A Novel Multi-carrier Scheme: Cyclic Block Filtered Multitone Modulation

Andrea M. Tonello[(1,2)]
[(1)]WiPli Lab - Wireless and Power Line Communications Lab, University of Udine, Italy
[(2)]WiTiKee s.r.l., Udine, Italy
email: tonello@uniud.it, web: www.diegm.uniud.it/tonello - www.witikee.com

***Abstract*—A novel filter bank modulation scheme is proposed. This architecture is based on the Filtered Multitone (FMT) modulation concept where well frequency confined prototype pulses are deployed. However, in the proposed scheme linear convolutions are replaced by circular convolutions. We refer to this new scheme as Cyclic Block Filtered Multitone Modulation (CB-FMT). Both the synthesis and the analysis filter banks in CB-FMT can be efficiently implemented in the frequency domain via the concatenation of Fast Fourier Transforms (FFTs). Furthermore, it is possible to design the prototype pulse so that the system is orthogonal and it does not experience any inter-symbol and inter-carrier interference in ideal conditions. To cope with the interference introduced by a frequency selective fading medium, simple frequency domain equalization can be used. Numerical results show that CB-FMT can provide better performance than FMT and OFDM in frequency selective fading with comparable or lower complexity.**

I. Introduction

Research and development of filter bank modulation schemes is quite large nowadays. This is because of the increasing demand of broadband telecommunication services both over wireline and wireless channels. Wide band channels are characterized by frequency selectivity which translates in time dispersive impulse responses that cause significant inter-symbol interference (ISI) in digital communication systems. The main idea behind filter bank modulation, also referred to as multi-carrier modulation, is to convert a sequence of data symbols at high rate, into a number of sub-sequences at low rate [1]. Each low rate sequence is transmitted through a sub-channel that has sufficiently narrow frequency response so that the equalization task can be simplified.

A popular multi-carrier scheme is orthogonal frequency division multiplexing (OFDM) [2] which has found application in the WLAN standard IEEE 802.11, in the WMAN standard WiMAX, in the ADSL standard, and in the power-line communication standard IEEE P1901. Another proposed technique is Filtered Multitone (FMT) modulation [3] that is a discrete time implementation of a multi-carrier system where sub-carriers are uniformly spaced and the sub-channel pulses are identical. In general, FMT privileges the sub-channel frequency confinement rather than the time confinement, as for example OFDM does. With frequency confined pulses the sub-channels are quasi-orthogonal to each other which allows the system to experience minimal inter-carrier interference (ICI), while the inter-symbol interference (ISI) introduced by the frequency selective (dispersive) channel can be mitigated with sub-channel equalization [4]-[5]. The high sub-channel frequency confinement renders FMT also a good candidate for multiuser asynchronous communications, e.g., in the wireless uplink, since loss of orthogonality is minimal also in the presence of time/frequency asynchronism between the user signals [6]. Typically, high sub-channel frequency confinement is obtained with the use of long prototype pulses that may increase the implementation complexity [7]-[8].

Our proposed novel multi-carrier modulation scheme aims at simplifying the implementation complexity and yielding high performance in frequency selective channels as those encountered in wireless mobile communications. The key idea is to consider the conventional FMT scheme but to substitute the linear convolutions in the synthesis and analysis filter banks with cyclic convolutions. This yields a new system that we refer to as Cyclic Block Filtered Multitone Modulation (CB-FMT), since, differently from conventional FMT, it transmits data symbols in blocks. Both the synthesis and the analysis filter banks in CB-FMT can be efficiently implemented in the frequency domain via discrete Fourier transforms (DFTs) and hence by Fast Fourier Transforms (FFT) followed, at the receiver, by either zero forcing (ZF) or minimum mean square error (MMSE) equalization.

