# Efficient Architectures for Multiuser FMT Systems and Application to Power Line Communications

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***Abstract*—In this letter we study filtered multitone modulation (FMT) for broadband multiuser power line communications. We address the implementation problem, and we derive a novel efficient digital implementation of both the synthesis and the analysis filter bank. A simple fractionally spaced multiuser receiver is also proposed.**

***Index Terms*—FMT modulation, multiuser systems, OFDM, power line communications.**

## I. Introduction

IN this letter we deal with filtered multitone modulation (FMT) for transmission over multiple access channels and its application over typical power line channels that are characterized by high frequency selectivity. An efficient implementation of single user FMT has been proposed in [1]. Recently, we have shown in [2] that an alternative and efficient implementation is possible. In this paper we generalize the idea in [2] to the multiuser context, and we devise several efficient architectures for both the synthesis and the analysis filter bank (Section III). Furthermore, we propose a simplified fractionally spaced receiver that simultaneously detects all sub-channels of the asynchronous users with lower complexity compared to traditional single user receivers that require one synchronous filter bank per user. The pulse design is addressed in Section IV. In Section V, we evaluate the complexity of the architectures presented, and compare them to OFDMA. Finally, in Section VI, we report a performance comparison in typical power line channels considering also narrow band interference.

## II. Multiuser FMT Modulation System

The notation and system parameters are the following. $N_U$ is the number of users, $M$ is the total number of sub-channels, $K_u \subseteq \{0, ..., M-1\}$ is the set of $M_u = |K_u|$ tones assigned to user $u$, $T$ is the sampling period, $W = 1/T$ is the transmission bandwidth, $f_k = k/(MT)$ is the $k$-th sub-carrier frequency, $f_k - f_{k-1} = 1/(MT)$ is the sub-carrier spacing, $a^{(u,k)}(\ell T_0)$ is the sequence of data symbols transmitted by user $u$ on sub-channel $k$, $T_0 = NT$ is the sub-channel symbol period, $g(nT)$ is the prototype pulse of the FMT modulator, $R = M/T_0$ is the aggregate transmission rate in symb/s.

Paper approved by G. H. Im, the Editor for Equalization and Multicarrier Techniques of the IEEE Communications Society. Manuscript received July 10, 2007; revised January 11, 2008 and March 13, 2008.

Part of this work has been presented at IEEE International Symposium on Power Line Communications (ISPLC) 2007, March 2007, Pisa, Italy.

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Digital Object Identifier 10.1109/TCOMM.2009.05.070328

The users are frequency multiplexed through the assignment of a sub-set of the $M$ available tones such that the complex FMT signal transmitted by user $u$ can be written as follows

$$x^{(u)}(iT) = \sum_{k \in K_u} \sum_{\ell \in \mathbb{Z}} a^{(u,k)}(\ell T_0) g(iT - \ell T_0) e^{j2\pi f_k iT}, \quad (1)$$

where $i \in \mathbb{Z}$ and the data symbols $a^{(u,k)}(\ell T_0)$ belong to the M-QAM signal set.

The low pass signal (1) is digital-to-analog converted and transmitted over the multiple access communication channel. The received discrete time low pass signal can be written as follows

$$y(iT) = \sum_{u=0}^{N_U - 1} y^{(u)}(iT - D^{(u)}) + \eta(iT), \quad (2)$$

where $y^{(u)}(iT)$ is the $u$-th user contribution to the received signal after propagation through the channel. $D^{(u)}$ is the delay of user $u$. $\eta(iT)$ is the Gaussian noise contribution.

In multiuser FMT the receiver can be implemented with a bank of single user receivers. Each single user receiver compensates the propagation delay of the desired user, and runs a filter bank that is matched to the transmitter filter bank. The analysis filter bank for user $u$ deploys the prototype pulse $h(nT)$, and outputs the following stream of samples at rate $1/T_0$ for sub-channel $k$

$$z^{(u,k)}(\ell T_0) = \sum_{i \in \mathbb{Z}} y(iT + D^{(u)}) e^{-j2\pi f_k iT} h(\ell T_0 - iT), \quad (3)$$

where $k \in K_u$. We assume the analysis prototype pulse to be real and matched to the synthesis prototype pulse.

