Performance of Single User Detectors in Multitone Multiple Access Asynchronous Communications

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Abstract **- We consider a multitone multiple access wireless system where users are multiplexed by the assignment of sub-sets of the available tones. The information signals of distinct users are multicarrier modulated and propagate through asynchronous frequency selective fading channels, i.e., uplink. In this scenario inter-symbol, inter-carrier, and multiple access interference arise. We consider simple single user detection and study its performance in terms of outage probability. In particular we investigate the effect on outage probability of the choice of the sub-channel transmit filters and the tone allocation strategy.**

I. INTRODUCTION

In this paper we consider a multiple access system where several users deploy multicarrier modulation and access the media in a frequency division mode using a sub-set of the available tones [4]-[7]. Each user transmits its signal through multiple sub-channels that are shaped with an appropriate transmit filter (*prototype filter*). When the sub-carriers are uniformly spaced, the system can be efficiently implemented via Fast Fourier Transform (FFT) and digital low rate filtering. We refer to the system as FMT-MA (*filtered multitone multiple access*) since it is a multiuser extension of the FMT modulation concept [1]. When the transmit filters have rectangular impulse response the system is referred to as DMT-MA (*discrete multitone multiple access*). In such a case the system is often named multiuser OFDM [3], [6], [7].

The signals belonging to distinct users propagate through independent frequency selective fading channels, and experience time and carrier frequency misalignments (uplink communications). In this scenario, inter-symbol (ISI), inter-carrier (ICI), and multiple access interference (MAI) components arise at the output of receive filter bank. The interference components are a function of the prototype filters, the tone allocation strategy, and the propagation conditions, i.e., channel time/frequency characteristics and time/frequency offsets [5].

The deployment of band-limited filters has the potential of minimizing the ICI and MAI. This is particularly advantageous when simple single user detection is used, since its performance is essentially determined by the signal-to-interference power ratio (assuming high enough signal-to-noise ratio). Reliable performance can be granted only for sufficiently large SIRs. Therefore, we study the *outage probability* [9], i.e., the probability that the SIR is smaller than a given threshold, assuming a frequency selective slowly fading channel and random time and carrier frequency offsets.

We show that the deployment of strictly band limited filters results in improved performance compared to time limited filters. From a practical implementation standpoint Gaussian shaped, or Nyquist filters, are a good option [4].

II. TRANSMISSION MODEL

A. Multitone Multiple Access

N_U users are multiplexed by allocating M sub-carriers (tones) in the available spectrum of width $W=1/T$. Each user is assigned a subset of such tones according to a given tone allocation strategy. If the sub-carriers f_k are uniformly spaced, then $f_k = k/T_1$ with $T_1 = MT$. Several carrier allocation methods are possible [6]. For instance, the spectrum can be partitioned into a number of blocks of tones. A given block is assigned to a given user. We refer to it as *block allocation*. Another strategy is to interleave the carriers across users, and we refer to it as *interleaved allocation*. Each user deploys multicarrier modulation over its set of tones.

B. Multicarrier Modulation

We consider a filter bank implementation of each transmitter [4]-[5] (Fig. 1). The sequence of information bits belonging to user u, is mapped into a sequence of complex M-QAM symbols. Then, it is S/P converted and formatted into M sub-sequences $\{a^{u,k}(lT_0)\}$, $k = 0,...,M-1$, $T_0 = NT$. If K_u is the number of tones assigned to user u, then only K_u out of M sub-sequences differ from zero. Let Γ_u be the set of tones indices assigned to user u. Each data sequence is filtered with an interpolation sub-channel filter $g_S(t)$ and sub-carrier modulated. The sub-carrier modulator outputs are summed together to obtain the multicarrier lowpass signal of user u. Finally, the lowpass signal is RF modulated with a bandpass modulator and transmitted over the air. Note that in general $T_0 \geq T_1$, such that the sub-carriers are non-minimally spaced.

If we include the sub-carrier modulator in the transmit filter impulse response, $g_T^k(t) = g_S(t)e^{j2\pi f_k t}$, and we define $\tilde{a}^{u,k}(lT_0) = a^{u,k}(lT_0)e^{j2\pi f_k lT_0}$, the lowpass multicarrier signal of user u reads $x^u(t) = \sum_{k\in\Gamma_u}\sum_l \tilde{a}^{u,k}(lT_0)g_T^k(t-lT_0)$.

