Multiuser Detection and Turbo Multiuser Decoding for Asynchronous Multitone Multiple Access Systems

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Abstract - **We consider multicarrier multiple access architectures that are based on combining discrete multitone (DMT) or filtered multitone (FMT) modulation with multiplexing of tones across users. Communications are asynchronous, e.g., uplink cellular communications. Optimum multiuser detection is formulated for demodulation of multitone signals that experience time and frequency asynchronism and propagate through frequency selective fading channels. Channel coding is also considered and practical decoding based on iterative (turbo) multiuser detection and decoding is investigated. Simplified detection schemes are also devised.**

I. INTRODUCTION

Multicarrier modulation has been widely investigated for application to broadband wireless communication systems. However, most of the work has focused on single user applications, or synchronous multiuser applications, i.e., downlink communications. In this paper we consider an asynchronous multiple access communication system that deploys multicarrier modulation [9]. In particular we consider a special form of multicarrier modulation where parallel information streams modulate sub-carriers spaced by the (parallel stream) symbol rate. Users are multiplexed in frequency by assigning sub-sets of carriers to them. When communications are asynchronous, e.g., uplink cellular communications, the receiver sees a multitude of signals that have propagated through independent frequency selective fading channels and have non-zero relative time offsets, and frequency offsets. A digital implementation of the system is possible and is here referred to as multitone multiple access (MTMA). The transmitter is implemented by using an inverse fast Fourier transform (IFFT), followed by a bank of polyphase filters and P/S conversion. The polyphase filters are obtained by the polyphase decomposition of a prototype filter. In general the sub-channels may overlap in time and in frequency, and the system is referred to as filtered multitone multiple access (FMT-MA) [7]. For instance, the prototype filter can be a squared-root-raised-cosine filter, or a Gaussian shaped filter. The system is referred to as a discrete multitone multiple access (DMT-MA), or also multiuser OFDM, when the prototype filter is a rectangular time-window [5]. An overview of FMT modulation for application in broadband digital subscriber lines can be found in [2]. Multiple access FMT architectures are described in [7], [9].

Some investigation of the effect of the time offsets in a DMT system with sub-optimal single user detection was done in [3]. A unified study of the joint effect of time offsets, frequency offsets, and multi-path fading in a DMT-MA system with single user detection was done in [5] based on which capacity evaluations were carried out in [6]. The performance of sub-optimum single user detection in more general MTMA architectures was presented in [10]. In this paper we are interested on formulating the optimal detector for both DMT and FMT multiuser architectures. We pursue the concept of joint detection of signals transmitted over multiple-input multiple-output channels, which has found application also in the context of optimum detection of asynchronous CDMA signals [15] and equalization in multiple transmit-receive antenna systems [8]. The detector is capable of delivering the a posteriori probabilities of the transmitted symbols/bits (soft outputs) and can include soft/hard a priori information. Thus, when the users deploy channel coding, it can be concatenated in an iterative (turbo) fashion with the channel decoders. Finally, simplified detection schemes are also described.

II. SYSTEM MODEL

We consider a system where N_U users wish to communicate with a receive station, i.e., uplink communications. The total available spectrum is $W=1/T$ inside which N carries are allocated. Modulation and multiplexing of the users' signals are obtained through a combination of frequency division multiplexing and multicarrier modulation. Essentially, each user is assigned a sub-set of carriers that are used for multicarrier modulation.

A. Multitone Modulation

We here describe a digital implementation of multicarrier modulation that is referred to as multitone modulation [2].