## III. Efficient Implementation

Differently from OFDM, FMT requires sub-channel filtering. In [1] an efficient implementation for single user FMT has been described. Herein, we propose an alternative and novel efficient implementation of both the synthesis and analysis filter banks.

### A. *Efficient Synthesis Filter Bank*

An efficient way of implementing the synthesis stage is depicted in Fig. 1. It is obtained by computing the polyphase decomposition of (1) with period $T_2 = M_2 T$ where

$$M_2 = l.c.m.(M, N) = K_2 M = L_2 N \quad (4)$$

and $l.c.m.(M, N)$ denotes the least common multiple between $M$ and $N$. The $i$-th polyphase component of the signal

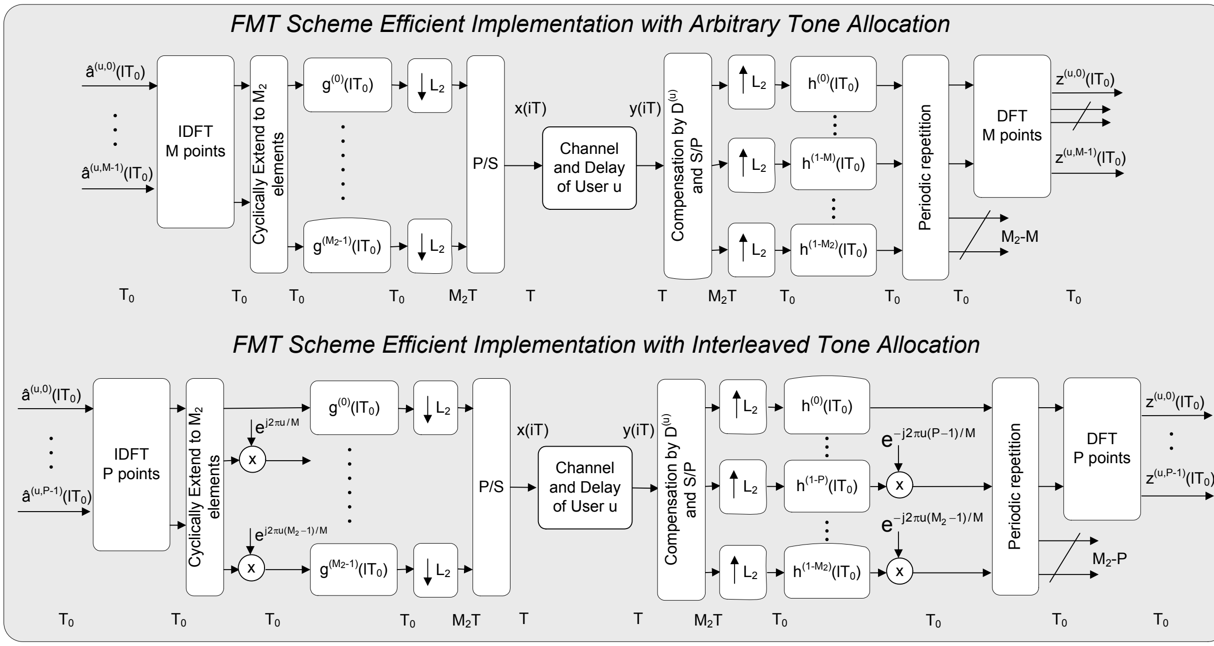


Fig. 1. Efficient FMT scheme for user $u$ with arbitrary and interleaved tone allocation.