The paper is organized as follows. In Section II, we describe the conventional FMT system. Then, we present the CB-FMT scheme in Sections III, and its efficient frequency domain implementation in Section IV. In Section V, we derive the criteria under which the CB-FMT scheme is orthogonal, and we present the main steps to design an orthogonal system. In Section VI, we describe the equalization stage, while, in Section VII, we analyze the computational complexity showing that it is lower than conventional FMT. In Section VIII, we compare CB-FMT with FMT and OFDM in terms of average signal-to-interference ratio (SIR) and average symbol error rate (SER) in typical wireless fading channels. Finally, in Section IX, the conclusions follow.

II. Conventional FMT scheme

In Fig.1, the conventional FMT modulation system is depicted. The transmitter generates the signal $x(n)$ as

$$x(n) = \sum_{k=0}^{K-1} \sum_{\ell \in \mathbb{Z}} a^{(k)}(N\ell) g(n - N\ell) W_K^{-nk}, \qquad (1)$$

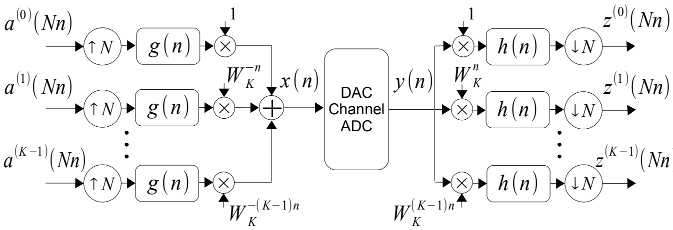

Fig. 1. Conventional Filtered Multitone (FMT) transceiver scheme.

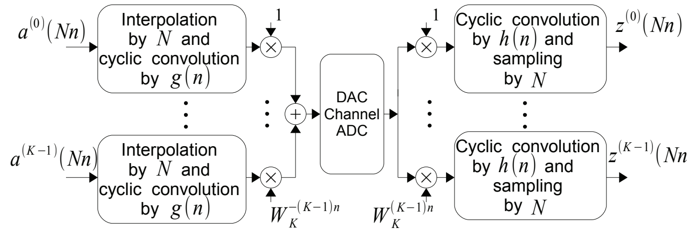

Fig. 2. Cyclic Block Filtered Multitone (CB-FMT) transceiver scheme.

where $a^{(k)}(Nn)$ is the sequence of complex data symbols, e.g., M-QAM, $g(n)$ is the prototype pulse, and $W_K^{-nk} = e^{j\frac{2\pi}{K}nk}$.

According to (1), the low rate signals $a^{(k)}(Nn)$ are upsampled by N and filtered by the prototype pulse $g(n)$. The obtained signals are modulated by complex exponential functions, then summed yielding $x(n)$. The signal $x(n)$ has nominal bandwidth $1/T$ where T is the sampling period in seconds. Then, $x(n)$ is digital-to-analog converted, upconverted to RF, and transmitted over the radio channel.

The received low-pass signal is analog-to-digital converted to obtain

$$y(n) = x * h_{CH}(n) + \eta(n),$$

where $*$ denotes linear convolution, $h_{CH}(n)$ is the discrete time equivalent channel response, and $\eta(n)$ is the additive white Gaussian noise contribution.

The signal at the output of the analysis filter bank can be written as

$$z^{(k)}(Nn) = \sum_{\ell \in \mathbb{Z}} y(\ell)\, W_K^{\ell k}\, h(Nn-\ell). \quad (2)$$

According to (2), the received signal $y(n)$ is demodulated by complex exponential functions, and the obtained signals are filtered by $h(n)$ and sampled by N. Since the main characteristic of FMT is to deploy a well frequency confined prototype pulse, a common choice is to use a truncated root-raised-cosine pulse for both the synthesis and the analysis banks so that the filters are matched.

III. Cyclic Block FMT Modulation

To obtain the CB-FMT scheme we propose to consider the transmission of a block of L data symbols (one block per sub-channel) and K sub-channels. Furthermore, without loss of generality, we consider a causal FIR prototype pulse with number of samples equal to $M_2 = LN$, with L integer. This is a non-limiting condition since the pulse can be zero-padded. Then, we exchange the linear convolution in (1) with the M_2-point circular convolution [9] to obtain

$$x(n) = \sum_{k=0}^{K-1} \sum_{\ell=0}^{L-1} a^{(k)}(N\ell) g((n-N\ell))_{M_2}\, W_K^{-nk}, \quad (3)$$
$$n \in \{0, \cdots, M_2 - 1\},$$

where $g((n-N\ell))_{M_2}$ denotes the M_2 periodic repetition of $g(n)$ shifted by $N\ell$ (the cyclic shift).