Let us consider a sequence of information bits belonging to user u. This is mapped into a sequence of complex M-PSK/M-QAM symbols, and S/P converted and formatted into N data sequences $\{a^{u,k}(lT_0)\}$, $T_0=NT$, $k=0,...,N-1$. Assuming that K_u tones are assigned to user u, then only K_u data sub-sequences out of N differ from zero. We assume the carriers to be uniformly allocated in the spectrum, i.e., $\overline{f}_k = f_k + f_{RF}$, with $f_k=k/T_0$, $k=0,...,N-1$. The data sequences are filtered and carrier modulated. The complex multicarrier signal transmitted by user u, before RF modulation, is

$$x^u(t) = \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} a^{u,k}(lT_0) g_T^k(t - lT_0) . \qquad (1)$$

The impulse response of the *k-th* sub-channel transmit filter is the frequency shift of a prototype filter $g_1(t)$: $g_T^k(t) = g_1(t)e^{j2\pi f_k t}$. A digital implementation of (1) is depicted in Fig. 1. The N

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parallel data sequences are passed through an N-point IDFT. The IDFT outputs are filtered with a polyphase filter bank. The impulse response of the *k-th* sub-channel filter is $g_1^k(lT_0) = g_1(kT + lT_0)$. The filter bank outputs are

$$x^{u,k}(lT_0) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} a^{u,n}(mT_0) e^{j\frac{2\pi}{N}nk} g_1^k((l-m)T_0) . \qquad (2)$$

The N discrete time signals (2) are P/S converted, D/A converted and eventually further filtered with a broadband filter $g_{TX}(t)$ before RF modulation.

The prototype filter is designed with the goal of minimizing the inter-carrier (ICI), inter-symbol (ISI), and multiple access (MAI) interference [7], [9]. In general it is not limited in time and frequency, and the system can be referred to as filtered multitone multiple access system (FMT-MA). However, if the prototype filter is a rectangular window of duration NT, the system collapses to a discrete multitone multiple access system (DMT-MA).

B. *Media Access Protocol*

Distinct users access the media at the same time and in the same spectrum. Multiplexing is obtained by allocating a subset of the available N tones to each user. A number of tone allocation methods are possible [5], [6], [9]. For instance, the spectrum can be partitioned into a number of blocks of carriers. A given block is assigned to a given user. We refer to it as *block allocation*. Another strategy is to interleave the carriers across users, and we refer to it as *interleaved allocation*.

C. *Asynchronous Multiple Access Channel Model*

The signal of a given user propagates through a linear channel with base band impulse response $g_{ch}^u(t)$. At the receiver side, down conversion and filtering, with a filter matched to $g_{TX}(t)$, accomplishes RF demodulation. Let us define the overall channel impulse response for user u as $g_E^u(t) = g_{TX} * g_{ch}^u * g_{RX}(t)$. Then, the composite received signal at the input of the base band detector can be written as

$$y(t) = \sum_{u=1}^{N_U} \sum_{k=0}^{N-1} e^{j(2\pi\Delta f_{u,k}t + \phi_{u,k})} y^{u,k}(t - \Delta t_{u,k}) + \eta(t) . \qquad (3)$$

The model (3) follows from the following assumptions.

- The complex received signal belonging to sub-channel k of user u in the absence of time, frequency and phase offsets is

$$y^{u,k}(t) = \sum_{l=-\infty}^{\infty} a^{u,k}(lT_0) g_R^{u,k}(t - lT_0) \qquad (4)$$

where the equivalent impulse response of the *k-th* sub-channel is defined as

$$g_R^{u,k}(t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nk} g_1^n(mT_o) g_E^u(t - nT - mT_0) . \qquad (5)$$

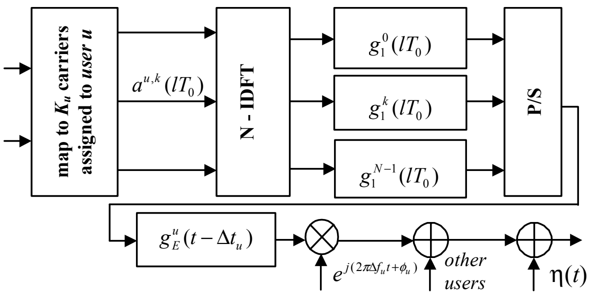

Fig. 1. Multitone transmission model of user u assuming $\Delta t_{u,k}$=Δt_u and $\Delta f_{u,k}$=Δf_u for all sub-channels of user u.

