Analysis of the Uplink of an Asynchronous Multi-user DMT OFDMA System Impaired by Time Offsets, Frequency Offsets, and Multi-path Fading

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Abstract

We study through analysis the joint effect of time offsets, frequency offsets, and multi-path fading in the uplink of an asynchronous multi-user system for wireless communications deploying discrete multi-tone modulation and demodulation. We derive analytical expressions for the multiple access interference, and we quantify its detrimental effects through the evaluation of both the average SINR, and the symbol-error-rate performance. As a result it is shown that the MAI strongly depends not only on the aforementioned impairments but also on the tone assignment algorithm used to multiplex the users. Based on our analysis, the insertion of appropriate time and frequency guard intervals and the accurate selection of the tone assignment algorithm effectively reduce the MAI, so that a proper trade off with spectral efficiency can be met to optimize system performance.

1 Introduction

This paper deals with a multiple user system based on discrete multi-tone modulation (DMT) and orthogonal frequency division multiple access (OFDMA). Multi-carrier modulation is an interesting choice for future wireless communication systems with high spectral efficiency and performance requirements. The advantages include spectral efficiency, frequency diversity, robustness against frequency selective fading, simplified equalization, and simple implementation of modulation/multiplexing with an FFT based approach [1-2]. Multi-carrier modulation can be combined with a media access scheme based on either TDMA, FDMA, or CDMA. Herein, we consider a frequency division approach (OFDMA) where distinct sub-carriers are assigned to distinct users [2].

The major drawback of the DMT/OFDM schemes is that they are sensitive to time misalignments and frequency offsets. In the presence of such offsets, inter-symbol and inter-carrier interference arise and limit the performance. For this reason most of the work on OFDM has been done for the synchronous downlink.

In the uplink, the signals transmitted by users at different distances from the base station (BS) are received with different time delays. The presence of multi-path (i.e. frequency selective) fading introduces echoes, such that the received signal is the superposition of multiple replicas of the signals transmitted by all users. Each replica has a delay equal to the sum of the delay due to multi-path and the delay due to the misalignment. Furthermore, a frequency offset is present among users whenever their local oscillators are misadjusted and/or movements introduce a frequency Doppler shift. Some investigation of the effect of time offsets and multi-path fading was carried out in [3]. However, in [3] neither the effect of the frequency offset nor the influence of a general tone assignment algorithm was taken into account. These effects were studied for a Gaussian channel in [4], and capacity considerations were derived in [5].

In this paper we generalize the analysis of the OFDMA based up-link system in [4] by including a frequency selective fading channel. We show through analysis that inter-symbol and multiple access interference (MAI) arise in this link in the presence of time offsets, frequency offsets, and multi-path fading. The detrimental effect is quantified by first calculating the average signal energy to noise plus interference ratio (SINR). Then, symbol error rate (SER) performance is considered and evaluated numerically for several system scenarios. It is found that the SINR and SER are a function not only of the aforementioned impairments but also of the tone assignment algorithm used to multiplex the users. A comparison between tone allocation strategies is made, and the beneficial effect of using time and frequency guards is quantified. The MAI due to the time offsets and the multi-path fading can be completely removed by inserting a guard time longer than the sum of the maximum channel delay spread and the propagation delay. Furthermore, if a block tone assignment scheme is deployed, the insertion of a frequency-guard between adjacent blocks of tones can significantly lower the MAI due to the frequency offset and the excess time offset from propagation and multi-path. In order to maximize the spectral efficiency for large outdoor cell sizes the use of time and frequency guards

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can still be practical especially if it is applied together with a centralized base station time and frequency alignment control [6].

This paper is organized as follows. In section 2 the transmitter and channel model are described. Section 3 deals with the receiver. Evaluation of the average SINR and SER is carried out in section 4 and 5. Results for several system scenarios are reported in section 6. Finally the conclusions follow.

2 Transmitter and Channel Model

We consider a wireless communication system where N_U users (terminals) communicate with the base station through a noisy multi-path fading channel. Each user's information is modulated with a DMT/OFDM scheme. The assignment of sub-sets of the available equally spaced tones multiplexes the users. Thus, the users share the same bandwidth at the same time, however orthogonality is achieved by assigning distinct tones to distinct users. For instance, K_u tones can be assigned to the *u-th* user as depicted in figures 1 and 2. In the *interleaved tone assignment scheme* (figure 1) the tones of distinct users are regularly interleaved across the overall set of N tones. In the *block tone assignment scheme* (figure 2) disjoint blocks of K_u contiguous tones are assigned to each user.

