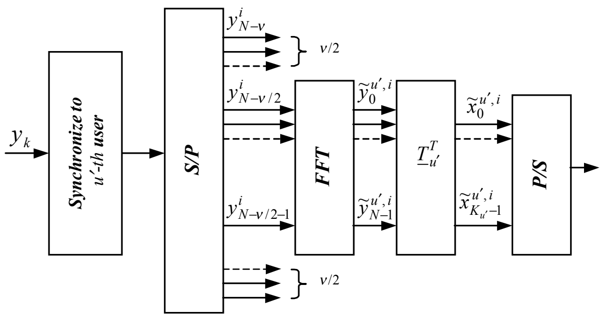

Figure 3. Baseband DMT-OFDMA receiver synchronized to user *u′*.

The *k-th* sample of the *i-th* received block can be written as

$$y^i_k=\sum_{u=0}^{N_U-1} e^{-j\frac{2\pi}{N}\Delta f_u k} X^{u,i,l_u}_k+\eta^i_k \tag{2}$$

where $i=-\infty,...,\infty$, $k=0,...,N-1$, and η^i_k is the thermal noise contribution, while X^{u,i,l_u}_k is the *k-th* element of the *N*-point window shifted by l_u

$$...,X^{u,i-1}_{N-1},X^{u,i}_{N-\nu},...,\left[X^{u,i}_{N-\frac{\nu}{2}},...,X^{u,i}_{N-1},X^{u,i}_0,...,X^{u,i}_{N-\frac{\nu}{2}-1}\right],...,X^{u,i}_{N-1},... \tag{3}$$

$$\underset{-|l_u|}{\leftarrow}\quad\underset{+|l_u|}{\rightarrow}$$

From this point on, we consider the sets of tones that are assigned to distinct users to be disjoint. Thus, let $\Gamma_{u'}=\{n^{u',i}_0,...,n^{u',i}_{K_{u'}-1}\}\subset\{0,...,N-1\}$ be the set of $K_{u'}$ tone indices univocally assigned to user *u′*. Then, the set of decision variables for the *i-th* block of user *u′* is [4]

$$\tilde{y}^{u',i}_n=\frac{1}{N}\sum_{k=0}^{N-1} y^i_k e^{-j\frac{2\pi}{N}nk}=c^{u',i}_n+\sum_{u=0,\,u\neq u'}^{N_U-1}\tilde{c}^{u,i}_n+w^{u',i}_n \tag{4}$$

where $n\in\Gamma_{u'}$ and

$$\tilde{c}^{u,i}_n=\frac{1}{N}\sum_{k=0}^{N-1} X^{u,i,l_u}_k e^{-j\frac{2\pi}{N}(n+\Delta f_u)k} \quad (5) \qquad w^{u',i}_n=\frac{1}{N}\sum_{k=0}^{N-1}\eta^i_k e^{-j\frac{2\pi}{N}nk} \quad (6)$$

Thus, each sub-channel output is the sum of the symbol transmitted on that sub-carrier $c^{u',i}_n$, a noisy term $w^{u',i}_n$, and a multiple access interference (MAI) term

$z_n^{u',i} = \sum_{u \neq u'} \tilde{c}_n^{u,i}$. The MAI term differs from zero whenever at least one of the other users has a time delay $|l_u| > \nu/2$ and/or a frequency offset $\Delta f_u \neq 0$.

3. Multiple-access Interference Channel Representation

In the presence of time and frequency asynchronous users the link of user u' can be modeled as shown in Figure 4, where the indices $n_l \in \Gamma_{u'}$, $l=0,...,K_{u'}-1$.

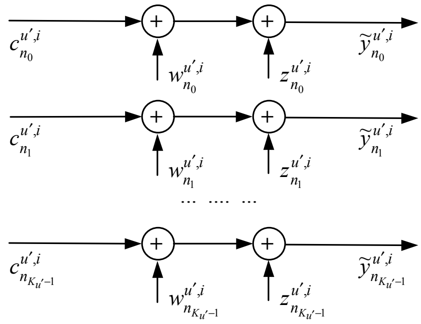

Figure 4. MAI channel model for the link of user u'.