- A carrier frequency offset $\Delta f_{u,k}$ and a phase offset $\phi_{u,k}$ can be present. The frequency offsets are due to the oscillator drifts, and Doppler from movements. They are assumed small compared to the overall bandwidth $1/T$. The signal of sub-channel k of user u can have a time offset $\Delta t_{u,k}$, relatively to a local reference, that is due to a certain propagation delay. In practical scenarios the frequency/time offsets can be considered identical across sub-channels of a given user, i.e., they differ only across users, as shown in Fig. 1.
- The propagation medium $g_{ch}^u(t)$ is static for a given time-window and may change only after some multitone data blocks. In general it is frequency selective thus responsible for time dispersion.
- The thermal noise $\eta(t)$ is assumed to be a stationary white Gaussian process with zero mean and spectral density $2N_0$.

III. OPTIMUM MULTIUSER MULTICARRIER DETECTION

The detector goal is to reconstruct the sequence of transmitted data symbols of all users by observing the complex low pass signal (3). This is a problem of jointly detecting multiple transmitted signals [8], [9], [11], [12], [15]. Assuming to observe the received signal over a finite time window $t \in I$, the optimum detector decides in favor of a transmitted data sequence $\underline{\hat{a}} = \{\hat{a}^{u,k}(lT_0)\}$, $lT_0 \in I$, T_0=NT, u=$1,...,N_U$, k=$0,...,N$-1, that maximizes the a posteriori probability $P(\underline{a} \,|\, y(t)) = p(y(t) \,|\, \underline{a}) P(\underline{a}) / p(y(t))$.

In what follows we assume the transmitted data symbols to be independent. If coding is applied, this is a reasonable assumption when the coded bits/symbols are interleaved. If no side information is available the symbols can be considered equally likely, and the decision rule collapses to the usual maximum likelihood one. Under the AWGN assumption, the channel probability density function conditioned on the channel, the time/frequency/phase offsets of all users, is

$$\begin{aligned} p(y(t) \,|\, \underline{\hat{a}}) &\sim e^{-\frac{1}{4N_0} \int_{t\in I} \left| y(t) - \sum_{u=1}^{N_U} \sum_{k=0}^{N-1} e^{j(2\pi\Delta f_{u,k} t + \phi_{u,k})} \sum_{l=-\infty}^{\infty} \hat{a}^{u,k}(lT_0) g_R^{u,k}(t - \Delta t_{u,k} - lT_0) \right|^2 dt} \\ &= e^{-\frac{1}{4N_0}\Omega(\underline{\hat{a}})} \end{aligned} \qquad (6)$$

We can decompose $\Omega(\underline{\hat{a}})$ (see also [8], [9], [13]) as

$$\begin{aligned} \Omega(\underline{\hat{a}}) \sim -\mathrm{Re}\{ \sum_{lT_0\in I} \sum_{u=1}^{N_U} \sum_{k=0}^{N-1} \hat{a}^{u,k^*}(lT_0)[2z^{u,k}(lT_0) + \\ - \sum_{u'=1}^{N_U} \sum_{k'=0}^{N-1} \sum_{l'=-\infty}^{\infty} \hat{a}^{u',k'}(l'T_0) s^{u,u',k,k'}(lT_0, l'T_0)]\} \end{aligned} . \qquad (7)$$

In (7) the *z-parameters* and *s-parameters* are defined as

$$z^{u,k}(lT_0) = \int_I y(t) e^{-j(2\pi\Delta f_{u,k}t + \phi_{u,k})} g_R^{u,k^*}(t - \Delta t_{u,k} - lT_0) dt \qquad (8)$$

$$s^{u,u',k,k'}(lT_0,l'T_0)=\int_I e^{-j(2\pi(\Delta f_{u,k}-\Delta f_{u',k'})t+\phi_{u,k}-\phi_{u',k'})}\cdot \\ \cdot g_R^{u,k^*}(t-\Delta t_{u,k}-lT_0)g_R^{u',k'}(t-\Delta t_{u',k'}-l'T_0)dt \quad . \tag{9}$$