transmitted by user $u$ is obtained as follows

$$\begin{aligned} x^{(u,i)}(mT_2) &= x^{(u)}(iT + mT_2),\ i = 0, \ldots, M_2 - 1,\ m \in \mathbb{Z} \\ &= \sum_{\ell \in \mathbb{Z}} \sum_{k=0}^{M-1} \hat{a}^{(u,k)}(\ell T_0) e^{j\frac{2\pi}{M}ik} g(iT + mT_2 - \ell T_0) \\ &= \sum_{\ell \in \mathbb{Z}} A^{(u,i)}(\ell T_0) g^{(i)}(mL_2T_0 - \ell T_0), \end{aligned} \tag{5}$$

where

$$\hat{a}^{(u,k)}(\ell T_0) = \begin{cases} a^{(u,k)}(\ell T_0) & k \in K_u \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

and the coefficients $\{A^{(u,i)}(\ell T_0)\}_{i=0,\ldots,M_2-1}$ are obtained by the $M$-point inverse discrete Fourier transform (IDFT) of $\{\hat{a}^{(u,k)}(\ell T_0)\}_{k=0,\ldots,M-1}$, followed by a cyclic extension to form a block of $M_2$ elements ($K_2$ times repetition). Furthermore, the $i$-th polyphase component of the filter is

$$\begin{aligned} g^{(i)}(mL_2T_0 - \ell T_0) &= g(iT + mT_2 - \ell T_0), \\ i &= 0, \ldots, M_2 - 1,\ m \in \mathbb{Z}. \end{aligned} \tag{7}$$

Therefore, as shown in Fig. 1, the FMT signal of user $u$ can be efficiently synthesized through:

a) S/P conversion, tone mapping, and $M$-point IDFT;
b) cyclic extension of the outputs;
c) low-rate filtering with the pulses $g^{(i)}(\ell T_0) = g(iT + \ell T_0)$;
d) sampling with period $L_2T_0$, and P/S conversion.

If the tones are interleaved across the users, i.e., user $u$ deploys the tones with index $kN_U + u$, $k = 0, ..., P-1$, the implementation can be simplified further since the block $\{A^{(u,i)}(\ell T_0)\}_{i=0,\ldots,M_2-1}$ is obtained by running an IDFT with $P = M/N_U$ points, followed by a cyclic extension adding $M_2 - P$ coefficients, and a phase rotation. In formulae, the coefficients $A^{(u,i)}(\ell T_0)$ are obtained as

$$\begin{aligned} A^{(u,i)}(\ell T_0) &= e^{j\frac{2\pi}{M}iu} \sum_{k=0}^{P-1} a^{(u,k)}(\ell T_0) e^{j\frac{2\pi}{P}ik}, \\ u &= 0, \ldots, N_U - 1,\ i = 0, \ldots, M_2 - 1,\ P = M/N_U. \end{aligned} \tag{8}$$

### B. Efficient Analysis Filter Bank

The analysis filter bank for user $u$ can be implemented via a polyphase decomposition of the received signal with period $T_2 = M_2T$. Since users are asynchronous, when demodulating user $u$ the filter bank has to compensate for its delay. Thus, the polyphase received signal is written as

$$\begin{aligned} y^{(u,i)}(mL_2T_0) &= y(iT + mL_2T_0 + D^{(u)}), \\ i &= 0, \ldots, M_2 - 1,\ m \in \mathbb{Z}. \end{aligned} \tag{9}$$

Then, the filter bank output is obtained as follows

$$z^{(u,k)}(\ell T_0) = \sum_{i=0}^{M_2-1} Z^{(u,i)}(\ell T_0) e^{-j\frac{2\pi K_2}{M_2}ik},\ k \in K_u \tag{10}$$

with

$$Z^{(u,i)}(\ell T_0) = \sum_{m \in \mathbb{Z}} y^{(u,i)}(mL_2T_0) h^{(-i)}(\ell T_0 - mL_2T_0). \tag{11}$$

According to (11) the following efficient implementation is devised:

a) S/P convert the received signal;
b) interpolate the $M_2$ polyphase components of the input signal by a factor $L_2$;
c) analyze them with the low-rate filters with impulse response $h^{(-i)}(\ell T_0) = h(\ell T_0 - iT)$;
d) apply an $M_2$-point discrete Fourier transform (DFT), and sample the outputs with index $K_2k,\ k \in K_u$.