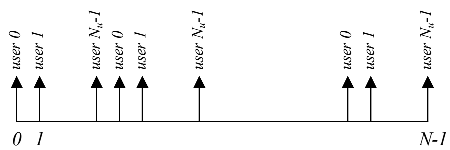

Figure 1: Interleaved tone assignment.

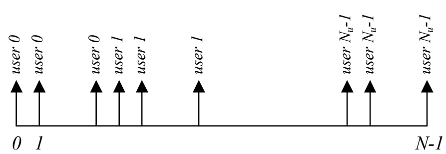

Figure 2: Block tone assignment.

The base-band processing at each transmitter is described in figure 3. The information bit stream of the *u-th* user is first mapped to complex symbols x_n^u with a spectrally efficient modulation scheme (e.g. M-PSK, M-QAM). After S/P conversion, the *i-th* block of K_u complex symbols $\underline{x}^{u,i} = [x_0^{u,i} \ldots x_{K_u-1}^{u,i}]^T$ is transformed into a block of N symbols $\underline{c}^{u,i} = \underline{T}_u \underline{x}^{u,i}$ by the excitation matrix $\underline{T}_u$ of size N by K_u. Basically, the excitation matrix permutes the elements of $\underline{x}^{u,i}$ into the elements of $\underline{c}^{u,i}$ whose indices correspond to assigned tones, and inserts zeros in correspondence of the other tones. DMT modulation is implemented through a N-point FFT, yielding

$$X_k^{u,i} = \sum_{n=0}^{N-1} c_n^{u,i} e^{j\frac{2\pi}{N}nk} \quad k = 0,\ldots,N\text{-}1\,. \tag{1}$$

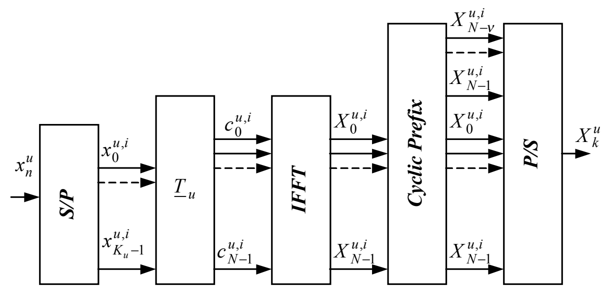

Figure 3: Baseband DMT-OFDM transmitter for user u.

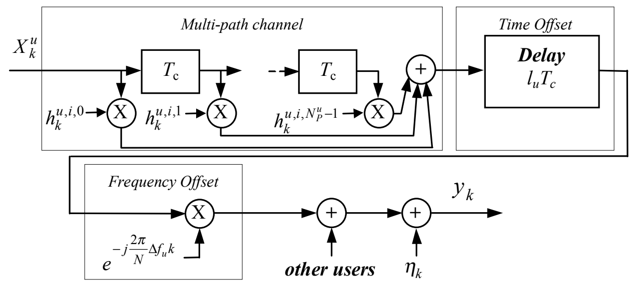

Figure 4: Baseband equivalent channel model for user u.

Hence, a cyclic prefix (guard time) of $\nu = \nu_1 + \nu_2$ symbols is inserted. The cyclic prefix is used to reduce the MAI in the presence of asynchronous users and to compensate for the effect of multi-path fading. The distinction between ν_1 and ν_2 is made by imagining that ν_1 counteracts the time misalignments of the terminals while ν_2 the delayed echoes of the multi-path channel. Finally, after P/S conversion the *u-th* user transmits the OFDM modulated symbol stream at rate $F_c = 1/T_c = (N+\nu)/T$.