From (8), $z^{u,k}(lT_0)$ corresponds to the matched filter output sample of the frequency and phase compensated received signal. The filter is matched to the equivalent *k-th* sub-channel impulse response of user *u*, and samples are taken at rate *1/T*$_0$. From (9), $s^{u,u',k,k'}(lT_0,l'T_0)$ is obtained by cross-correlating with appropriate frequency/phase offset compensation the equivalent *k-th* sub-channel impulse response of user *u*, and the equivalent *k′-th* sub-channel impulse response of user *u′*.

Therefore, according to (7) interference components are present. They can be distinguished as follows. *Inter-symbol interference* on a given carrier *k* assigned to a given user *u*. *Inter-carrier interference* between carriers *k*, *k′* assigned to user *u*. *Multiple-access interference* between carriers *k*, *k′*, assigned respectively to user *u* and *u′*, over symbols transmitted at signaling periods lT_0, and $l'T_0$. These interference components are a function of the prototype filter shape, of the channel, and the time/frequency offsets [7], [9].

To proceed let us define the following index relations: $m=u+kN_U+lM_1-1$, with $M_1=NN_U$, $l(m)=m \text{ div } M_1$, $k(m)=(m \bmod M_1) \text{ div } N_U$, $u(m)=(m \bmod M_1) \bmod N_U+1$ for $k=0,\ldots,N-1$, $u=1,\ldots,N_U$, $l=-\infty,\ldots,\infty$, $m=-\infty,\ldots,\infty$. Then, we have $\hat{a}_m=\hat{a}^{u(m),k(m)}(l(m)T_0)$, $z_m=z^{u(m),k(m)}(l(m)T_0)$ and $s_{m,m'}=s^{u(m),u(m'),k(m),k(m')}(l(m)T_0,l(m')T_0)$. Since $s_{m,m'}=s^*_{m',m}$ we can rewrite (7) as follows

$$\Omega(\underline{\hat{a}}) \sim -\sum_m \mathrm{Re}\{\hat{a}^*_m[2z_m-\hat{a}_m s_{m,m}-2\sum_{m'>0}\hat{a}_{m-m'}s_{m,m-m'}]\}\,. \tag{10}$$

Further, let us define the state at time *m* as $S_m=\{\hat{a}_m,\ldots,\hat{a}_{m-L}\}$ for some finite L, and let $P(S_m\,|\,S_{m-1})=P(\hat{a}_m)$ be the a priori state transition probability. Then, under the assumption of independent data symbols $P(\underline{\hat{a}})=\prod_m P(S_m\,|\,S_{m-1})$. It follows that the a posteriori probability $P(\underline{a}\,|\,y(t))$ can be factored such that the search of the MAP transmitted sequence can be implemented with a BCJR [1] algorithm that uses the transition probability:

$$\gamma(z_m,S_m\,|\,S_{m-1}) \sim e^{-\frac{1}{4N_0}\Omega_m(S_m|S_{m-1})}P(S_m\,|\,S_{m-1}) \tag{11}$$

with $\Omega_m(S_m\,|\,S_{m-1})=-\mathrm{Re}\{\hat{a}^*_m[2z_m-\hat{a}_m s_{m,m}-2\sum_{m'>0}\hat{a}_{m-m'}s_{m,m-m'}]\}$. An estimate of the a priori transition probability can be obtained from the channel decoders (Section IV).

The details of the BCJR algorithm have been extensively described in literature (e.g., [1], [8], [16]). Here, we just point out that when we move on the underlying trellis structure we increase either the user index, the frequency index, or the time index by one step. Therefore, the trellis can be imagined as a hyper trellis with a 3-D structure. The complexity of the algorithm is determined by the number of states that is equal to $|\Sigma|=M^L$, and the number of transitions to/from each state that is equal to *M*, with *M* being the constellation size. Therefore, the complexity grows exponentially with the number of users, of carriers, and channel memory.

