Bit-Loading Algorithms for OFDM with Adaptive Cyclic Prefix Length in PLC Channels

Salvatore D'Alessandro[(1)], Andrea M. Tonello[(2)], and Lutz Lampe[(3)]
[(1,2)] DIEGM - Università di Udine, Udine, Italy, e-mail: salvatore.dalessandro@uniud.it, tonello@uniud.it
[(3)] University of British Columbia, Vancouver, Canada, e-mail: lampe@ece.ubc.ca

***Abstract*— The achievable rate of OFDM systems is not necessarily maximized when deploying a cyclic prefix (CP) as long as the channel impulse response. Even though for shorter CP lengths both inter-symbol and inter-carrier interference do occur, the rate gain offered by a shortened CP may exceed the losses due to interference. In this paper we address the problem of CP length adaptation and we propose practical bit-loading algorithms that include the CP adaptation.**

Keywords—Home networks, OFDM, PLC, resource allocation.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been adopted for a number of wireless and wireline communication systems that include power line communication (PLC) systems, e.g., IEEE 802.11x [1], HomePlug AV (HPAV) [2] and Universal Powerline Association (UPA) [3] compliant devices. An attractive feature of OFDM is that with the use of the cyclic prefix (CP) the channel can be orthogonalized such that simple one tap equalization can be used.

It has recently been shown that to maximize the achievable rate in an OFDM system the CP length has not necessarily to be equal to the channel length [4]-[5]. This is because the insertion of the CP introduces a loss in transmission rate that equals $M/(M+\mu)$ where M is the number of OFDM sub-channels and μ is the CP length in number of samples. Furthermore, the channel impulse response realizations may be different for different links and/or may vary with time. It is therefore reasonable to consider the design and adaptation of the CP length to the specific channel realization. We have presented in [4] several metrics for the choice of the CP length over PLC channels such that system capacity is maximized.

In this paper we extend the work in [4] and we propose bit-loading algorithms suitable for the CP length adaptation. For specific relevance to PLC, we consider transmission parameters similar to those used by the HPAV standard [2]. In the bit-loading algorithms that we discuss practical constraints are taken into account. A first constraint is the use of finite size constellations. Another constraint is the use of identical constellations over all sub-channels, as for instance it is done in the IEEE 802.11 WLAN standard [1].

This second approach is motivated by the idea of simplifying the resource allocation algorithm and minimizing the feedback necessary for the transmitter adaptation. The effect of varying the number of sub-channels is also investigated.

The organization of the paper is as follows. In Section II we compute the achievable bit-rate in a cyclic prefixed OFDM system when the CP lasts less than the channel. Then, in Section III we describe two bit-loading algorithms that perform joint bit-loading and CP length adaptation. In Section IV we quantify the gains obtained by the proposed algorithms compared to bit-loading with a fixed CP length that is equal to the channel duration. The conclusions are given in Section V.

II. CYCLIC PREFIXED OFDM

We consider an OFDM system [6]-[7] with M sub-channels, and a CP of length $\mu = N - M$ samples, where N is the normalized sub-channel symbol period (OFDM symbol duration in samples) assuming that the sampling period T is equal to the time unit in the system. The normalized sub-carrier frequencies are defined as $f_k = k/M$, for $k = 0,...,M-1$.

The OFDM signal is transmitted over a channel that has an equivalent discrete time complex impulse response

$$g_{ch}(n) = \sum_{p=0}^{\upsilon-1} \alpha_p \delta_{\mathbb{Z}}(n-p), \qquad (1)$$

where α_p denote the complex channel coefficients, and the discrete time delta impulse is defined as $\delta_{\mathbb{Z}}(n) = 1$ for $n = 0$, and zero otherwise. Moreover, we assume $\upsilon \le M$ so that the channel is shorter than the useful OFDM symbol duration. The channel duration can be longer than the CP. As a result, at the receiver side, after synchronization, CP discarding, and discrete Fourier transform computation, the signal for sub-channel k can be written as

$$z^{(k)}(\ell N) = G_{CH}^{(k)} a^{(k)}(\ell N) + I^{(k)}(\ell N) + W^{(k)}(\ell N) \qquad (2)$$

where $a^{(k)}(\ell N)$ is the data symbol transmitted on that tone at time instant ℓN, $G_{CH}^{(k)}$ is the channel transfer function at frequency f_k, $I^{(k)}(\ell N)$ is the inter-symbol (ISI) plus inter-carrier (ICI) interference term that arises because of the loss of orthogonality due to an insufficient CP, and $W^{(k)}(\ell N)$ is the additive noise contribution.