After RF conversion, and channel propagation, the signals of all users are superimposed and received at the base station. In a wireless up-link scenario the users are asynchronous and are received with a time and frequency offset relatively to, say, user u'. Further, multiple replicas of the transmitted signal are received in the presence of a frequency selective fading channel. All these impairments are here modeled with a discrete time equivalent channel as shown in figure 4. Ideal sampling at rate F_c after RF down conversion is assumed. A complex Gaussian channel tap $h_k^{u,i,p}$ weights each of the N_P^u resolvable paths. The relative time delay of the *u-th* user with respect to the *u′-th* user is $l_u T_c$. The constant normalized frequency offset between the *u-th* transmitter oscillator at frequency f_u and the local oscillator at frequency $f_{u'}$ is $\Delta f_u = (f_{u'} - f_u) N T_c$. The thermal noise contribution is given by η_k.

3 DFT Based Receiver

In order to reconstruct the transmitted information symbol stream of all users, we adopt a bank of N_U single user detectors identical to the one shown in figure 5. Demodulation for the *u′-th* user is accomplished by first

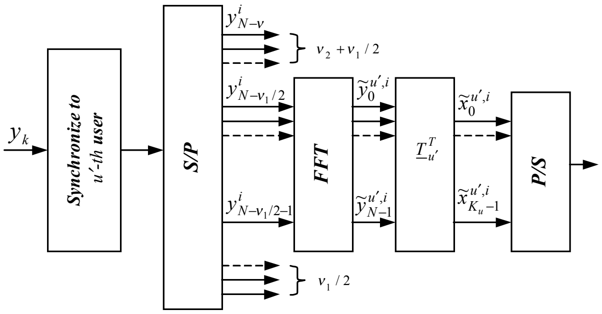

Figure 5: Baseband DMT-OFDM receiver synchronized to user u'.

acquiring time and frequency synchronization with user u' (that is equivalent to setting $l_{u'}=0$ and $\Delta f_{u'}=0$). Then, blocks of $N+\nu$ samples are extracted. A window of N samples is set starting from the middle of the cyclic prefix of length ν_1. Finally, a N-point FFT is applied. From the FFT output we extract blocks of K_u decision variables, based on which the transmitted information bit stream is reconstructed.

Now, consider the symbols in (1), and let X_k^{u,i,l_u} be the *k-th* element of the N-point window shifted by l_u defined as

$$\ldots, X_{N-1}^{u,i-1}\, X_{N-\nu_2-\nu_1}^{u,i} \cdots \left[X^{u,i}_{N-\frac{\nu_1}{2}} \cdots X^{u,i}_{N-1}\, X_0^{u,i} \cdots X^{u,i}_{N-\frac{\nu_1}{2}-1} \right] \cdots X^{u,i}_{N-1} \cdots \quad (2)$$

$$\underset{-|l_u|}{\leftarrow} \quad \underset{+|l_u|}{\rightarrow}$$

Then, the *k-th* sample of the *i-th* received block at the output of the synchronization stage of user u' is

$$y_k^i = \sum_{u=0}^{N_U-1} e^{-j\frac{2\pi}{N}\Delta f_u k} \sum_{p=0}^{N_P^u-1} h_k^{u,i,p} X_{k-p}^{u,i,l_u} + \eta_k^i , \qquad (3)$$

with $i=-\infty,..,+\infty$, $k=0,...,N-1$.

To proceed we assume to assign to distinct users disjoint sets of tones. Thus, let $\Gamma_{u'} = \{n_0^{u',i} \ldots n_{K_{u'}-1}^{u',i}\} \subset \{0 \ldots N-1\}$ be the set of $K_{u'}$ tone indices univocally assigned to user u'. Further, the fading is assumed slow, i.e. $h_k^{u,i,p} = h^{u,i,p}$ is constant over the sampling window length for all u. Finally, the channel maximum delay spread is assumed smaller than the cyclic prefix ν_2, i.e. $N_P^u - 1 \le \nu_2$. Then, the set of decision variables that allow recovering the transmitted symbols $c_n^{u',i}$ of user u' is

$$\tilde{y}_n^{u',i} = \frac{1}{N}\sum_{k=0}^{N-1} y_k^i e^{-j\frac{2\pi}{N}nk} = c_n^{u',i} H_n^{u',i} + \sum_{u=0, u\ne u'}^{N_U-1} \tilde{c}_n^{u,i} + w_n^{u',i} \quad (4)$$