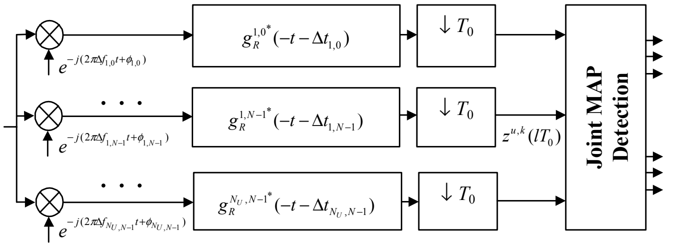

Fig. 2. Multiuser-multicarrier detector.

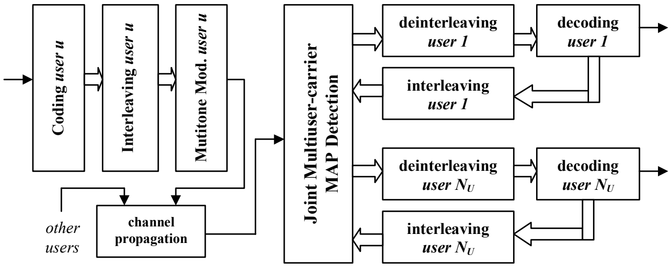

Fig. 3. Coded and interleaved MTMA with turbo detection and decoding.

It should also be noted that the MAP algorithm can be implemented in the logarithm domain [16]. Further, the practical implementation of the algorithm requires an estimate of the channel impulse responses, as well as of the time/frequency/phase offsets of all users.

A. Digital Implementation

We point out that efficient digital implementations of the detector front-end can be derived [9], [11], [12]. For instance in DMT-MA the z-parameters are obtained by N-point DFTs of the matched filter outputs. Further, the s-parameters are computed through a two-dimensional DFT of the propagation channel cross-correlations.

IV. TURBO MULTIUSER-CARRIER DETECTION AND DECODING FOR CODED SYSTEMS

A. Channel Coding

Each user is independently channel coded. Several approaches are possible such as deploy trellis codes where coding and modulation are jointly performed, or interleaved codes where coding and modulation are separately done. Further, coding for a given user can be done independently on each carrier, or across carriers. Coding across carriers allows for exploiting the temporal and frequency diversity.

Here we consider the deployment of bit interleaved codes (Fig. 3). Essentially the information bit stream is encoded and interleaved. The encoder can be block, convolutional, or turbo. The encoded bit stream is parsed into a number of sub-streams equal to the number of assigned carriers. Each sub-stream is mapped into M-QAM data symbols. Finally, multicarrier modulation takes place.

B. Turbo Detection and Decoding

When interleaved codes are deployed, practical decoding is accomplished through the turbo decoding approach.

Essentially, we run multiuser-carrier detection, then after de-interleaving we run channel decoding (Fig. 3). Further, we can iteratively concatenate demodulation and decoding by passing feedback information from the decoders [4], [8].

The multiuser detector has now to provide the a posteriori probabilities of the coded bits, which are then de-interleaved and passed to the decoders. The decoders can be implemented with a soft-in soft-out algorithm (e.g., MAP decoder for convolutional codes) and are capable to deliver new a posteriori probabilities of the coded bits. These are interleaved and passed back to the multiuser detector where they are used as an estimate of the a priori transition probabilities.

V. SIMPLIFIED MULTIUSER-CARRIER DETECTION

In order to simplify the complexity of multiuser-carrier detection several approaches can be pursued.

- Introduce some level of time and frequency synchronization across users. For instance a control loop can be deployed to help the terminals to adjust their local oscillators, and timing [14].
- Design the prototype filters such that the interfering components are lowered [7]. Further, apply appropriate tone allocation strategies in conjunction with time and frequency guards, e.g., DMT with cyclic prefix, [5]-[6].
- Simplify the MAP multicarrier-user detector through reduced state techniques [9]. Reducing the number of states implies that some of the branches in the full state 3-D trellis are cut. This is done dynamically as we move through the 3-D trellis, by retaining only a certain amount of states. This is equivalent to force hard decisions on some of the past (in the user-frequency-time sense) transmitted symbols.