Assuming Gaussian inputs, additive white Gaussian background noise (AWGN), and a single tap equalizer the capacity of the system can be evaluated using the formula for

The work of this paper has been partially supported by the European Community's Seventh Framework Programme FP7/2007-2013 under grant agreement n° 213311, project OMEGA – Home Gigabit Networks.

Proc. of IEEE ISPLC 2009, Dresden, Germany, March 2009

parallel Gaussian channels, which yields

$$C(\mu)=\frac{1}{(M+\mu)T}\sum_{k\in K_{ON}}\log_2\left(1+\frac{SINR^{(k)}(\mu)}{\Gamma}\right)\quad [bit/s], \quad (3)$$

where the sub-channel signal-over-noise-plus interference power ratio is defined as

$$SINR^{(k)}(\mu)=\frac{P_U^{(k)}(\mu)}{P_W^{(k)}+P_I^{(k)}(\mu)}, \quad (4)$$

with

$$\begin{aligned} &P_U^{(k)}(\mu)=E\left[\left|H^{(k)}a^{(k)}(\ell N)\right|^2\right],\\ &P_I^{(k)}(\mu)=E\left[\left|I^{(k)}(\ell N)\right|^2\right], \text{ and } P_W^{(k)}=E\left[\left|W^{(k)}(\ell N)\right|^2\right]. \end{aligned} \quad (5)$$

In (5), $E[\cdot]$ denotes the expectation operator. The set of sub-channel indexes that are effectively used is denoted with K_{ON}. We emphasize that here a simple one tap equalizer is used. The system performance can be improved using a time domain equalizer and/or frequency domain interference mitigation. Obviously, the use of these techniques increases the system complexity. Details on the SINR computation can be found in [4]. Γ represents a gap factor from the Shannon capacity curve that accounts for the deployment of practical modulation and coding schemes [6],[8]. We assume it equal to 9 dB when showing numerical results. We further assume a power spectral density (PSD) constraint as it is typically the case in power line or in wireless systems, e.g., ultra wide band (UWB) systems. Therefore, the power is uniformly distributed across the sub-channels.

Relation (3) shows that the capacity is a function of the CP duration. For a fixed number of sub-channels, although the SINR increases as the CP increases, the data rate decreases. Therefore, the optimal approach to choose the CP length should target the capacity maximization. In other words, the optimal CP for a given channel maximizes (3), and it is chosen according to

$$\mu_{OPT}=\arg\max_{\mu}\left\{\frac{1}{(M+\mu)}\sum_{k\in K_{ON}}\log_2\left(1+\frac{SINR^{(k)}(\mu)}{\Gamma}\right)\right\}. \quad (6)$$

Clearly, the upper limit for the choice of μ is equal to the channel impulse response duration ν in number of samples.

III. Resource Allocation Strategies

Bit-loading is a strategy that allows OFDM to efficiently use the channel resources and to approach the capacity limit given in (3). Some OFDM systems load the sub-channels with symbols belonging to different constellations, while others use the same constellation across all the sub-channels. Further, there is a limit in the constellation order that can be used. This is not taken into account by the capacity formula (3).

In this section we propose two allocation algorithms that perform bit-loading and optimal CP length computation taking into account practical constraints. In the first algorithm we assume that the constellations can vary across the sub-channels but a constraint on the constellation order is considered. In the second algorithm we assume that the constellation cannot vary across the sub-channels. This constraint significantly reduces the amount of feedback for bit-loading and it has been, for example, adopted in the wireless IEEE 802.11 standard [1].

It should be noted that when bit-loading is applied some of the sub-channels may not transmit any information, or in other words they can be switched off. Clearly, the PSD mask constraint can be satisfied with zero power allocation to some sub-channels. However, the SINRs change since the interference power experienced in the active sub-channels changes as a result of switching off some of the sub-channels. One more bit-loading iteration can increase the transmission rate. That is, at the first step we perform bit-loading under the PSD constraint by first calculating the SINRs assuming that all sub-channels are switched on. At this step we determine the set of active sub-channels K_{ON}. Then, we repeat the procedure recalculating the SINRs according to (4) using the set of active sub-channels that has been determined at the previous step.