where $n \in \Gamma_{u'}$ and

$$H_n^{u',i} = \sum_{p=0}^{N_P^{u'}-1} h^{u',i,p} e^{-j\frac{2\pi}{N}np} , \qquad (5)$$

$$\tilde{c}_n^{u,i} = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{p=0}^{N_P^u-1} h^{u,i,p} X_{k-p}^{u,i,l_u} e^{-j\frac{2\pi}{N}(n+\Delta f_u)k} , \qquad (6)$$

$$w_n^{u',i} = \frac{1}{N}\sum_{k=0}^{N-1} \eta_k^i e^{-j\frac{2\pi}{N}nk} . \qquad (7)$$

It follows that each sub-channel output is the sum of the symbol $c_n^{u',i}$ transmitted on that sub-carrier weighted by the complex factor $H_n^{u',i}$, a noisy term $w_n^{u',i}$, and a multiple access interference (MAI) term $z_n^{u',i} = \sum_{u\ne u'} \tilde{c}_n^{u,i}$. The *u-th* user interference contribution on the *n-th* sub-carrier is given by (6). The MAI term differs from zero whenever at least one of the other users has a frequency offset $\Delta f_u \ne 0$, and/or a time delay such that $l_u < N_P^u - 1 - \nu_1/2 - \nu_2$ or $l_u > \nu_1/2$.

Thus, a simple way of counteracting the joint effect of multi-path fading and time misalignments is to insert a guard time whose length is larger than the maximum time misalignment plus the multi-path channel delay spread. Since the time delay is a function of distance, this is a practical solution for systems with limited cell radius. However, we could always think of having some degree of centralized control that confines the time offset within the guard time length [6]. Vice versa the frequency offset detrimental effect can be lowered, but not completely eliminated, by the insertion of frequency guards (sect. 6).

4 Average SINR Evaluation

To quantify the performance degradation due to MAI we can evaluate the average signal energy over noise plus interference ratio seen on sub-carrier n (univocally assigned to user u')

$$SINR(n \,|\, \underline{l}, \underline{\Delta f}) = P_n^{u'} \left(\sigma^2 + \sum_{u=0, u\ne u'}^{N_U-1} M_u(n \,|\, l_u, \Delta f_u) \right)^{-1} . \qquad (8)$$

In (8) $P_n^{u'} = E[|c_n^{u',i}|^2 |H_n^{u',i}|^2]$ is the average received power of the *n-th* sub-carrier (averaged over the signal constellation, and the fade distribution); $\sigma^2 = E[|w_n^{u',i}|^2]$ is the thermal noise power; $M_u(n \,|\, l_u, \Delta f_u) = E[|\tilde{c}_n^{u,i}|^2]$ is the average power of the MAI generated on the *n*-th sub-carrier by the *u-th* user conditioned on its time and frequency offsets.

Note that in first approximation the MAI can be considered Gaussian distributed. For brevity only results are reported. These are obtained under the following assumptions. The transmitted symbols are i.i.d with zero mean. Thus the MAI is zero mean too. The multi-path channel profile models a slow fading scenario, where the rays are independent, each with zero mean, and power

$\Omega_p^u = E[|h^{u,i,p}|^2]$. Finally, the cyclic prefix is chosen such that $N_P^u - 1 \le \nu_2$ for all users' channels (i.e. the guard time ν_2 is longer than the maximum delay spread). The calculation yields

$$M_u(n|l_u,\Delta f_u) = \sum_{k\in\Gamma_u} \frac{P_k^u}{N^2} \frac{A(n-k,\Delta f_u,l_u)}{\sin^2\left(\frac{\pi}{N}(n+\Delta f_u - k)\right)}. \qquad (9)$$

In (9) Γ_u is the set of K_u tones assigned to the *u-th* user (interferer). A (see equation 10) is a term that is a function of the sub-carrier index *n-k*, of the frequency offset, of the interval of definition of the time misalignment, and of the channel profile. Curves representing the SINR for several system scenarios and system parameters are reported in section 6.