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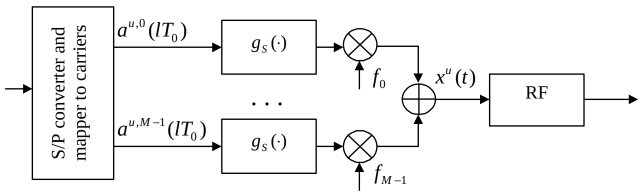

Fig. 1. Multitone multiple access transmitter of user *u*.

The prototype filter $g_S(t)$ is designed with the goal of minimizing the ICI, ISI, and MAI [4], [5].

C. Asynchronous Multiple Access Channel

In an asynchronous multiple access channel the signals of distinct users suffer of distinct time misalignments and carrier frequency offsets relatively to a reference at the receiver. The time offsets are due to the propagation delays of transmitters at different distance from the receiver. The frequency offsets are due to misadjusted oscillators and Doppler from movement. Further, in a wireless uplink scenario the signals of distinct users propagate through distinct frequency selective fading channels.

At the receiver side, we first run RF down conversion and lowpass filtering with a broadband filter with nominal bandwidth *W*. Let $g_E^u(t)$ be the equivalent lowpass impulse response that includes the channel and the broadband filters in the RF modulator/demodulator. Then, at the receiver side the composite lowpass signal can be written as

$$y(t)=\sum_{u=1}^{N_U}\sum_{k\in\Gamma_u}e^{j(2\pi\Delta f_{u,k}t+\Delta\phi_{u,k})}\sum_{l=-\infty}^{\infty}\tilde{a}^{u,k}(lT_0)g_R^{u,k}(t-lT_0-\Delta t_{u,k})+\eta(t) \quad (1)$$

$$g_R^{u,k}(t)=\int_R g_T^k(\tau)g_E^u(t-\tau)d\tau\,. \quad (2)$$

In (2) $g_R^{u,k}(t)$ represents the overall impulse response of the *k-th* sub-channel that includes the sub-channel transmit filter. In (1) $\eta(t)$ is the thermal noise contribution that is assumed to be a zero-mean stationary white Gaussian process. The frequency offset of the sub-channel *k* of user *u* is denoted with $\Delta f_{u,k}$. It is assumed to be smaller than the sub-carrier spacing $1/T_1$. The time offset of the sub-channel *k* of user *u* is denoted with $\Delta t_{u,k}$. Finally, $\Delta\phi_{u,k}$ is a phase offset.

It should be noted that although we have assumed distinct time and frequency offsets for all sub-channels and users, in practical scenarios they may differ only across distinct users, i.e., $\Delta t_{u,k}=\Delta t_u$ and $\Delta f_{u,k}=\Delta f_u$.

D. Tapped Delay Line Fading Channel Model

For practical purposes a frequency selective slow fading channel can be modeled with a tapped delay line with N_P taps [8], i.e., $g_E^u(t)=\sum_{p=1}^{N_P}\alpha^u(p)\delta(t-\tau_p)$. The tap gains are assumed to be independent Gaussian with zero mean. The tap delays are assumed identical for all users.

III. RECEIVER

The goal of the receiver is to reconstruct the sequence of transmitted data symbols of all users. Optimal as well as sub-optimal detection algorithms are studied in [4]. In this paper we consider simple single user detection. Single user detection is based on acquiring carrier and timing synchronization with a given user at the time, followed by equalization/data detection at single user level.

A. Sub-channel Matched Filtering

Let us assume to detect user *u*. To this purpose we acquire carrier and time synchronization with that user and pass the received signal through a bank of filters $g_M^k(t)$. Each receive filter is time, frequency, and phase matched to one sub-channel. The receive filter output of user *u* and sub-carrier *k* at time lT_0 is

$$z^{u,k}(lT_0)=\int_R y(t)e^{-j(2\pi\Delta f_{u,k}t+\Delta\phi_{u,k}+2\pi f_k lT_0)}g_M^k(t-lT_0-\Delta t_{u,k})dt\,. \quad (3)$$