Similarly to the synthesis stage, the analysis filter bank in CB-FMT works with circular convolutions, i.e., the k-th filter bank output at time Nn is obtained as follows

$$z^{(k)}(Nn) = \sum_{\ell=0}^{M_2-1} y(\ell) W_K^{\ell k} h((Nn-\ell))_{M_2} \quad (4)$$
$$n \in \{0, \cdots, L-1\},$$

where $h((Nn-\ell))_{M_2}$ denotes the circular shifted version of the analysis pulse.

In Fig.2 a general CB-FMT transceiver is depicted. It should be noted that each of the K sub-channels conveys a block of L data symbols over a time period equal to LNT seconds. Therefore, the transmission rate equals

$$R = \frac{K}{NT} \quad \text{symbols/sec.}$$

In the next sections we will show that it is possible to design a perfect reconstruction filter bank with respect to the circular convolution such that $z^{(k)}(Nn) = a^{(k)}(Nn)$ in the absence of noise and with an ideal channel.

IV. Frequency Domain Implementation of CB-FMT

The CB-FMT scheme can be efficiently implemented exploiting the Fast Fourier Transform (FFT) algorithm. This is illustrated in what follows.

Let us assume $M_2 = LN = QK$, with L, N, Q, K integer numbers. Then, if we compute the M_2-point DFT of the transmitted signal $x(n)$ defined as $X(i) = \sum_{n=0}^{M_2-1} x(n) W_{M_2}^{ni}$, we will obtain the following input-output relation in the frequency domain

$$X(i) = \sum_{k=0}^{K-1} A^{(k)}(i-Qk) G(i-Qk), \quad (5)$$

where $G(i)$ is the M_2-point DFT of the prototype pulse $g(n)$, and $A^{(k)}(i)$ is obtained by computing an L-point DFT of the data block $a^{(k)}(Nn)$, $n \in \{0, \cdots, L-1\}$.

It can be noted that if the DFT of the prototype pulse has only Q coefficients that differ from zero, e.g., $G(i) = 0$ for $i \in \{Q, \cdots, M_2 - 1\}$, then (5) becomes

$$X(i) = A^{(k)}(i-Qk) G(i-Qk), \quad (6)$$
$$i \in \{Qk, \cdots, Q(k+1)-1\},$$
$$k \in \{0, \cdots, K-1\},$$