The re-computation of the SINRs can be used in both the proposed bit-loading algorithms that we describe in detail in the following.

A. Bit-loading and CP Design

In the following, we present a simple bit-loading algorithm that satisfies the implementation constraints. It is derived from a modification of the one proposed by Chow and Cioffi and that is described in [6]. The algorithm distributes uniformly the power over the sub-channels (tones), it computes the SINRs according to (4) and it determines the optimal CP length. Then, bit-loading is computed by rounding the real number of bits to the integer number of bits that can be transmitted by the nearest available constellation. If some tones are switched off, we recompute the SINRs and we do another bit-loading iteration.

The procedure implemented by the algorithm is the following:

a) Set *RateTemp=0*, *iteration*=0 and $K_{ON}=\{0,...,M-1\}$ or equal to the set of active tones according to the transmission mask.

b) Distribute uniformly the power among the sub-channels belonging to K_{ON} to meet the PSD constraint, and compute the $SINR^{(k)}(\mu)$ as in (4) for $k\in K_{ON}$. Compute the optimal cyclic prefix as follows:

$$\mu_{opt}=\arg\max_{0\le\mu<\nu}\left\{\frac{1}{M+\mu}\sum_{k\in K_{ON}}\log_2\left(1+\frac{SINR^{(k)}(\mu)}{\Gamma}\right)\right\}. \quad (7)$$

c) Determine bit-loading on sub-channel $k\in K_{ON}$ as follow

$$\bar{b}^{(k)}=\left[\left[\min\left\{\log_2\left(1+\frac{SINR^{(k)}(\mu_{opt})}{\Gamma}\right),\log_2\left(M_{QAM\_\max}\right)\right\}\right]\right], \quad (8)$$

where $[[\bullet]]$ denotes the operation of rounding the bits in each sub-channel to the nearest available constellation towards the one of smallest order or to zero if the achievable rate is smaller than 0.5.

d) Compute the transmission rate (in bit/s) as

$$R(\mu_{opt}) = \frac{1}{(M+\mu_{opt})T} \sum_{k \in K_{ON}} \bar{b}^{(k)} . \quad (9)$$

If for some sub-channels $\bar{b}^{(k)} = 0$ and *iteration*=0, set $RateTmp = R(\mu_{opt})$ and *iteration*=1 and determine the set of active sub-channels K_{ON}, i.e., the set of sub-channels for which $\bar{b}^{(k)} \neq 0$. Then, repeat the procedure from step b) Otherwise, keep the last bits allocation and use the optimal CP given by the previous iteration.

B. Allocation of Identical Constellations and CP Design

We now consider an OFDM system that is allowed to transmit identical constellations on all the sub-channels. In the following we denote with S the set of constellation orders that can be used, e.g., $S = \{\gamma : \gamma = 2,4,16,...,M_{QAM\_\max}\}$ where $M_{QAM\_\max}$ is the highest constellation order.

The allocation problem becomes the problem of selecting the constellation and choose the CP length such that rate is maximized. From (3), the *k*-th sub-channel could be loaded with a constellation of order

$$\gamma^{(k)}(\mu) = \lfloor\lfloor 1 + SINR^{(k)}(\mu)/\Gamma \rfloor\rfloor \quad (10)$$

where $\lfloor\lfloor \bullet \rfloor\rfloor$ denotes the operation of rounding to the nearest available constellation lower than the argument. Now, due to the fact that all sub-channels have to be loaded with the same constellation, for each constellation γ belonging to S and for each CP, we can compute the number of sub-channels that are allowed to transmit. That is, we compute $M_{ON}(\gamma,\mu)$ that corresponds to the number of sub-channels for which $\gamma^{(k)}(\mu)$ is equal to or larger than γ, as follows

$$M_{ON}(\gamma,\mu) = \sum\nolimits_{k \in K_{ON}} \xi^{(k)}(\mu), \quad (11)$$

with

$$\xi^{(k)}(\mu) = \begin{cases} 1 & \gamma^{(k)}(\mu) \geq \gamma \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Thus, the achievable rate can be computed as

$$R(\gamma,\mu) = \frac{M_{ON}(\gamma,\mu)}{(M+\mu)T} \log_2(\gamma). \quad (13)$$