A. Iterative Sub-trellis Detection

A special case of reduced complexity multicarrier detection is obtained when we split the original 3-D trellis into independent sub-trellises. Decisions made on each trellis can be iteratively exchanged among them. Exchange of information can be done in parallel, or in serial mode. For instance, single carrier detection is obtained by deploying a bank of NN_U parallel detectors. Each is time and frequency synchronized to a given carrier and performs only sub-channel equalization. The decisions made by the parallel detectors, at a first detection stage, are reused in a new parallel detection stage. The similarity to parallel interference cancellation in CDMA systems is clear.

A more drastic reduction in complexity is obtained by using single carrier detectors that make symbol by symbol decisions and use decision feedback on all other interfering data symbols. This approach basically consists on iteratively cancel the interference terms using hard decisions from other independent detection stages. Symbol decisions can be directly included in the metric (11). We refer to this simplified method as *iterative per-symbol detection*. If channel coding is deployed the detectors and the decoders can be concatenated by performing cancellation with decisions from the decoders. In this case we refer to it as *turbo per-symbol decoding*. To limit error propagation it may be beneficial to use soft symbol decisions. The performance of this simplified approach is evaluated in the next section for both an uncoded and a coded system.

VI. SIMULATION RESULTS

As an example we report the performance of simplified iterative per-symbol detection/decoding in a multiuser scenario with an AWGN channel in the presence of severe time/frequency asynchronism, and in a single user scenario in the presence of multipath fading.

A. Uncoded Multiuser System

We consider an uncoded system with N=16 total carriers. Two users are multiplexed by assigning 8 carriers each. The allocation scheme is either block or interleaved. We assume that the users' signals experience a time misalignment Δt_u that is uniformly distributed in [-8T, 8T], and a frequency offset Δf_u that is uniformly distributed in [-0.125/16T, 0.125/16T], relatively to a fixed reference point at the receiver. Note that we assume all sub-channels of a given users to experience identical time/frequency offsets. We compare the system that deploys prototype filters that are rectangular in time (DMT-MA), with the system that uses filters that are Gaussian shaped with a normalized bandwidth equal to 0.33 (FMT-MA) [7]. In DMT no cyclic prefix is added. The Gaussian filters are a good choice to minimize the temporal and frequency overlapping across sub-channels.

No channel coding is considered. Each user transmits a frame of 160 information bits that are mapped to QPSK symbols with Gray coding. These frames of information symbols correspond to 10 OFDM symbols over which the time/frequency offset is constant, and changes randomly over the next 10 OFDM symbols. Propagation is through an AWGN channel with no fading.

In Fig. 4 we report bit error rate performance. The reference curve is represented by the BER achieved when all users are synchronous (Bound), or identically when interference cancellation takes place with perfect extrinsic knowledge (optimal multicarrier detection).

The other curves have been obtained with iterative per-symbol detection (Section V). Only FMT yields good performance with interleaved tone multiplexing. DMT achieves better performance with the block tone allocation strategy since less MAI is introduced. With 8 iterations FMT is about 0.5 dB from the bound at BER=10^{-3} with both multiplexing schemes. Finally, note that all curves exhibit an error rate floor.

B. Coded Multiuser System

The system scenario is identical to the one in Section VI.A however we add channel coding. Two users are independently coded. Frames of 80 information bits are encoded with a rate 1/2 convolutional code with memory 2. The resulting 160 coded bits are randomly interleaved, mapped to QPSK symbols, and assigned to the sub-carriers. We use the simplified turbo per-symbol detection/decoding of Section V. The feedback from the two decoders to the detector is hard. With only four detection/decoding iterations performance is greatly improved. It interesting to note that with coding the interleaved multiplexing scheme is capable of yielding better performance than the block scheme as the number of iterations increases.