If we expand (3) we can write

$$z^{u,k}(lT_0)=\sum_{u'=1}^{N_U}\sum_{k'=0}^{M-1}\sum_{l'=-\infty}^{\infty}a^{u',k'}(l'T_0)s^{u,u',k,k'}(l,l')+n^{u,k}(lT_0) \quad (4)$$

with $n^{u,k}(lT_0)$ filtered thermal noise, and

$$s^{u,u',k,k'}(l,l')=e^{j\theta_0}\sum_{p=1}^{N_P}\alpha^{u'}(p)\psi_{TM}^{u,u',k,k'}(lT_0-l'T_0+\Delta t_{u,k}-\Delta t_{u',k'}-\tau_p) \quad (5)$$

$$\theta_0=2\pi(\Delta f_{u',k'}-\Delta f_{u,k})(lT_0+\Delta t_{u,k})+\Delta\phi_{u',k'}-\Delta\phi_{u,k}+2\pi T_0(f_{k'}l'-f_k l) \quad (6)$$

$$\psi_{TM}^{u,u',k,k'}(\lambda)=\int_R e^{j2\pi(\Delta f_{u',k'}-\Delta f_{u,k})t}g_T^{k'}(t+\lambda)g_M^k(t)dt \quad (7)$$

being the sub-channels cross correlation. The interference parameters in (5) may differ from zero as a function of the prototype filters, the sub-carrier spacing, the sub-carrier allocation among users, the time/frequency offsets, the propagation media [5]. We can distinguish among self-interference components, and multiple access interference components, thus we can write

$$z^{u,k}(lT_0)=\underbrace{z_S^{u,k}(lT_0)}_{V_0a^{u,k}(lT_0)+ISI+ICI}+\underbrace{z_I^{u,k}(lT_0)}_{MAI}+n^{u,k}(lT_0) \quad (8)$$

where $V_0=s^{u,u,k,k}(l,l)$. Single user detection is based on running equalization for just one user and neglecting the presence of the multiple access interference. In general equalization has to be performed in both time and frequency when both self-ISI and self-ICI components are present [4].

IV. DESIRED SIGNAL AND MAI STATISTICS

The performance of single user detection is limited by the MAI. Thus, it depends upon the SIR power ratio. In Section VI we study the outage probability. To this purpose we need to determine the signal and MAI power statistics.

A. Statistics of the Signal Power

Let us assume the data symbols to be i.i.d. with zero mean and power $P_{u,k} = E[|a^{u,k}(lT_0)|^2]$, while the channel taps to be independent complex Gaussian with zero mean and power $\Omega_{u,p} = E[|\alpha^u(p)|^2]$. The average power of the desired signal computed with respect to the data symbols is

$$S(u,k) = \sum_{l'=-\infty}^{\infty} \sum_{k' \in \Gamma_u} P_{u,k'} |s^{u,u,k,k'}(l,l')|^2 = \sum_{p=1}^{N_P} \sum_{p'=1}^{N_P} \alpha^u(p) \alpha^{u^*}(p') \Psi_u(p',p) \quad (9)$$

$$\Psi_u(p',p) = \sum_{l'=-\infty}^{\infty} \sum_{k' \in \Gamma_u} P_{u,k'} \psi_{TM}^{u,u,k,k'}(l'T_0 - \tau_p) \psi_{TM}^{u,u,k,k'^*}(l'T_0 - \tau_{p'}) . \quad (10)$$

Let $\underline{\alpha}_u$ be the column vector with elements $\alpha^u(p)$ and $\underline{\Psi}_u$ be the matrix of size $N_P \times N_P$ and elements $\Psi_u(p',p)$. Since $\underline{\Psi}_u$ is Hermitian we can find a unitary matrix $\underline{V}$ and a diagonal matrix $\underline{D}_u$ such that $\underline{V}\,\underline{\Psi}_u \underline{V}^H = \underline{D}_u$, [2]. The diagonal elements of $\underline{D}_u$ are the real non-negative eigenvalues $\lambda_{u,i}$ of $\underline{\Psi}_u$, while the rows of $\underline{V}$ are an orthonormal basis of eigenvectors of $\underline{\Psi}_u$. Therefore,

$$S(u,k) = \underline{\alpha}_u^H \underline{\Psi}_u \underline{\alpha}_u = \underline{\beta}_u^H \underline{D}_u \underline{\beta}_u = \sum_{i=1}^{N_p} \lambda_{u,i} |\beta_{u,i}|^2 \quad (11)$$