The *u'-th* user transmits its information through $K_{u'}$ parallel channels that experience additive multiple user interference and thermal noise. The thermal noise vector $\underline{w}^{u',i} = [w_{n_0}^{u',i},...,w_{n_{K_{u'}-1}}^{u',i}]^T$ is Gaussian distributed with zero mean and covariance $\underline{K}_{ww}^{u'} = E[\underline{w}^{u',i}(\underline{w}^{u',i})^H] = N_0 \underline{I}$, where $\underline{I}$ is the $K_{u'}$ by $K_{u'}$ identity matrix. In general the MAI is correlated in time (along index *i*) and in frequency (along the sub-carriers). However, we make the assumption of temporal uncorrelation, based on the fact that this is true for time offsets $|l_u| \leq \nu/2$, and is a valid approximation for small time offsets exceeding the guard interval.

Now, for a capacity evaluation standpoint, we constrain to consider input signals with Gaussian distribution that have zero mean and covariance $\underline{K}_{cc}^{u'} = E[\underline{c}^{u',i}(\underline{c}^{u',i})^H]$. Inputs signals of distinct users are considered independent. It follows that the overall MAI is Gaussian distributed since each user generates a MAI contribution that is Gaussian. The mean of the MAI is zero and the covariance is $\underline{K}_{zz}^{u'} = E[\underline{z}^{u',i}(\underline{z}^{u',i})^H]$. In conclusion, the link of user u' can be modeled as $K_{u'}$ additive Gaussian noise channels that are correlated. The time index *i* can be omitted and we can write in vector notation

$$\underline{\tilde{Y}}^{u'} = \underline{C}^{u'} + \underline{Z}^{u'} + \underline{W}^{u'} \qquad (7) \qquad\qquad \underline{Z}^{u'} = \sum_{u=0,\, u \neq u'}^{N_U - 1} \underline{A}(u,u')\underline{C}^{u} \qquad (8)$$

where for instance $\underline{C}^{u'} = [c_{n_0}^{u'},...,c_{n_{K_{u'}-1}}^{u'}]^T$, and $\underline{A}(u,u')$ is the interference matrix of user *u* on user u'. The interference matrix is a function of the system parameters and in particular of the relative time and frequency offset of user *u* with respect to user u'.

If we assume that the input covariance of each user is equal to $\underline{K}^{u}_{cc} = diag(P^{u}_{n_0},...,P^{u}_{n_{K_u-1}})$, and that all users have a time delay $|l_u| \leq \nu/2$ relatively to user u', then the interference covariance seen by user u' computed over carrier indices n and $n+p$ (belonging to $\Gamma_{u'}$) can be easily evaluated, yielding

$$K^{u'}_{zz}(n, n+p) = \frac{1}{N^2} \sum_{\substack{u=0 \\ u \neq u'}}^{N_U-1} \sum_{c \in \Gamma_u} P^{u}_{c} \frac{2 - e^{j2\pi\Delta f_u} - e^{-j2\pi\Delta f_u}}{1 - e^{j\frac{2\pi}{N}(n+p+\Delta f_u - c)} - e^{j\frac{2\pi}{N}(-n-\Delta f_u + c)} + e^{j\frac{2\pi}{N}p}} \quad (9)$$

The result in (9) will be used in the next section for the link capacity evaluation. Note that there is no dependency on the time offset since we have considered delays satisfying $|l_u| \leq \nu/2$. It has to be said that more general expressions can be calculated to include delays $|l_u| > \nu/2$.

4. Capacity Evaluation

The presence of asynchronous users (interferers) induces some degradation on the performance of the single user receiver. The problem we are dealing with is to quantify such degradation as a function of the system parameters. In [4] we have used the signal energy over noise plus interference ratio as a figure for the performance degradation. In this paper we aim to evaluate the capacity of the *u′-th* link, and more in general the capacity region where jointly reliable communications are possible.