C. *Uncoded Single User System in Multipath Fading*

We now consider an uncoded system where a single user transmits its information via DMT with 8 sub-carriers and 4-PSK modulation. No cyclic prefix is added. Propagation is through both a flat Rayleigh fading channel, and a 2-rays channel with independent, equal power, and Rayleigh faded rays. The channel is static for the DMT block duration. The second ray is delayed by $3T$. In Fig. 6 we show that practical iterative per-symbol detection is capable of exploiting a significant fraction of the frequency diversity provided by the 2-ray channel. Note that if we inserted a cyclic prefix and we did conventional DMT detection the performance would be identical to the one in flat fading with no diversity exploitation [9], [11], [12]. We also report the performance of optimal multicarrier detection with perfect cancellation of all interfering data symbols (bound) in the 2-rays channel.

VII. CONCLUSIONS

We have proposed optimal MAP multiuser detection in asynchronous multitone multiple access systems. When interleaved codes are deployed the proposed multicarrier-user detector can be concatenated in an iterative fashion with the channel decoders. Simplified approaches are also described. Several simulation results show that simplified iterative per-symbol detection yields results close to the optimum. Further, results and performance analysis can be found in [11] and [12] where the time-variant channel is also considered.

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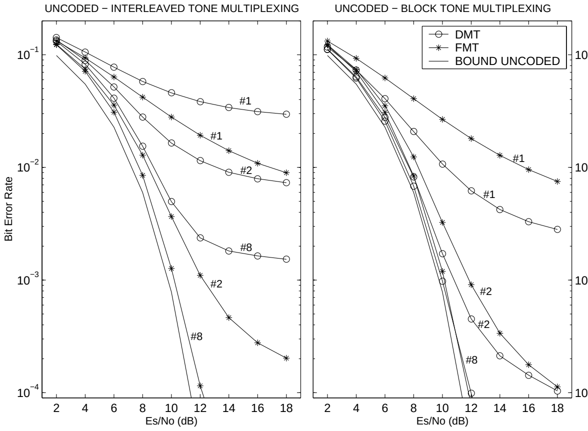

Fig. 4. Average BER with iterative per-symbol detection and optimal multiuser detection (bound) in uncoded DMT-MA and FMT-MA (Gaussian filters) with 2 time-frequency misaligned users in AWGN.

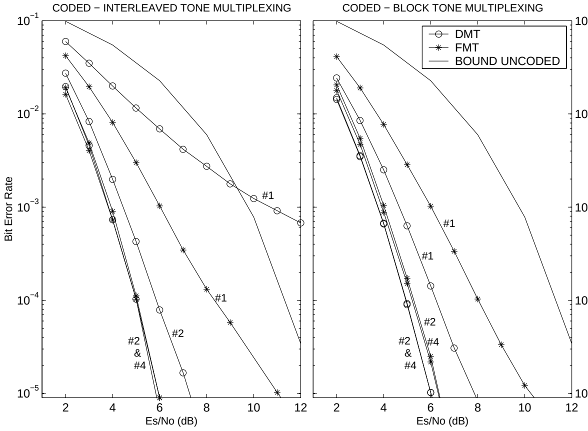

Fig. 5. Average BER with iterative per-symbol detection in convolutionally coded DMT-MA and FMT-MA with 2 time-frequency misaligned users in AWGN.

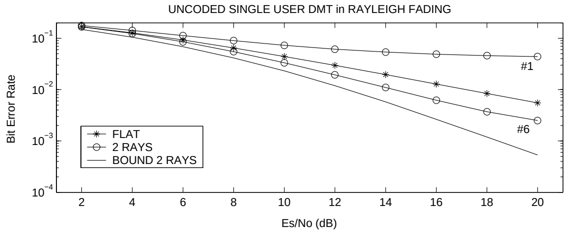

Fig. 6. Average BER performance of iterative per-symbol detection and optimal multicarrier detection (bound) in Rayleigh fading. Single user with DMT.