Since sampling the DFT output is equivalent to a periodic repetition of the input, an alternative implementation is obtained by making periodic with period $M$ the input block of size $M_2$ to the DFT to obtain the block of coefficients $\left\{Z_M^{(u,i)}(\ell T_0)\right\}_{i=0,\ldots,M-1}$. The ones with index $k \in K_u$ belong to the desired user $u$. The periodic transform is implemented as follows

$$Z_M^{(u,i)}(\ell T_0) = \sum_{n=0}^{K_2-1} Z^{(u,i+nM)}(\ell T_0),\ i = 0,\ldots,M-1. \tag{12}$$

This implementation is shown in Fig. 1. It should be noted that this receiver requires an analysis filter bank per user. This is because we assume that the users have different time delays and each filter bank needs to be synchronized with a different timing phase, one per user.

If the tones are interleaved across the users the analysis filter bank of user $u$ can be simplified further, as shown in Fig. 1, by deploying a $P$-point DFT on the block of coefficients $\left\{Z_P^{(u,i)}(\ell T_0)\right\}_{i=0,\ldots,P-1}$ that is obtained by an appropriate rotation, and a periodic transform as follows

$$Z_P^{(u,i)}(\ell T_0) = \sum_{n=0}^{M_2/P-1} e^{-j\frac{2\pi}{M}(i+nP)u} Z^{(u,i+nP)}(\ell T_0),$$
$$\text{with} \quad \frac{M_2}{P} = K_2 N_U, \quad i = 0,\ldots,P-1. \tag{13}$$

The rotation is deployed to shift the $M_2$-point DFT output according to the user tone allocation, while the periodic repetition is done to realize sampling at the output of the DFT.

### C. Fractionally Spaced Multiuser Receiver

One filter bank per user is required in the implementations above. It would be beneficial, for complexity purposes, to use a unique filter bank at the output of which we simultaneously get the set of $M$ channels that belong to the $N_U$ users. To achieve this goal, we propose to deploy two fractionally spaced analysis stages instead of $N_U$ filter banks. Each analysis stage is efficiently implemented as described in Section III-B. The two analysis stages work with the sequence of input samples $y_0(iT) = y(iT + \Delta_0)$, $y_1(iT) = y(iT + \Delta_1)$ where $\Delta_1 = \Delta_0 + T_0/2$. The first sampling phase can be chosen as $\Delta_0 = \left(\max_u\left\{\hat{\Delta}_u\right\} + \min_u\left\{\hat{\Delta}_u\right\}\right)/2 - T_0/4$, $\hat{\Delta}_u = T \cdot \text{reminder}(D^{(u)}/T, N)$. The filter banks outputs are processed with fractionally spaced linear sub-channel equalizers [3]. Increased performance at the expense of complexity can be obtained with more fractional branches.

## IV. Design of the Prototype Pulse

To approach filter bank orthogonality, we can use pulses that are band limited and have Nyquist autocorrelation. In [4] a design method based on a tradeoff between inter-symbol interference (ISI) and inter-carrier interference (ICI) is proposed. In this paper we synthesize the pulse in the frequency domain. First, we choose a pulse that belongs to the Nyquist class with frequency response $G^2(f)$, roll-off $\rho$ and Nyquist frequency $F_N = 1/(2T_0)$. Then we obtain the frequency components by sampling $\sqrt{G^2(f_n)}$ in

$$nF = n\frac{2F_N}{L},\ n \in \mathbb{Z},\ |n| \leqslant \left\lfloor \frac{K}{2} \right\rfloor,$$
$$\text{with} \quad L = QM_2/N = MK/N. \tag{14}$$

$M$ is chosen to be a power of 2, $Q$ is an integer, and $K = QK_2$ denotes the number of frequency components. We choose $K$ to be an odd integer larger than 3 so that the pulse is real and even. The impulse response of the prototype pulse is obtained using a $QM_2$-point IDFT. Herein, we propose a conventional raised-cosine spectrum, and we choose the roll-off as $\rho = (N-M)/M$.