Finally, to determine the optimal CP and constellation order we perform the following maximization

$$(\mu_{opt}, \gamma_{opt}) = \arg\max_{\mu,\gamma} R(\gamma,\mu). \quad (14)$$

It is interesting to note that with uncoded modulation and the same target symbol error rate $\widehat{P}_e$ over all sub-channels, the SINRs have to fulfil the following relation in order to select the constellation of order γ

$$SINR^{(k)}(\mu) \geq \frac{\gamma-1}{3}\left(Q^{-1}\left(\frac{\widehat{P}_e}{N_e^{\gamma}}\right)\right)^2, \quad (15)$$

with $N_e^{\gamma} = 4\left(1-(\gamma)^{-1/2}\right)$. In other words, we load for a given CP the constellation γ for all those tones for which (15) holds true. This can be proved by noting that the margin with uncoded modulation is given by

$$\Gamma = \left(Q^{-1}\left(\hat{P}_e / N_e^{\gamma}\right)\right)^2 / 3 . \quad (16)$$

Then, if we substitute (16) in (10) we obtain (15).

IV. Numerical Results

To obtain numerical examples we consider the statistical PLC channel model presented in [9]. It synthesizes the channel frequency response with a finite number of multipath components [10] as follows

$$G_{CH}^{+}(f) = A\sum_{i=1}^{N_P} g_i \; e^{-(\alpha_0+\alpha_1 f^k)d_i} \; e^{-j2\pi f(d_i/v_P)}, \quad 0 \leq f \leq B .$$

The number of such components N_P is drawn from a Poisson process. The attenuation factor is denoted with *A*. Further, the reflection factors g_i are considered to be uniformly distributed. The parameters have been chosen to fit responses obtained from measurements. They are the following: $k=1$, $\alpha_0 = 0.3\times10^{-2}$, $\alpha_1 = 4\times10^{-10}$, $v_P = 2\times10^8$, average path rate equal to 0.2, maximum path length of 800 m. The channel has been generated in the band 0-37.5 MHz which is the one deployed by HPAV. Further, it has been truncated to $5.57\mu s$.

The signal is transmitted with a power spectral density equal to -50 dBm/Hz (as in most of the commercial systems) and the background noise equals -140 dBm/Hz [11]-[12]. The channel is normalized such that the average path loss at zero frequency is equal to 70, 50 and 30 dB. Therefore the SNR at zero frequency is respectively equal to 20, 40, and 60 dB.

Fig. 1 shows two channel impulse responses. These are representative of a best case (BeC) and a worst case (WoC) responses. Details on how we have chosen the BeC and the WoC channels can be found in [4]. For the BeC and the WoC channels the average channel path loss, averaged over the transmission band, is respectively equal to 1.3 and 11.6 dB. Therefore, the average SNR, in dB, can be computed as the difference between the SNR at zero frequency and the above defined average channel path loss.

In the following, we first present the simulations results for the algorithm described in Section III.A. Then, we present the results of the use of the algorithm described in Section III.B.

A. Bit-loading

For the numerical results we choose parameters similar to those employed in HPAV. We further vary the number of tones in the 0-37.5 MHz band fixing *M*=384, 768, 1536 and we switch off some tones at the band edges to obtain a useful transmission band from 2 MHz to 28 MHz. Thus, the maximal number of useful tones is respectively equal to 266,532,1065.

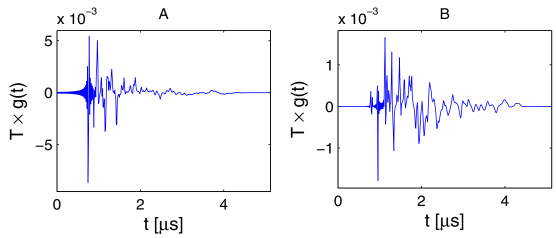

Fig. 1. (A) and (B) represent respectively the best and the worst channel impulse response.