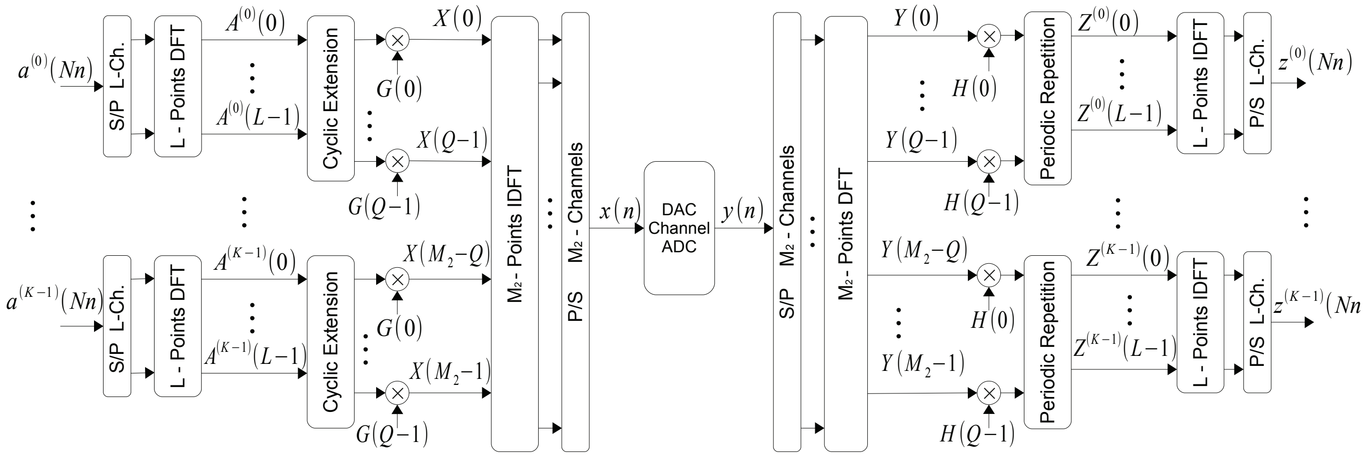

Fig. 3. Efficient implementation of the Cyclic Block Filtered Multitone (CB-FMT) transceiver scheme.

that is, the pulse frequency domain coefficients weight the block of symbols $A^{(k)}(i-Qk)$, for $i \in \{0, \cdots, Q-1\}$.

The above results suggest a very simple frequency domain implementation of the CB-FMT synthesis stage. The realization is shown in Fig.3. It comprises a L-point DFT for each sub-channel, followed by a cyclic extension to Q coefficients. The obtained streams are weighted by the Q DFT components of the pulse. Finally, a M_2-point IDFT yields the signal $x(n)$.

The analysis stage can also be implemented in the frequency domain as

$$z^{(k)}(Nn) = \sum_{p=0}^{L-1} Z^{(k)}(p-Qk) W_L^{-(p-Qk)n}, \tag{7}$$

where the L-point DFT of the signal $z^{(k)}(Nn)$ has the following input-output relation

$$Z^{(k)}(p-Qk) = \sum_{q=0}^{N-1} Y(p+Lq) H(p+Lq-Qk), \tag{8}$$

where $Y(i)$ is the M_2-point DFT of the signal $y(n)$, and $H(i)$ is the M_2-point DFT of the analysis prototype pulse $h(n)$.

According to (7) and (8), the received signal $y(n)$ is processed by a M_2-point DFT yielding the coefficients $Y(i)$ that are then weighted by the DFT coefficients of the prototype pulse $h(n)$, i.e., $H(i)$. Then, a periodic repetition with period N is performed, and finally, the L-point IDFT is applied to each sub-block of L coefficients, yielding the signal $z^{(k)}(Nn)$.

V. Orthogonality Conditions and Prototype Pulse Design

A. Orthogonality Conditions

In this section we aim to derive the orthogonal conditions for CB-FMT. In general, a filter bank system will be said orthogonal if it has the perfect reconstruction property and if the receiver filter bank is matched to the analysis filter bank.

We recall that $M_2 = NL = KQ$. Orthogonality in CB-FMT is achieved with a finite impulse response (FIR) prototype pulse that satisfies the following proposition.

Proposition 1. The sufficient conditions that the prototype pulse of a CB-FMT system ($g(n) = h^*(-n)$) must satisfy to attain the orthogonality property are:

1. the M_2-point DFT of the prototype pulse $g(n)$ has only Q non zero coefficients;
2. the prototype pulse $g(n)$ is orthogonal to its cyclic translations of multiples of N, i.e.,

$$\begin{aligned}\left[g \otimes g_-^*\right](Nn) &= \sum_{\ell=0}^{M_2-1} g((\ell))_{M_2} g^*((\ell - Nn))_{M_2} \\ &= \delta(Nn),\end{aligned} \tag{9}$$

where $\otimes$ denotes circular convolution, and we defined the Kronecker delta $\delta(n) = 1$ if $n = 0$, and zero otherwise. We can rewrite the relation in the frequency domain by applying the M_2-point DFT at both sides of the equation (9), yielding

$$\sum_{k=0}^{N-1} G(i-Lk) G^*(i-Lk) = 1 \quad \text{(i.e., a constant).}$$

Proof. Condition 1. is sufficient to avoid ICI since the sub-channels are spaced by $M_2/K = Q$ DFT points. Condition 2. grants zero ISI in each sub-channel since it is the Nyquist criterion for discrete time periodic signals.