where $\beta_{u,i} = \underline{\alpha}_u^H \underline{V}_i^H$ and $\underline{V}_i$ *i-th* row of $\underline{V}$. Since the rows of $\underline{V}$ are orthonormal, $\beta_{u,i}$ is a sequence of independent zero-mean Gaussian variables. Let us define $\mathrm{X}_{u,i} = \lambda_{u,i} E[|\beta_{u,i}|^2]$. It follows that the *pdf* of $S(u,k)$ can be found via the partial fraction expansion of the characteristic function. Assuming N_λ distinct eigenvalues each with multiplicity m_i, the calculation yields in general

$$p_S(a) = \sum_{i=1}^{N_\lambda} \sum_{k=1}^{m_i} A_{i,k} \frac{a^{k-1}}{(k-1)!} e^{-\frac{a}{\mathrm{X}_{u,i}}} \quad (12)$$

where $A_{i,k}$ are the coefficients of the expansion. If the eigenvalues are distinct the *pdf* reads

$$p_S(a) = \sum_{i=1}^{N_\lambda} \mathrm{X}_{u,i}^{N_\lambda - 2} e^{-\frac{a}{\mathrm{X}_{u,i}}} \prod_{j=1, j\neq i}^{N_\lambda} \frac{1}{\mathrm{X}_{u,i} - \mathrm{X}_{u,j}} . \quad (13)$$

If the eigenvalues are all identical we get an Erlang *pdf* of order N_P (Chi-square with N_P degrees of freedom).

B. MAI Statistics

We assume the multiple access interference to be Gaussian distributed with zero mean. The instantaneous MAI power $I(u,k)$ seen by user *u* on sub-carrier *k*, conditioned on the time/frequency offsets of all interferers, is exponential distributed, i.e., the *pdf* is

$$p_I(a \,|\, \underline{\Delta t}, \underline{\Delta f}) = M_I^{-1}(u,k \,|\, \underline{\Delta t}, \underline{\Delta f}) e^{-\frac{a}{M_I(u,k|\underline{\Delta t},\underline{\Delta f})}} \quad (14)$$

with $a \geq 0$. The average power of the MAI computed with respect to the data symbols seen by user *u* on sub-carrier *k*, is

$$M_I(u,k \,|\, \underline{\Delta t}, \underline{\Delta f}, \underline{\alpha}) = \sum_{u' \neq u} \sum_{k' \in \Gamma_{u'}} P_{u',k'} \sum_{l'=-\infty}^{\infty} |s^{u,u',k,k'}(l,l')|^2 . \quad (15)$$

If we average over the channel profile we get

$$M_I(u,k \,|\, \underline{\Delta t}, \underline{\Delta f}) = \sum_{u'} \sum_{k' \in \Gamma_{u'}} P_{u',k'} \sum_{p=1}^{N_P} \Omega_{u',p} \Psi'(\Delta t_{u,k} - \Delta t_{u',k'} - \tau_p) \quad (16)$$

$$\Psi'(\Delta t_{u,k} - \Delta t_{u',k'} - \tau_p) = \sum_{l=-\infty}^{\infty} |\psi_{TM}^{u,u',k,k'}(lT_0 + \Delta t_{u,k} - \Delta t_{u',k'} - \tau_p)|^2 . \quad (17)$$

V. DMT-MA AND FMT-MA

In this paper we consider the deployment of rectangular pulses in the time domain and in the frequency domain. We refer to the former as DMT-MA and to the latter with FMT-MA [4]-[5]. To proceed let us define the following quantities:

$$\lambda \triangleq lT_0 - l'T_0 + \Delta t_{u,k} - \Delta t_{u',k'} \qquad \nu \triangleq f_k - f_{k'} + \Delta f_{u,k} - \Delta f_{u',k'} . \quad (18)$$

A. Interference Components in DMT-MA with Guard Time

In DMT-MA with guard time (cyclic prefix) [5] the transmit and receive filters are respectively

$$g_T^k(t) = \frac{1}{\sqrt{T_0}} \mathrm{rect}\left(\frac{t - T_0/2}{T_0}\right) e^{j2\pi f_k (t-\mu T)} \quad (19)$$

$$g_M^k(t) = \frac{\sqrt{T_0}}{T_1} \mathrm{rect}\left(\frac{t - T_0/2}{T_1}\right) e^{-j2\pi f_k (t-\mu T/2)} . \quad (20)$$