Within the model in Section 2, the capacity of the *u′-th* user link is evaluated by maximizing the mutual input-output information $I(\underline{\tilde{Y}}^{u'}; \underline{C}^{u'} | \underline{C}^{0},...,\underline{C}^{u'-1}, \underline{C}^{u'+1}...)$, over all possible input covariance matrices $\underline{K}^{u'}_{cc}$ subject to the trace constraint $trace(\underline{K}^{u'}_{cc}) = \sum_{n \in \Gamma_{u'}} P^{u'}_{n} \leq P$. Since we consider Gaussian inputs, the capacity in bit/s/Hz is obtained as [6]

$$C(u' | \underline{K}^{u'}_{zz}) = \frac{1}{K_{u'}} \max_{\underline{K}^{u'}_{cc}} \left\{ \log_2 \left(\frac{|\underline{K}^{u'}_{cc} + \underline{K}^{u'}_{ww} + \underline{K}^{u'}_{zz}|}{|\underline{K}^{u'}_{ww} + \underline{K}^{u'}_{zz}|} \right) \right\} \quad (10)$$

where $|\underline{A}|$ is the determinant of the matrix $\underline{A}$, and $K_{u'}$ is the number of tones assigned to user u′ [(1)]. Note that (10) is conditioned on the covariance of the interference, which in turn is a function of the input covariance of the interferers, their time/frequency offsets, and their set of assigned tones. For a particular system realization these parameters can be considered constant.

All information rates $R_{u'} \leq C(u' | \underline{K}^{u'}_{zz})$ allow communications with arbitrarily small probability of error over the *u′-th* user link. Further, we do not consider the case where cooperation among decoders of distinct users exists. In this case a way to proceed would be to augment the dimensions of the model in (7)-(8).

To get insight, we constrain the input covariance to be diagonal for all users. With this hypothesis, knowing the frequency offset, and the set of assigned tones of each

(1): The standard capacity formula for real vectors is *(1/2)log₂[.]*. Since we are dealing with Gaussian complex vectors it can be shown that the factor (1/2) vanishes out.

user, (9) gives the covariance matrix of the interference (when $|l_u| \le \nu/2$ for all u). Thus, (10) can be computed, yielding an achievable rate for a given user link. In this case, suppose to assign an equal number of tones to all users, and to distribute equally the power among tones, then the individual link capacities can in general differ. This is due to the fact that each of the links sees on its set of tones a different MAI contribution. In this scenario, the conditional capacity region, which is the closure of the set of all rate vectors $\underline{R}=[R_0,...,R_{N_U-1}]$ that satisfy $R_u \le C(u|\underline{K}_{zz}^u)$ for $u=0,...,N_U-1$, can be thought to be an hyper-parallelepiped. This capacity region (inner bound) is contained into the capacity region (outer bound) defined by all rate vectors whose components satisfy $R_u \le (1/K_u)\log_2 \prod_{n\in\Gamma_u}(1+P_n^u/N_0)$ for $u=0,...,N_U-1$. This outer bound is certainly achieved by the synchronous system where the MAI detrimental effect disappears.

5. Capacity as a Random Variable

In general, even if we constrain the input covariance matrices to be the same for all users, simple meaningful closed expressions that define the conditional capacity region (i.e. conditioned on the time and frequency offsets) cannot be found. This is because the interference covariance in (9) still has a dependency on how we assign the tones to the users. Furthermore, the time and frequency offsets are random parameters. Thus, to proceed we follow the approach of treating the link capacity (i.e. rate that grants reliable communications over a user link) a random variable. We consider the time and frequency offsets of all users independent and equally distributed. Then, we fix the tone assignment scheme together with the power allotment to sub-carriers, and we constrain the input covariance matrices of all users to be diagonal.