The constellations employed are 2-PAM, 4,8,16,64,256,1024-QAM. The gap factor is set to $\Gamma = 9$ dB. The channels used are the BeC and the WoC channels above presented. The baseline system uses a fixed CP length equal to 5.57 μs (209 samples) that is equal to the channel duration. The same CP duration is deployed in the HPAV system.

Figs. 2-4 show the bit rate as a function of the CP length for an SNR equal to 20, 40 and 60 dB for the BeC and the WoC channels using the bit-loading algorithm of Section III.A. The number of tones equals 384 in Fig.2, 768 in Fig. 3, and 1536 in Fig. 4. The CP has been increased by step of 5 samples. The figures show that the achievable rate is a convex function of the CP length. Therefore, an optimal CP value can be determined. It is interesting to note that the optimal CP length varies not only as a function of the channel response but also as a function of the SNR.

Now, in Fig. 5 we show the bit rate achieved with the optimal CP as a function of the number of tones. We also report the rate that is obtained via bit-loading and a fixed CP equal to 5.57 μs (209 samples). In all SNR cases and for all number of tones, the rate gains obtained optimizing the CP range from 5% to 20% w.r.t. to the conventional choice of a CP equal to the channel length (209 samples). The gains due to CP adaptation diminish as we increase the number of tones *M*. These results are simply justifiable observing that if we increase *M*, the impact of the CP length on the achievable rate decreases. It is also interesting to note that in some cases the CP optimization in a system with a lower number of tones yields similar or even higher rate than that obtained with a higher number of tones and CP equal to 5.57 μs (209 samples). For instance, for SNR=20 dB the rate with *M*=384 and optimal CP allocation is higher than the rate with *M*=768 and CP=5.57 μs, and similar to the rate with *M*=1536 and CP=5.57 μs (essentially the parameters of HPAV). Therefore, the results suggest that in certain cases it is possible to maximize rate and lower the implementation complexity by the optimal design of the CP.

We further note that only for the WoC channel and an SNR of 20 dB, the recomputation of the bit-loading, because some tones were switched off in the first iteration, has yielded some small improvement. This can be explained by the fact that number of tones switched off by the bit-loading algorithm is typically small. This translates into only little sub-channel SINR variations.

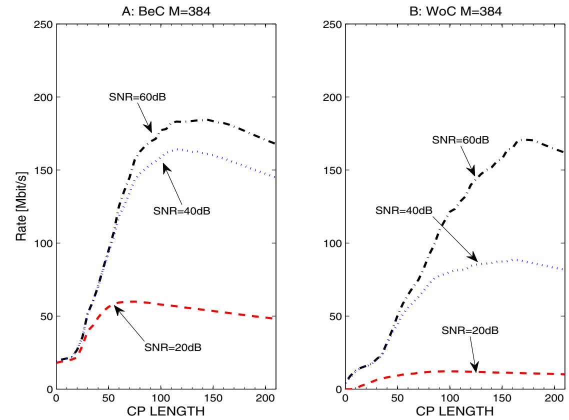

Fig. 2. Bit rate for an SNR equal to 20, 40 and 60 dB. (A) shows the results for the BeC channel, while (B) shows the results for the WoC channel. The number of tones is equal to 384.

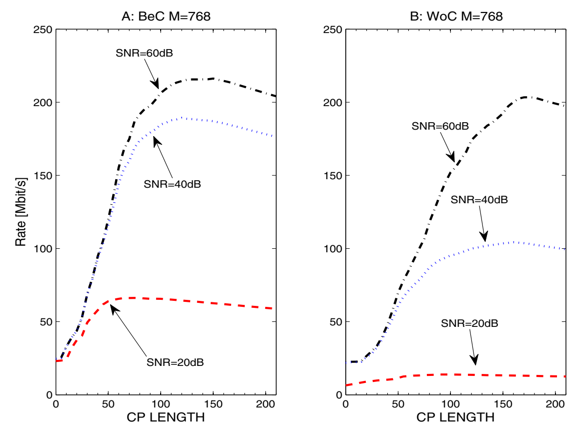

Fig. 3. Bit rate for an SNR equal to 20, 40 and 60 dB. (A) shows the results for the BeC channel, while (B) shows the results for the WoC channel. The number of tones is equal to 768.