The sub-carriers are $f_k \triangleq k/T_1$ and $T_1 \triangleq T_0 - \mu T$. The evaluation of the interference parameters yields the following result:

If $|\lambda| \leq \mu T/2$, $\quad \psi_{TM}^{u,u',k,k'}(\lambda) = e^{j\theta_1} \mathrm{sinc}(T_1 \nu)$. (21)

If $|\lambda| > \mu T/2$ and $|\lambda| \leq T_1 + \mu T/2$

$$\psi_{TM}^{u,u',k,k'}(\lambda) = e^{j\theta_2} \frac{T_1 + \mu T/2 - |\lambda|}{T_1} \mathrm{sinc}\big((T_1 + \mu T/2 - |\lambda|)\nu\big) . \quad (22)$$

Otherwise, $\psi_{TM}^{u,u',k,k'}(\lambda) = 0$.

$\theta_{1,2}$ are a function of λ and ν. Note that in the absence of frequency offsets and with overall delay $|\lambda| \leq \mu T/2$, (21) is non-zero only for *u*=*u*′, and *k*=*k*′.

B. Interference Components in FMT-MA

In an ideal FMT-MA system [5] the sub-channel tx filter is

$$g_T^k(t) = \frac{1}{\sqrt{T_0}} \mathrm{sinc}\left(\frac{t}{T_0}\right) e^{j2\pi f_k t} \quad (23)$$

while the receive filter is $g_M^k(t) = g_T^{k^*}(-t)$. The sub-carriers are $f_k \triangleq k/T_1$ with $T_1 \le T_0$. The evaluation of the interference parameters yields the following result

$$\text{If } |\nu| \le 1/T_0,\ \psi_{TM}^{u,u',k,k'}(\lambda) = e^{j\theta}\left(1-|\nu|T_0\right)\operatorname{sinc}\left(\left(\frac{1}{T_0}-|\nu|\right)\lambda\right) \quad (24)$$

$$\theta = 2\pi\lambda(f_{k'}+\nu/2). \quad (25)$$

Otherwise, $\psi_{TM}^{u,u',k,k'}(\lambda) = 0$.

Note that with frequency offsets $|\Delta f_{u,k}| < 0.5/T_1$ and $f_k = k/T_1$, only adjacent sub-channels may overlap.

C. Desired Signal Statistics in DMT-MA

In a DMT-MA system that deploys a guard time longer than the maximum channel echo delay no self-ICI and no self-ISI components are present (assuming all sub-channels of a given user to have identical $\Delta t_{u,k}$ and $\Delta f_{u,k}$). In fact the *k-th* sub-channel received signal component belonging to user *u* is:

$$z_S^{u,k}(lT_0) = a^{u,k}(lT_0)\sum_{p=1}^{N_P}\alpha^u(p)e^{-j2\pi f_k\tau_p}. \quad (26)$$

Therefore, the instantaneous power $S(u,k)$ of the desired signal on sub-carrier *k*, is exponential distributed with average $M_S(u,k) = P_{u,k}\sum_{p=1}^{N_P}\Omega_{u,p}$. Note that the single user detector simplifies to a simple coherent symbol detector.

D. Desired Signal Statistics in FMT-MA

In an FMT-MA system no self-ICI components are present, however self-ISI is in general present. The *k-th* sub-channel signal component of user *u* is (assuming all of its sub-channels to have identical $\Delta t_{u,k}$ and $\Delta f_{u,k}$):

$$z_S^{u,k}(lT_0) = \sum_{l'=-\infty}^{\infty} a^{u,k}(lT_0 - l'T_0)\sum_{p=1}^{N_P} e^{-j2\pi f_k\tau_p}\alpha^u(p)\operatorname{sinc}\left(\frac{l'T_0-\tau_p}{T_0}\right). \quad (27)$$

The evaluation of the desired signal power (averaged over the data symbols), on sub-carrier *k*, yields

$$S(u,k) = \sum_{p=1}^{N_P}\sum_{p'=1}^{N_P}\alpha^u(p)\alpha^{u^*}(p')\underbrace{P_{u,k}e^{-j2\pi(\tau_p-\tau_{p'})f_k}\operatorname{sinc}\left(\frac{\tau_p-\tau_{p'}}{T_0}\right)}_{\Psi_u(p',p)}. \quad (28)$$

Note that the presence of self-ISI requires the deployment of an equalizer per sub-channel [1]. The equalizer complexity is limited due to the fact that in general $|\tau_p| < T_0$. Further, the statistics of $S(u,k)$ in FMT-MA are given by (12).