Our aim is now to compute the complementary cumulative distribution function (ccdf) of the link capacity. This is defined as the probability that the capacity of a given user is greater than a fixed value under all possible system realizations where all users have a time and frequency offset independently distributed

$$ccdf_{u'}(\overline{C}) = P[C(u') > \overline{C}] \quad (11)$$

If the tone assignment scheme of each user satisfies symmetry rules, the power allotment is also the same, and the system is fully loaded, then the ccdfs in (11) are the same for all users. Thus, rate $R_{u'} = \overline{C}$ is achievable by the *u′-th* link with probability given by (11), and the same applies to the other links, individually considered.

Following the same approach it is possible to define the joint link capacities ccdf

$$ccdf_U(\overline{C}_0,...,\overline{C}_{N_U-1}) = P[C(u=0)>\overline{C}_0,...,C(u=N_U-1)>\overline{C}_{N_U-1}] \quad (12)$$

Now, the capacity region takes on a probabilistic significance. It can be interpreted as the region determined by all rate vectors for which jointly reliable communications are possible with a given probability. This, capacity region is outer bounded by the capacity region that is determined in the absence of MAI (achieved with probability 1 in the synchronous case).

Finally, we emphasize that (11) and (12) still depend upon the tone allocation strategy, and the power allotment to tones.

6. System Scenarios and Capacity Performance Comparison

Evaluation of the ccdfs (11) and (12) was carried out numerically through Monte Carlo simulations. As a result it was possible to evaluate the capacity performance of several systems that differ on the tone allocation and power allotment strategy.

6.1 Single link capacity complementary cumulative distribution functions

We start considering the ccdf of the single user link (i.e., Equation 11). The system under investigation is a fully loaded system scenario characterized by the following parameters: fixed number of overall carriers $N=256$, $N_U=16$ users, $K_u=16$ tones per user. A guard time is inserted such that the MAI effect due to time misalignments is null. Further, the bandwidth penalty is also ignored assuming that N is much larger than the cyclic prefix. Obviously, this assumption depends upon the cell size. However, we could always think of having some degree of centralized control that confines the time offset within the guard time length [8]. Vice versa the frequency offset detrimental effect cannot be completely canceled out by the insertion of frequency guards [4]. Thus, we have considered users with frequency offsets that are independent and uniformly distributed in $(-\Delta f_{\max}, +\Delta f_{\max})$.

In Figures 5 and 6 the users are allocated with an *interleaved tone assignment* scheme [4]. In other words, the tones of distinct users are regularly interleaved across the overall set of N tones. All sub-carriers of all users have the same power. The signal-to-thermal-noise ratio is set to 10 dB and 30 dB and several maximum frequency offsets are considered. We plot a vertical line in correspondence of the capacity that is achieved by the synchronous system. This represents a capacity upper bound. For SNR=10 dB the upper bound is 3.46 bit/s/Hz, while for SNR=30 dB is 9.97 bit/s/Hz.

Now, looking for instance at Figure 5, we set a percentage, say 90%, then we read the minimum capacity we can provide to a user link with that probability over all system realizations. Note that, since the system is fully loaded, the tones are regularly assigned, and the power allotment to sub-carriers is the same, all N_U links have the same ccdf. The same applies in the other schemes described in this section. From Figure 5 it can be seen that the capacity lower bound is about 0.5 bit/s/Hz from the upper bound for 90% of the system realizations when Δf_{max}=0.1 and the SNR=10 dB. The capacity loss dramatically increases to about 2.5 bit/s/Hz for Δf_{max}=0.5. As the SNR increases (Figure 6), the capacity performance is dominated by the MAI, such that the difference between the lower and the upper bound is more pronounced.