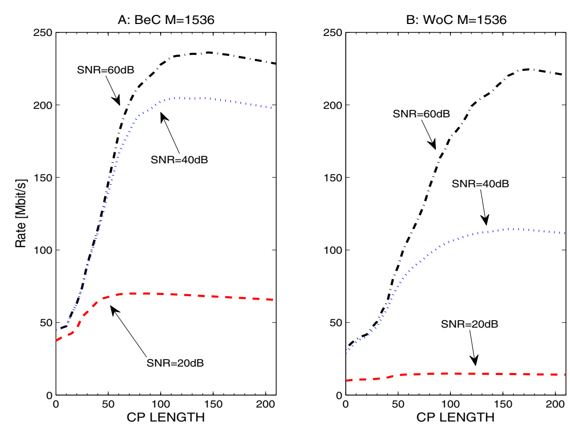

Fig. 4. Bit rate for an SNR equal to 20, 40 and 60 dB. (A) shows the results for the BeC channel, while (B) shows the results for the WoC channel. The number of tones is equal to 1536.

Table I lists the bit rate for a number of tones equal to 384 and 1536 for the BeC and WoC channels. Therein, we also list the achievable rates without considering the constellation constraint. In other words, the achievable rates given by the use of (3) and (6).

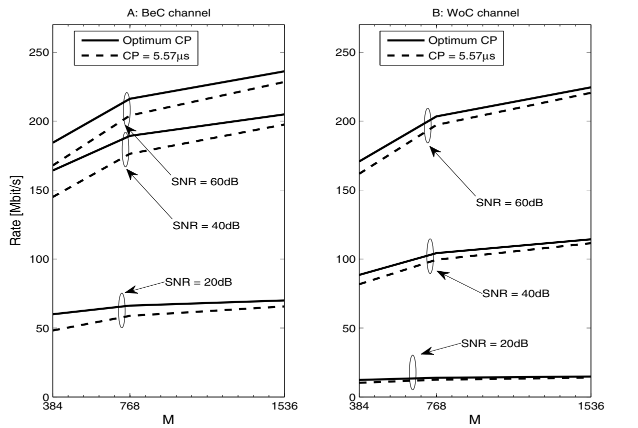

Fig. 5. Bit rate with the bit-loading algorithm as a function of number of tones M=384, 768 and 1536 both with optimal CP and with sub-optimal CP of duration equal to the channel length. The channel employed is (A) the BeC channel and (B) the WoC channel.

Table I. Summary of achievable rate obtained using the bit-loading algorithm with M=384 and 1536 tones, with and without constellation constraints and for the best and worst channels.

Bit Rate [Mb/s]	No Constellation constraint		Constellation constraint	
	Optimal CP M=384/1536	CP=5.57 µs M=384/1536	Optimal CP M=384/1536	CP=5.57µs M=384/1536
BeC SNR=60dB	279.9 / 368.7	266.2 / 362.4	184.3 / 236.1	167.8 / 228.4
BeC SNR=40dB	170.9 / 217.4	154.7 / 210.6	164.2 / 204.8	145 / 197.5
BeC SNR=20dB	61.9 / 72.2	49.5 / 67.3	60 / 70	48.1 / 66.6
WoC SNR=60dB	206.7 / 279	197.7 / 268.9	170.8 / 224.5	161.8 / 220.5
WoC SNR=40dB	94.6 / 122.5	88.2 / 119.5	88.5 / 114.3	81.7 / 111.5
WoC SNR=20dB	13.4 / 16.3	11.5 / 15.5	12.3 / 14.8	10.2 / 14.1

B. Allocation of Identical Constellations

In Fig. 6 we report the bit rate as a function of the CP length when the bit loading algorithm uses the same constellation on all tones. We consider an OFDM system with 384 tones and an SNR of 20 and 40 dB. The channel employed is the BeC channel. The SER target is equal to 10^{-6}. The CP has been increased in steps of 5 samples. The set of constellations employed is $S=\{\gamma:\gamma=2-PAM,4,16,64-QAM\}$.

As we can see in Fig. 6 for an SNR equal to 20 dB the optimal joint choice of the CP and the constellation order is $\mu=60$, and $\gamma=4$. For these values the system reaches a rate of 34 Mbit/s. When the SNR equals 40 dB, the best choice is $\mu=125$, and $\gamma=64$ yielding a rate equal to 108.7 Mbit/s.