- $-\nu_1/2-\nu_2 > l_u \ge -N+\nu_1/2$

$$A = \sum_{p=0}^{N_P^u-1} \Omega_p^u (\sin^2 \frac{\pi}{N}(p-l_u-\frac{\nu_1}{2}-\nu_2)(\Delta f_u+n-k) + \sin^2\frac{\pi}{N}(p-l_u-\frac{\nu_1}{2}-\nu_2-N)(\Delta f_u+n-k))$$

- $N_P^u-1-\nu_1/2-\nu_2 > l_u \ge -\nu_1/2-\nu_2$

$$A = \sin^2(\pi\Delta f_u)\sum_{p=0}^{\nu-|l_u|}\Omega_p^u + \sum_{p=\nu-|l_u|+1}^{N_P^u-1}\Omega_p^u(\sin^2\frac{\pi}{N}(p-l_u-\frac{\nu_1}{2}-\nu_2)(\Delta f_u+n-k)+\sin^2\frac{\pi}{N}(p-l_u-\frac{\nu_1}{2}-\nu_2-N)(\Delta f_u+n-k))$$

- $\nu_1/2 \ge l_u \ge N_P^u-1-\nu_1/2-\nu_2$

$$A = \sin^2(\pi\Delta f_u)\sum_{p=0}^{N_P^u-1}\Omega_p^u$$

- $N_P^u-1+\nu_1/2 \ge l_u > \nu_1/2$

$$A = \sin^2(\pi\Delta f_u)\sum_{p=|l_u|+1}^{N_P^u-1}\Omega_p^u + \sum_{p=0}^{|l_u|}\Omega_p^u(\sin^2\frac{\pi}{N}(p-l_u+\frac{\nu_1}{2})(\Delta f_u+n-k)+\sin^2\frac{\pi}{N}(p-l_u+\frac{\nu_1}{2}+N)(\Delta f_u+n-k))$$

- $N+\nu_1/2 \ge l_u > N_P^u-1+\nu_1/2$

$$A = \sum_{p=0}^{N_P^u-1}\Omega_p^u(\sin^2\frac{\pi}{N}(p-l_u+\frac{\nu_1}{2})(\Delta f_u+n-k)+\sin^2\frac{\pi}{N}(p-l_u+\frac{\nu_1}{2}+N)(\Delta f_u+n-k)) \qquad (10)$$

5 Symbol Error Rate Performance

Under the hypothesis of the MAI being Gaussian distributed, the probability of symbol error is here approximated as follows.

Let us define $\mathrm{H}^{u'} = E[|H_n^{u',i}|^2] = \sum_{p=0}^{N_P^{u'}-1}\Omega_p^{u'}$ and let $\gamma_s = |H_n^{u',i}|^2\, SINR(n|\underline{l},\underline{\Delta f})/\mathrm{H}^{u'}$ be the conditional average SINR, i.e. conditioned on the channel profile of user u' and on the time/frequency offsets of all interferers. Let $P_{e,s}(n|\gamma_s)$ be the conditional probability of symbol error on the *n-th* sub-channel that is achieved with a given γ_s. Then, averaging over the distribution of γ_s yields the average symbol error rate of the *n-th* sub-channel, $P_{e,s}(n)$. To exemplify the procedure, consider M-PSK modulation coherently detected. Since $|H_n^{u',i}|$ is Rayleigh distributed, then, the result of first averaging over the distribution of $|H_n^{u',i}|$ is from [7, chapter 14]

$$P_{e,s}(n|\underline{l},\underline{\Delta f}) \approx \frac{(M-1)}{(M\log_2 M)\sin^2(\pi/M)\bar{\gamma}_s/2}. \qquad (11)$$

where $\bar{\gamma}_s = SINR(n|\underline{l},\underline{\Delta f})$.

A final integration over each $p_l(l_u)$, i.e. pdf of the delay of user *u*, and $p_f(\Delta f_u)$, i.e. pdf of the frequency offset of user *u*, yields the result. Note that the time and frequency offsets are considered independent. Further, averaging of (11) is done only on the time and frequency offsets of the interferers and not on their channel profile. The channel profile of the interferers concurs to the calculation of the average SINR in (10). Finally, observe that $P_{e,s}(n)$ is in general not the same for all sub-channels. The global performance of the *u′-th* user link is then

$$P_{e,s}^{u'} = \frac{1}{K_{u'}} \sum_{n \in \Gamma_{u'}} P_{e,s}(n) . \qquad (12)$$

The evaluation of $P_{e,s}^{u'}$ is carried out numerically, and reported in the next section for several system scenarios.