In Figures 7 and 8 we consider the same system with, however, a *block tone assignment* where disjoint blocks of K_u contiguous tones are assigned to each user. Fixed the same outage probability (i.e. 10%), the block tone allocation scheme exhibits a strong capacity improvement over the interleaved. For instance, at 30 dB SNR the block scheme provides a minimum capacity ranging from about 8.4 to about 9.1 bit/s/Hz for Δf_{max} ranging from 0.5 to 0.1. On the other hand the interleaved scheme yields capacities ranging from 1 to 4.5 bit/s/Hz.

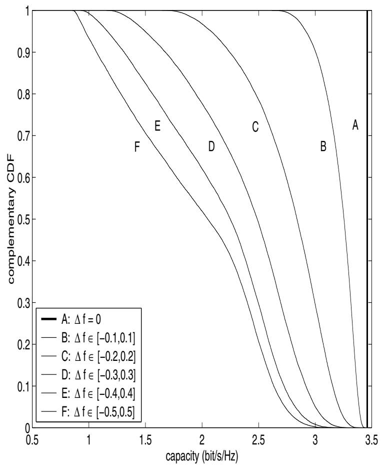

Figure 5. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16**. Interleaved tone assignment**. Equal power tones. SNR=10 dB.

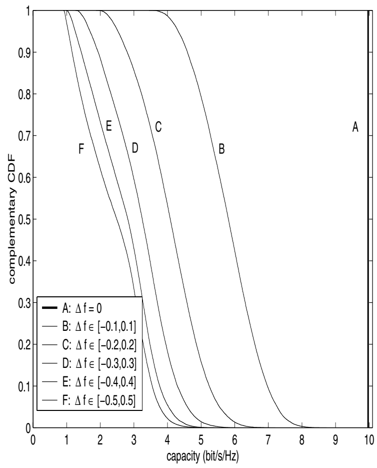

Figure 6. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Interleaved tone assignment**. Equal power tones. SNR=30 dB.

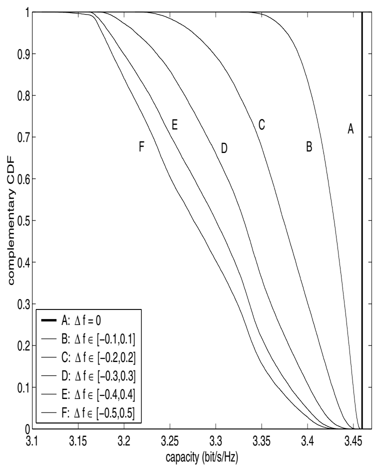

Figure 7. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Block tone assignment**. Equal power tones. SNR=10 dB.

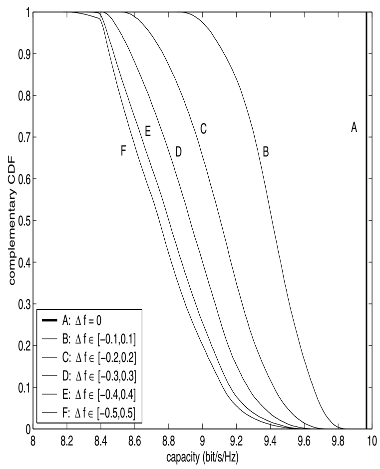

Figure 8. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Block tone assignment**. Equal power tones. SNR=30 dB.

At this point, a first approach to improve further the single link capacity can be to differentiate the power transmitted on the sub-carriers. We pursued this idea in a simplified manner. In Figure 9, the tones are block allocated. Then, the power of the first and last tone of each user block is set to zero. Basically, this corresponds to inserting frequency guards [4]. Unfortunately, this helps only for high Δf_{max} , as shown by comparison of Figure 9 with Figure 8. This is due to the fact that the diminished MAI level does not always compensate the capacity loss due to no transmission on two sub-carriers.

In a second experiment (Figure 10), the total available power is distributed in the following manner. The first and the last tone of each block have half power, while the power of two middle tones is increased by half. As Figure 10 confirms, this strategy shows some improvement over Figure 8.

It is clear that although we pursued an intuitive and simple method to allocate the power to sub-carriers, the results in Figure 10 confirm that distributing the power uniformly among the tones is not optimal. A systematic application of the water-filling method [6] should lead to more significant improvements.