In all SNR cases the rate gains obtained optimizing the CP range from 14% to 32% w.r.t. to the standard choice of a CP equal to the channel length. Clearly, the comparison with the previous bit-loading algorithm shows that lower rates can be achieved. However, this second algorithm is simpler and requires less amount of feedback to the transmitter.

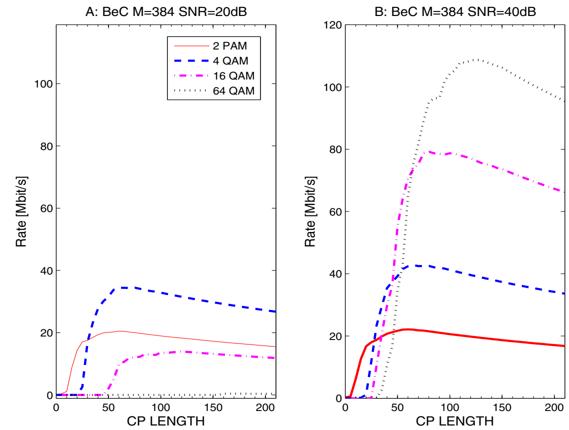

Fig. 6. Bit rate for the BeC channel using a constant constellation across all the tones with SER target equal to 10^{-6}. With (A) SNR equal to 20 dB and (B) SNR equal to 40 dB.

V. CONCLUSIONS

We have proposed two practical bit-loading algorithms that include the CP adaptation. The first algorithm adapts the constellations over the sub-channels jointly with the CP length. The second algorithm allocates identical constellations on all the sub-channel and adapts the CP length. The numerical results show that both algorithms yield significant improvements w.r.t. to the conventional choice of using a CP as long as the channel impulse response duration. The use of the same constellation on all the sub-channels provides smaller data rate w.r.t. to the full adaptation of the constellations. However, the former approach allows to diminish the amount of feedback to the transmitter for the adaptation.

REFERENCES

[1] IEEE Std.802.11, "Wireless LAN Medium Access Control and Physical Layer Specification," pp. 591-612, June 2007.

[2] Home Plug AV System Specifications, Version 1.0.09, Feb., 2007.

[3] Universal Powerline Association DHS Specific., Version 1.0, May 2006.

[4] A. M. Tonello, S. D'Alessandro, L. Lampe, "Bit, tone and Cyclic Prefix Allocation in OFDM with Application to In-Home PLC," *Proc. of IFIP Wireless Days2008*, Dubai, November 2008.

[5] S. Galli, H. Koga, N. Kodama, "Advanced Signal Processing for PLCs: Wavelet-OFDM," *Proc. of Int. Symp. Power Line Commununications* 2008, Jeju Island, Korea, pp.187-192, April 2008.

[6] J. M. Cioffi " chapter 4," *Advanced Digital Communication*, available at http://www.stanford.edu/class/ee379c/reader.html.

[7] A. M. Tonello and F. Pecile, "Analytical Results about the Robustness of FMT Modulation with Several Prototype Pulses in Time-Frequency Selective Fading Channels," *IEEE Trans. Wireless Comm.*, April 2008.

[8] I. Kalet, "The Multitone Channel," *IEEE Transaction on Communications,* vol. 37, no. 2, February 1989.

[9] A. M. Tonello, "Wideband Impulse Modulation and Receiver Algorithms for Multiuser Power Line Communications," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, Article ID 96747, 14 pages, 2007. doi:10.1155/2007/96747.

[10] M. Zimmermann, K. Dostert, "A Multipath Model for the Powerline Channel," *IEEE Trans. on Comm*., vol. 50, pp. 553-559, April 2002.

[11] M. Tlich et al., " A Broadband Powerline Channel Generator," *Proc. of ISPLC 2007*, Pisa, pp. 505-510, March 2007.

[12] F. Cañete, J.Cortés, L.Díez, and J. Entrambasaguas, "Analysis of the Cyclic Short-Term Variation of Indoor Power Line Channels," *IEEE J. Sel. Areas Commun*., vol. 24, no. 7, pp. 1327-1338.