VI. SUB-CHANNEL OUTAGE PROBABILITY

The performance of single user detection in a multitone multiple access system depends upon the signal-to-interference power ratio assuming sufficiently high signal-to-noise ratio. The *outage probability* [9] on a given sub-channel is defined as the probability that the signal-to-interference power ratio is smaller than a given threshold

$$P[S(u,k)/I(u,k) < \gamma]. \quad (29)$$

According to Section IV.B the interference power conditioned on the time/frequency offsets is exponential distributed. Therefore,

$$P\left[\frac{S(u,k)}{I(u,k)} < \gamma \mid \underline{\Delta t}, \underline{\Delta f}, S(u,k)\right] = e^{-\frac{S(u,k)}{\gamma M_I(u,k|\underline{\Delta t},\underline{\Delta f})}}. \quad (30)$$

Averaging over the *pdf* of the signal power, the time and frequency offsets yields the sub-channel outage probability.

A. Outage probability in DMT-MA

In DMT-MA the signal power is exponentially distributed. With the assumption of overall interference being Gaussian, the conditional outage probability is

$$P\left[\frac{S(u,k)}{I(u,k)} < \gamma \mid \underline{\Delta t}, \underline{\Delta f}\right] = \frac{\gamma M_I(u,k \mid \underline{\Delta t},\underline{\Delta f})}{\gamma M_I(u,k \mid \underline{\Delta t},\underline{\Delta f}) + M_S(u,k)}. \quad (31)$$

B. Outage Probability in FMT-MA

With the assumption of overall interference being Gaussian, the conditional (on the time/frequency offsets) outage probability can be evaluated in closed form. For instance, if we assume the eigenvalues of $\underline{\Psi}_u$ to be distinct, the outage probability conditioned on the time/frequency offsets reads

$$P\left[\frac{S(u,k)}{I(u,k)} < \gamma|\underline{\Delta t},\underline{\Delta f}\right] = \sum_{i=1}^{N_\lambda}\frac{\mathrm{X}_{u,i}^{N_\lambda-1}\gamma M_I(u,k|\underline{\Delta t},\underline{\Delta f})}{\mathrm{X}_{u,i}+\gamma M_I(u,k|\underline{\Delta t},\underline{\Delta f})}\prod_{j\ne i}\frac{1}{\mathrm{X}_{u,i}-\mathrm{X}_{u,j}} \quad (32)$$

VII. SYSTEM SCENARIO EXAMPLE

To quantify the above results, we consider a multitone multiple access system with *M*=64 tones uniformly spaced by $F_1=1/T_1$, *N*=76, μ=12, maximum number of users N_U=8 each with K_u=8 tones. The users have distinct equal power tones. The channel has N_P=3 equal power independent Rayleigh faded rays.

In Fig. 2 we plot the *cdf* of the desired signal for both DMT and FMT for three channels with increasing delay spread. The guard time in DMT compensates only Channel A. As the delay spread increases FMT is capable of exploiting the frequency diversity and exhibits better signal power statistics.

In Fig. 4-5 we plot the outage probability of the worst sub-channel (adjacent sub-channel) when there are two adjacent asynchronous users (interferers), see Fig. 3. The curves are obtained by Monte Carlo integration of the conditional outage when the time offsets and the frequency offsets of distinct users are independent and uniformly distributed in $[-\Delta t_{\max}, \Delta t_{\max}]$ and $[-\Delta f_{\max}, \Delta f_{\max}]$. All sub-channels of a given user experience the same time/frequency offsets. The channel tap delays correspond to Ch.A, i.e., $\tau_1 = -6T,\ \tau_2 = 0,\ \tau_3 = 6T$.