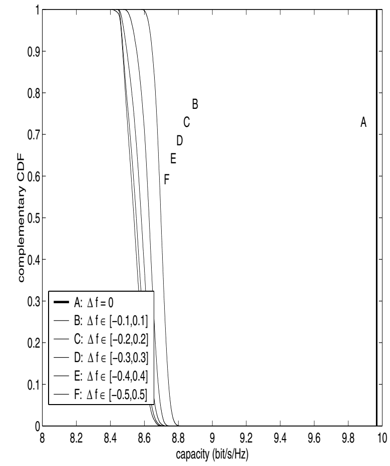

Figure 9. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Block tone assignment.** Power zero on first and last tone in each block. SNR=30 dB.

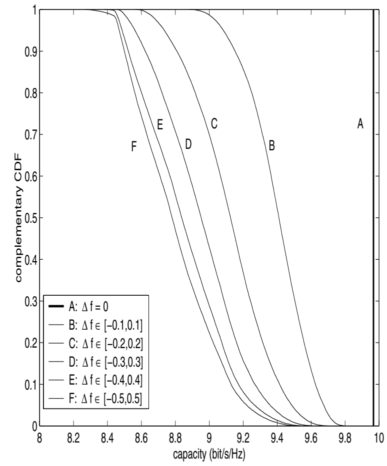

Figure 10. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Block tone assignment**. Half power on first and last tone, 3/2 power on two mid-tones. SNR=30 dB.

A second proposed method to improve capacity is based on *adaptively allocating* the tones to users. The main idea is that if we know for instance what the frequency offsets of the users are, we can assign block of tones in a way such that each block has adjacent blocks ordered with increasing frequency offset. This can be simply accomplished by imagining of allocating the N_U users in a circular queue of N_U cells. The user with the lowest offset is assigned to a cell. Then, pick other two users with the remaining lowest offset and allocate them one to the right and one to the left adjacent cells. Proceed until the cells are filled. This adaptive method of allocating tones in order of increasing frequency offset can be generalized to include time offsets exceeding the guard time.

The ccdf of the user link capacity with this adaptive scheme is plotted in Figures 11 and 12. A deep improvement is found over the schemes that we have considered so far. For instance at 30 dB SNR the lower bound in capacity granted with probability 90% loses less than 1 bit/s/Hz over the upper bound for all Δf_{max} from 0.5 to 0.1.

Clearly, the practicality of such an approach needs to be investigated.

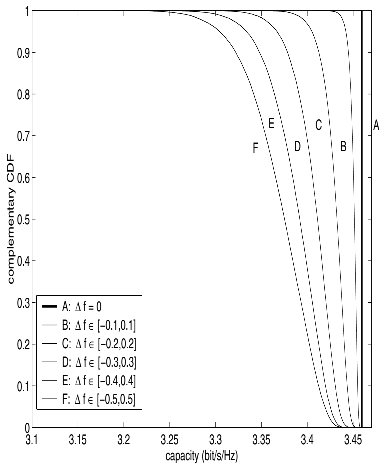

Figure 11. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Adaptive block tone assignment**. Equal power tones. SNR=10 dB.

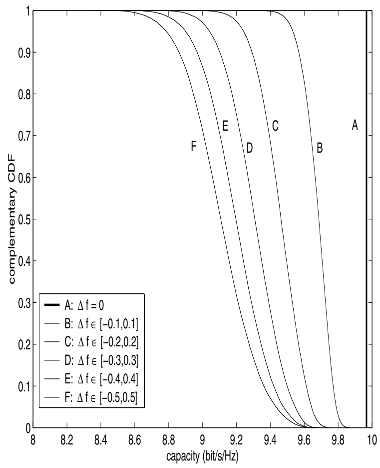

Figure 12. Ccdf of the capacity of a user link for several maximum frequency offsets. Fully loaded system with N=256, N_U=16, K_u=16. **Adaptive block tone assignment**. Equal power tones. SNR=30 dB.