B. Orthogonal Prototype Pulse Design

The frequency domain implementation suggest to synthesize the prototype pulse in the FD with a finite number of frequency components. This can be done by choosing a pulse that belongs to the Nyquist class with roll-off β, Nyquist frequency $1/(2NT)$, and frequency response $\hat{G}(f)$. Then, the frequency components of the prototype pulse are derived sampling the Nyquist pulse as follows: $G(i) = H(i) = \sqrt{\hat{G}(\frac{i}{M_2 T})}$. In Fig.4 some examples of orthogonal prototype pulse are reported.

To satisfy *Proposition 1*, the pulse $\hat{G}(f)$ must have only Q DFT components that differ from zero. Consequently, since the sub-channel data rate is $1/(NT)$ the in-band part (Nyquist

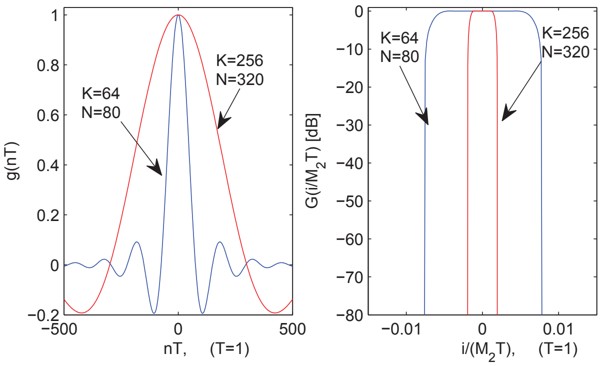

Fig. 4. Orthogonal prototype pulse considering roll-off $\beta = 0.2$, $L = K$ and $Q = N$.

band) of the pulse is defined by $M_2/N = L$ DFT points. Furthermore, we must choose the roll-off $\beta \leq (Q-L)/Q$ to prevent the pulse spectrum to exceed the bandwidth $1/(KT)$.

VI. Outer Cyclic Prefix and Equalization

Although the CB-FMT scheme is designed to be orthogonal, when signaling over frequency selective channels some interference may arise. More specifically, the multi-path channel yields Inter Block Interference (IBI). In order to prevent the system from suffering of IBI, analogously to OFDM we append a cyclic prefix to the M_2-point DFT output block as depicted in Fig. 5. This renders the transmitted signal $x(n)$ to be periodic with period M_2, and if the length of the CP (denoted as CP) is longer than the channel duration, the received signal $y(n)$ can be viewed as a cyclic convolution between $x(n)$ and the channel impulse response $h_{CH}(n)$. Then, the DFT outputs $Y(q)$ read

$$Y(p) = X(p)H_{CH}(p) + N(p) \tag{10}$$

where $H_{CH}(p)$ denotes the M_2-point DFT of the channel.

This enables the application of a simple frequency domain Zero Forcing (ZF) or Minimum Mean Squared (MMSE) equalizer as shown in Fig. 5. In particular, the receiver M_2-point DFT outputs are weighted by the coefficients

$$H_{EQ}(p) = \frac{H(rem[p,Q])H^*_{CH}(p)}{|H_{CH}(p)|^2 + \sigma^2_\eta K_{MMSE}} \tag{11}$$

where we have defined the remainder operation as $rem[p,Q] = p - \lfloor p/Q \rfloor Q$, while σ^2_η denotes the power of the received noise, and K_{MMSE} equals 1 if we consider a MMSE equalizer, 0 if we consider a ZF equalizer. It should be noted that the K CB-FMT sub-channels are not necessarily flat and the equalizer is capable to coherently collect their energy and therefore to exploit the sub-channel frequency diversity. Furthermore, with ZF equalization the system is perfectly orthogonal, i.e., no inter-channel and inter-symbol interference is present at the final L-point DFT outputs (see also Fig. 3).