In Fig. 4 we consider DMT-MA. The guard time fully compensates the ISI channel although it implies a data rate

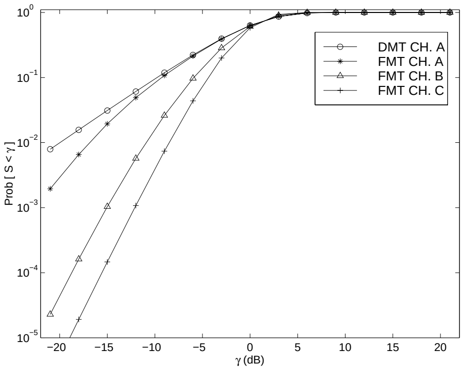

Fig. 2. Probability distribution function of desired signal power for FMT and DMT. Channels have 3 equal power Rayleigh faded rays with delays τ_p , Ch. A: {-6T, 0, 6T};, Ch. B: {-32T, 0, 32T}, Ch. C: {-64T, 0, 64T}.

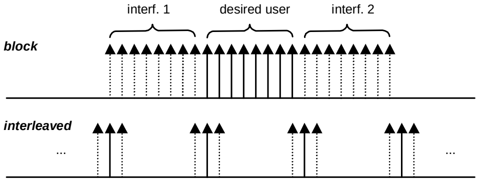

Fig. 3. Tone multiplex. with M=64. Three active users with 8 tones each.

loss of about 19%. The pair of time/frequency asynchronous interferers introduces MAI. The block tone allocation scheme yields lower MAI levels than the interleaved tone allocation. This translates in better outage probability performance with the former tone allocation scheme. Note that with the interleaved multiplexing the outage probability is identical for all sub-channels. With block multiplexing the other sub-channels have outage probability comprised between the best and the worst cases that are shown.

In Fig. 5 we consider FMT-MA. The sub-carrier spacing is identical to the one deployed for Fig. 4, i.e., $F_1 = 1/T_1$ and $T_1 < T_0$. Therefore, the sub-carriers are non-minimally spaced and the data rate is identical to the previous system. The ISI channel introduces self-ISI. Frequency asynchronous users that deploy adjacent sub-carriers introduce MAI. However, the non-minimal sub-carrier spacing translates into increased immunity to interferers that experience frequency offsets. Note that with the interleaved multiplexing the outage is identical for all sub-channels. With block multiplexing only the two outermost sub-carriers of the block may suffer from MAI in the scenario that we consider, i.e., have non-zero outage probability.

VIII. CONCLUSIONS

We have studied the outage probability in a multitone multiple access system with simple single user detection. It is shown that the deployment of band limited sub-channel transmit filters is a better option than time limited pulses. Lower MAI and lower outage probability are obtained at the expense of increased complexity of the single user detector. In fact with band limited filters the single user detector needs to run equalization on each sub-channel. Its complexity is however limited if a high number of sub-carriers is deployed.

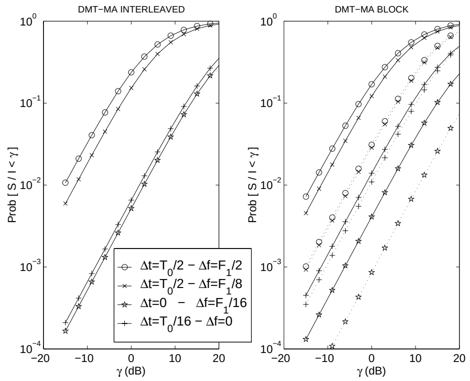

Fig. 4. Outage prob. for DMT-MA on worst sub-channel with uniform Δf and Δt for several $\Delta f_{\max}$, $\Delta t_{\max}$. Dotted curves are for the best sub-channel.

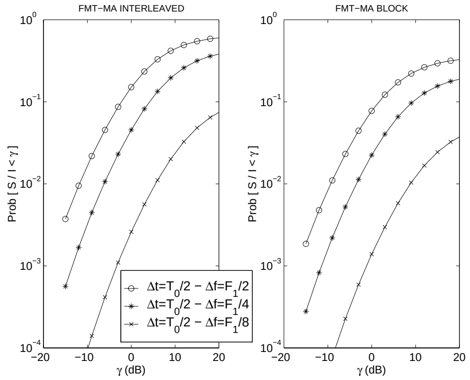

Fig. 5. Outage probability for FMT-MA on worst sub-channel.

In [4] we have devised other optimal/sub-optimal multiuser detectors for asynchronous multitone multiple access systems. Further, we have considered practical filter options, e.g., Gaussian shaped pulses, that can be considered band limited.

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