6.2 Joint link capacity complementary cumulative distribution functions

In this section we report some results obtained by numerically computing (12). We considered a fully loaded system with $N=64$, where 4 users have $K_u=16$ tones block assigned with equal power, and experience independent frequency offsets with uniform distribution in (-0.25,+0.25). Figure 13 shows the joint ccdf of two users out

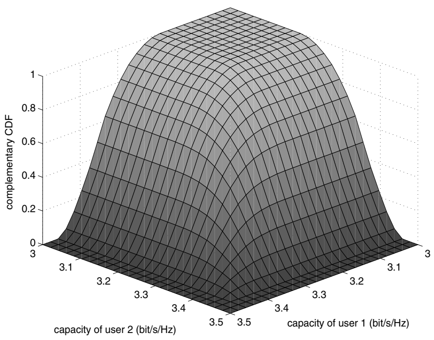

Figure 13. Joint ccdf of user 1 and 2 out of 4 users in a fully loaded system. **Block tone assignment**, N=64, N_u=4, K=16. Equal power tones. SNR=10 dB. $\Delta f_u \in U$[-0.25,0.25].

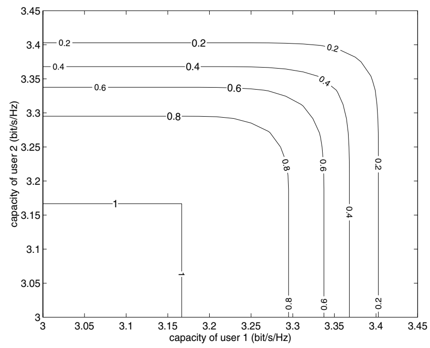

Figure 14. Contour lines of joint ccdf of user 1 and 2 out of 4 users in a fully loaded system. **Block tone assignment**, N=64, N_u=4, K=16. Equal power tones. SNR=10 dB. $\Delta f_u \in U$[-0.25,0.25].

of four. Figure 14 is the contour plot of the surface in Figure 13, each curve is the set of rate pairs for which joint reliable communications are possible with a given probability over all system realizations. The closure determined by such curves can be thought to be the set of all achievable rate pairs with probability at least equal to the corresponding contour label. The capacity region that has probability one is a square, and is the one that would be determined in the case all users experienced the maximum frequency offset in all realizations.

7. Conclusions

In this paper we have studied the problem of defining the achievable information rates for reliable communications of a set of time and frequency asynchronous users that share a Gaussian channel through a DMT OFDMA access scheme. Demodulation is accomplished with a bank of single user FFT based detectors, at the output of which multiple access interference arises. We have shown that each user link can be modeled with a multiple input multiple output correlated Gaussian interference channel. Consequently, we have determined an inner bound to both the capacity of a given user link, and the region of achievable information rates for joint reliable communications (i.e., capacity region). These bounds are conditioned on a system realization (i.e., time and frequency offsets), and depend on the tone allocation scheme and the power allotment to carriers. Thus, by treating these bounds as a random variable we have numerically computed their ccdfs for several fully loaded system scenarios characterized by different tone assignment schemes, and power allotment to sub-carriers.

Based on the results, the following conclusions and guidelines for maximizing capacity in a multi-user DMT OFDMA system are derived. The insertion of an appropriate cyclic prefix completely eliminates the MAI due to time misalignments. For large cell size some degree of centralized synchronization is required to reduce the prefix length, thus save bandwidth. Multiplexing the users with a block tone allocation yields improved capacity, compared to interleaving the users tones, in the presence of the irreducible frequency offset detrimental effect. Other capacity improvements can be obtained with a dynamic tone allocation that adaptively allocates the tones on a per user time/frequency offset base. Finally, the power of a user should be distributed in an ad hoc fashion among the user tones, following for example a water-filling approach.

8. References

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