6 Performance Results of Several System Scenarios

From the discussion in sections 4 and 5 it is clear that many system parameters determine the ultimate effect from time and frequency offsets.

As an example, we consider a hypothetical system whose main parameters are summarized in table 1. The cyclic prefix allows to fully compensate the channel delay spread as long as the round trip propagation delay is smaller than 5.36 µs, that is for cells with radius up to 0.8 Km. The channel power profile is exponential and identical for all users, i.e. $\Omega_p^u = (1-e^{-\beta})(1-e^{-\beta N_P})^{-1} e^{-\beta p}$ with $N_P=42$, $\beta = \ln(1.1)$. The system is fully loaded.

W=4.096 MHz	modulation: 4-PSK	max delay spread: 10 µs
N=192 - ν=64	total net bit-rate: 6.144 Mb/s	exponential channel power profile
T_c=0.24 µs	num. of users: N_U=16	
ν_1=22 (5.36 µs) ν_2=42 (10µs)	num. of tone/user: K_u=12 equal power tones	

Table 1: System parameters

First we show the average SIR performance (i.e. $\sigma^2=0$) as a function of the carrier index $k=0,...,K_u-1$, in the presence of only frequency offsets (figure 6) and only time offsets (figure 7). Note that the indices k are mapped to the carriers $n_k^u \in \Gamma_u$ according to the tone assignment scheme (e.g. block, interleaved). The worst case that is here considered has all users with identical time and frequency offsets. Since the system is fully loaded, the tones have equal power and are assigned with symmetric rules, the average SIR performance does not differ among users.

The interleaved scheme yields a worse SIR compared to the block scheme. The SIR for the block scheme can be further improved by assigning a frequency guard at the beginning and/or end of each block [4]. Basically this is equivalent to no symbol transmission on some of the tones at the lower or upper end of each block (e.g. FG=1 no symbols transmitted on the first tone, FG=2 no symbols transmitted on the first and last tone of each block).

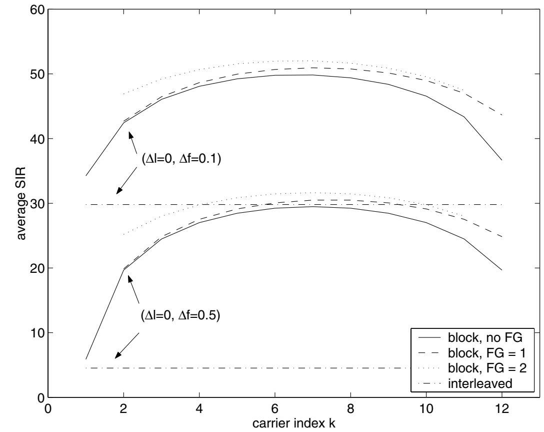

Figure 6: Average SIR on each of the 12 sub-carriers assigned to user u' when all the remaining users have no excess time offset Δl_u but all have identical frequency offset Δf_u relatively to user u' (i.e. worst case). Further, all users have equal power tones. The different curves are obtained for $\Delta f_u=0.1$ and $\Delta f_u=0.5$ with block, block with frequency guards, and interleaved tone assignments.

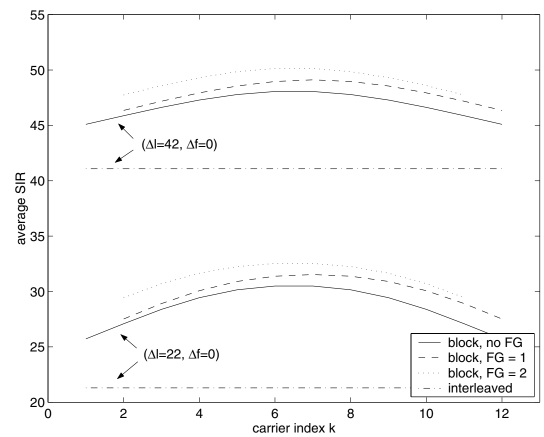

Figure 7: Average SIR on each of the 12 sub-carriers assigned to user u' when all the remaining users have no frequency offset Δf_u but all have identical excess time offset Δl_u relatively to user u' (i.e. worst case). Further, all users have equal power tones. The different curves are obtained for $\Delta l_u=22$ and $\Delta l_u=42$ with block, block with frequency guards, and interleaved tone assignments.