Pulse Parameters with $M = 32$

| $N$ | $R$ | $L$ | S/ISI (dB) | S/ICI (dB) |
|---|---|---|---|---|
| 33 | 0.97 | 32 | 41.5 | 32.7 |
| 34 | 0.94 | 16;48 | 41.6;61.0 | 29.0;55.3 |
| 35 | 0.91 | 32 | 59.9 | 53.3 |
| 36 | 0.89 | 8;24;40;56 | 45.2;59.3;67.2;73.0 | 25.1;51.8;63.3;70.7 |
| 37 | 0.86 | 32 | 66.5 | 62.1 |
| 38 | 0.84 | 16;48 | 58.7;76.1 | 49.6;74.3 |
| 39 | 0.82 | 32 | 71.1 | 67.9 |
| 40 | 0.80 | 4;12;20;28; 36;44 | 42.4;58.8;65.3;70.7; 75.1;78.7 | 21.1;48.0;59.6;67.2; 72.8;77.3 |
| 41 | 0.78 | 32 | 74.7 | 72.2 |
| 42 | 0.76 | 16;48 | 64.9;83.8 | 58.3;83.0 |
| 43 | 0.74 | 32 | 77.7 | 75.6 |
| 44 | 0.73 | 8;24;40 | 60.5;73.9;83.2 | 45.5;70.6;82.1 |
| 45 | 0.71 | 32 | 80.3 | 78.4 |
| 46 | 0.70 | 16 | 69.6 | 64.0 |
| 47 | 0.68 | 32 | 82.6 | 80.8 |

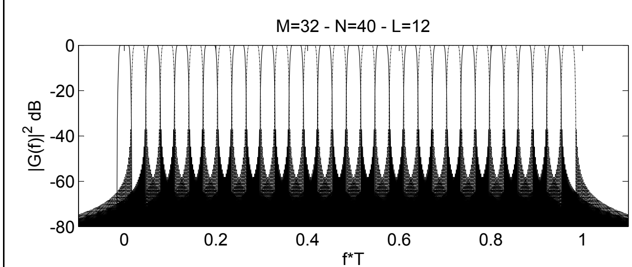


Fig. 2. Example of some possible choices of the parameters for the prototype pulse design and frequency response of the filter bank with $M = 32$, $N = 40$, $L = 12$.

In Fig. 2, we report possible choices of the parameters assuming $M = 32$ and $N$ up to 47. $R$ denotes the transmission rate, while $L$ is the length of the prototype pulse polyphase components. Both signal-to-ISI (S/ISI) and signal-to-ICI (S/ICI) power ratio improve by increasing $L$, or equivalently the number of frequency components. We also plot the frequency response of the filter bank for $M = 32$, $N = 40$, $L = 12$.

## V. Complexity Comparison

In Fig. 3 we report the results of the evaluation of the complexity for the schemes herein proposed in terms of

COMPLEXITY PER USER

| *Scheme* | *Number of Operations per Second* |
|---|---|
| OFDMA TX General | $(\alpha M \log_2 M)/(M + N_{CP})T$ |
| OFDMA TX Interleaved | $(\alpha P \log_2 P + M)/(M + N_{CP})T$ |
| OFDMA RX General | $(\alpha M \log_2 M)/(M + N_{CP})T$ |
| OFDMA RX Interleaved | $(\alpha P \log_2 P + P(2N_U - 1))/(M + N_{CP})T$ |
| FMT TX General | $(\alpha M \log_2 M + N(2\lfloor L_g/N \rfloor - 1))/T_0$ |
| FMT TX Interleaved | $(\alpha P \log_2 P + M_2 + N(2\lfloor L_g/N \rfloor - 1))/T_0$ |
| FMT RX General | $(\alpha M_2 \log_2 M_2 + M_2(2\lfloor L_h/M_2 \rfloor - 1))/T_0$ |
| FMT RX Interleaved | $(\alpha P \log_2 P + M_2(2\lfloor L_h/M_2 \rfloor - 1) + P(2K_2N_U - 1))/T_0$ |
| Fractionally Spaced FMT RX General | $2(\alpha P \log_2 M + K_2P(2\lfloor L_h/M_2 \rfloor - 1) + P(K_2 - 1))/T_0$ |

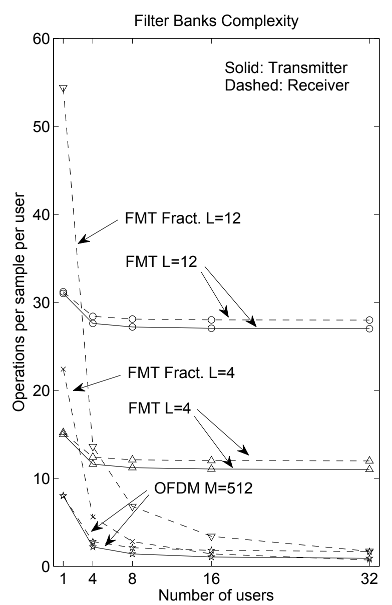


Fig. 3. Complexity of the derived schemes in terms of complex operations per second per user. The plot shows the complexity of multiuser FMT and OFDMA for the case of interleaved tone allocation. $\alpha$ is a factor larger than 1 that depends on the FFT implementation (set to 1 in the example). $L = L_{g,h}/N$ is the polyphase pulse length identical for the synthesis and analysis pulse. $N_{CP}$ is the cyclic prefix length and $P = M/N_U$.

number of complex operations (additions and multiplications) per second per user. A numerical comparison is also shown for various prototype pulse lengths. We consider the complexity of the analysis and the synthesis filter banks, only. Herein, for FMT we fix $M = 32$ and a prototype pulse of length $L = L_{g,h}/N = \{4, 12\}$, while for OFDM the number of tones is $M = 512$ which is required to obtain the same data rate once the cyclic prefix of length $N_{CP} = 128$ is added and uniform bit loading is assumed. The tones are interleaved among the users. OFDMA involves lower complexity than multiuser FMT when a single user receiver is used. This is due to the complexity introduced by sub-channel filtering that increases linearly with the pulse length. The complexity of the FMT receiver is significantly reduced with the proposed fractionally spaced receiver, and it becomes comparable to OFDMA as the number of users increases.

## VI. PERFORMANCE DISCUSSION

In this Section we discuss the performance of multiuser FMT in typical power line channels and compare it with a baseline conventional OFDMA system. The power line channel is generated according to the statistical model in [5]. It has duration equal to $80T = 4\mu s$. The overall bandwidth is $1/T = 20$ MHz. $N_U = 8$ asynchronous users are considered, and are multiplexed with an interleaved tone allocation. The users' time offset is uniformly distributed in $[0, D_{\max}]$, with $D_{\max} = \{0, T_0, 2T_0\}$, and $T_0 = 40T$. 4-PSK signalling is used. The FMT scheme deploys the prototype pulse in Fig. 2 and 32 tones. A 10-tap linear MMSE sub-channel equalizer is used to compensate for the sub-channel ISI. The OFDMA scheme uses 256, or 512 tones, and correspondingly a CP length equal to $N_{CP} = \{64, 128\}$. Thus, the two schemes have the same aggregate data rate equal to 16 Msymb/s. In our bit-error-rate evaluation we have considered three different receiver structures:

a) *Sub-channel Synchronized Receiver with $T_0$ Spaced Equalizer*. This receiver not only compensates the time offset of a given user, but it also uses an optimal time phase for each sub-channel. This receiver cannot be efficiently implemented.
b) *User Synchronized Receiver with $T_0$ Spaced Equalizer*. This receiver compensates the time offset $D^{(u)}$ for each user, but deploys a common sampling phase for all the sub-channels of user $u$ (Section III-B).
c) *Fractionally Spaced Receiver*. This receiver is described in Section III-C. It has the advantage of being the less complex since only two filter banks are required to detect all user signals.

Fig. 4 shows that the average bit-error rate (BER) performance of FMT is insensitive to the users' asynchronism. The fractionally spaced receiver achieves performance close to the optimal sub-channel synchronized receiver. On the contrary, the baseline OFDMA system exhibits high error floors, as it is well known, due to the loss of orthogonality in the presence of channel time dispersion and user time offsets in excess of the cyclic prefix duration.

### A. Narrow Band Interference

In power line communications, narrow band interference (NBINT) is a relevant impairment. We model the NBINT with a Gaussian process having a raised cosine spectrum with roll-off 0.5, and normalized bandwidth (with respect to the sub-carrier spacing) $B_{d,norm} = B_d MT = \{0.4, 1.6\}$ [6]. We fix $M = 32$ for both FMT and OFDM, and the interferer is centred at frequency $f_d$, with $f_d$ being uniformly distributed in $[0, 1/T)$. To better understand the effect of NBINT we assume no channel frequency selectivity. We fix to $30dB$ the margin between AWGN and NBINT. In Fig. 4, a single user scenario is considered. In AWGN the two systems have identical performance. However, FMT has a significant higher immunity to NBINT than OFDM. This is due to the better sub-channel spectral confinement.

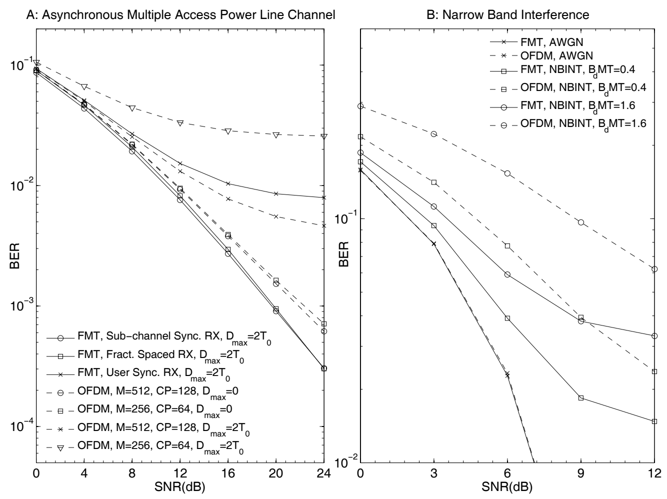


Fig. 4. Performance comparison between multiuser FMT and OFDMA with 8 asynchronous users. FMT with $M = 32$, $N = 40$ and maximum user delay equal to $2T_0$. OFDM with $M = \{256, 512\}$, $CP = \{64, 128\}$ and maximum user delay equal to 0 or $2T_0$ (left plot). Performance comparison between FMT and OFDM with $M = 32$ for the single user scenario with narrow band interference, NBINT (right plot).

## VII. Conclusions

We have proposed novel efficient filter bank architectures for multiuser FMT systems, and we have shown that they yield reduced complexity. A simple fractionally spaced multiuser receiver is also proposed. It achieves the performance of optimal sub-channel synchronized receivers yet requiring much smaller complexity. The performance of the multiuser scheme has been evaluated in typical power line channels. The results show that multiuser FMT has better performance than conventional OFDMA. It has similar complexity because it can be implemented with the efficient scheme herein proposed and can deploy a smaller number of sub-channels.

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