Now consider the user link SER performance. Averaging of (11) is numerically done under the assumption of time offsets and frequency offsets uniformly distributed and independent among users. Each user has the same average SER performance. Comparing figure 8 with 9 the superiority of the block tone assignment scheme is clear. Further SER improvements are possible with the insertion of frequency guards as shown in figure 10, where the error floor due to the MAI significantly drops by just deploying one frequency guard. However, this goes together with some loss of bandwidth or net bit rate.

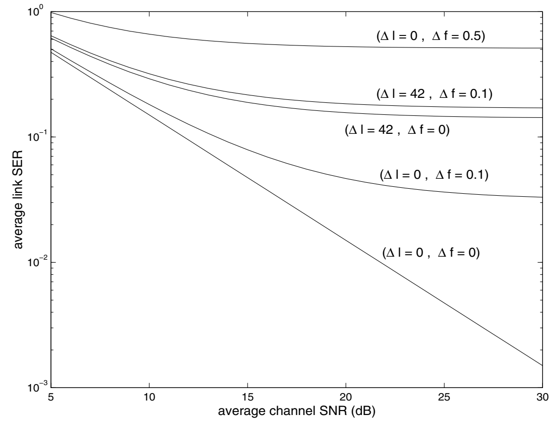

Figure 8: User link average symbol error rate versus SNR for several values of maximum frequency offset and maximum time offset. Interleaved tone assignment. Equal power tones.

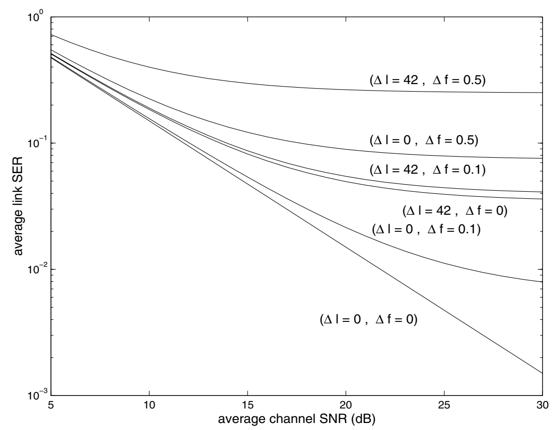

Figure 9: User link average symbol error rate versus SNR for several values of maximum frequency offset and maximum time offset. Block tone assignment. Equal power tones.

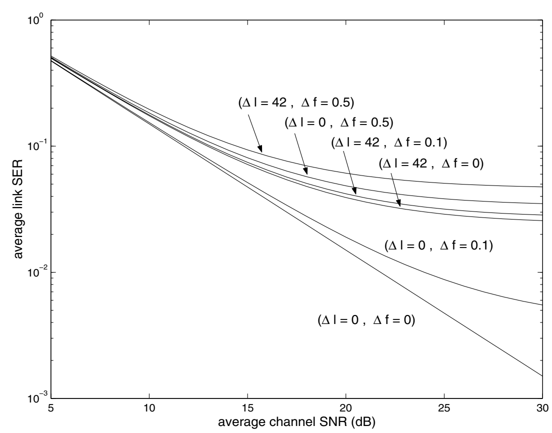

Figure 10: User link average symbol error rate versus SNR for several values of maximum frequency offset and maximum time offset. Block tone assignment with one frequency guard (FG=1). Equal power tones.

Conclusions

We have investigated the uplink of a multi-user DMT OFDM system impaired by time offsets, frequency offsets, and multi-path fading. These impairments introduce multiple access interference that strongly depends on the tone assignment scheme that is used to multiplex the users.

Analytical expressions for the average SINR are obtained, and numerical evaluation of the average SER (symbol error rate) is carried out. The block tone assignment scheme is more robust than the interleaved, and yields improved SINR/SER performance.

Further improvements are possible by deploying time and frequency guards. Thus, a proper trade off with spectral efficiency can be met to optimize system performance.

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