VII. Computational Complexity

The transmitter and the receiver deploy K FFTs of L-points and one M_2-point IFFT. Furthermore, the signals along

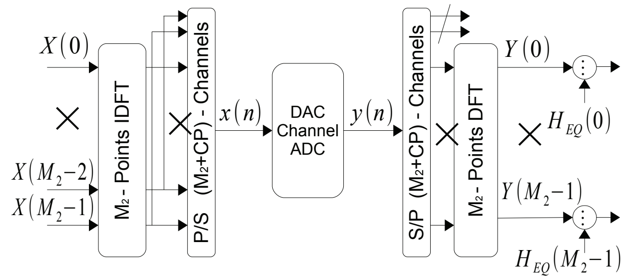

Fig. 5. Outer cyclic extension scheme with frequency equalizer.

the filter banks are weighted via M_2 multiplications. The complexity of the cyclic extension and periodic repetition can be neglected. Since we transmit a block of L symbols, it results that the number of complex operations $[cop]$ (additions and multiplications) per transmitted sample is equal to (both at the transmitter and at the receiver)

$$\frac{K\alpha L\log_2(L) + \alpha M_2\log_2(M_2) + M_2}{LN} \qquad \left[\frac{cop}{sample}\right].$$

To provide some numbers, we compare the complexity of CB-FMT with conventional FMT [8] assuming for both systems $K = 64$, $N = 80$, and a filter of length $M_2 = 64N$ in CB-FMT (assuming also $L = K$, $Q = N$) and $20N$ in FMT. If we further assume that a M-point FFT block has complexity equal to $\alpha M\log_2(M)$ $[cop]$ where $\alpha = 1.2$, we obtain that the complexity is equal to $\{44.8, 21.4\}$ $[cop]$ respectively for conventional FMT and CB-FMT. This shows the gain in complexity for CB-FMT yet using a longer pulse.

VIII. Performance in Fading Channels

In order to evaluate the performance of the proposed CB-FMT scheme, we consider transmission over a wireless dispersive fading channel having impulse response $h_{CH}(n) = \sum_{p=0}^{N_p-1} \alpha_p\, \delta(n-p)$ where α_p are assumed to be independent complex Gaussian variables with power $\Omega_p = \Omega_0\, e^{-p/\gamma}$, γ is the normalized delay spread, and the channel is truncated at $-10dB$.

We first evaluate the robustness in terms of Signal-to-Interference Power Ratio (SIR) versus delay spread γ, comparing CB-FMT without the outer CP, w.r.t. conventional FMT. It should be noted that in the absence of a CP the channel introduces a loss of orthogonality. The SIR is evaluated by first computing the sub-channel SIR for a given channel realization. Then, we average the SIR over the sub-channel and over the fading channel realizations. The simulation has been done considering CB-FMT with $K = \{16, 64, 256\}$, $N = (5/4)K$, $L = K$, $Q = N$, and $CP = 0$. As a baseline, we have considered conventional FMT with $K = 64$, and $N = (5/4)K$ that deploys a root-raised cosine pulse with roll-of factor equal to 0.2 and length $L_f = \{4, 12, 24\}N$.

Now, Fig.6 shows that CB-FMT has considerable better SIR performance than FMT especially for low values of normalized delay spread γ. Furthermore, in CB-FMT the SIR increases significantly as the number of sub-channels K increases.

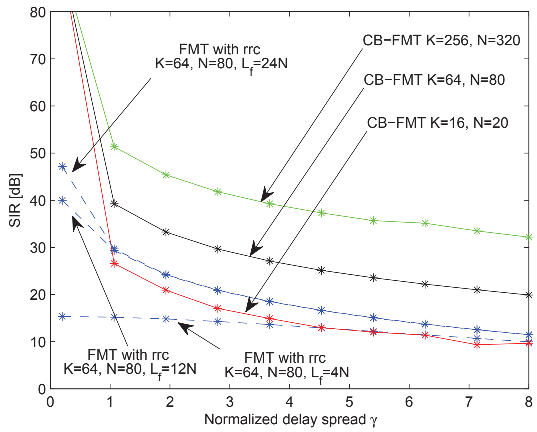

Fig. 6. SIR vs. normalized delay spread γ.

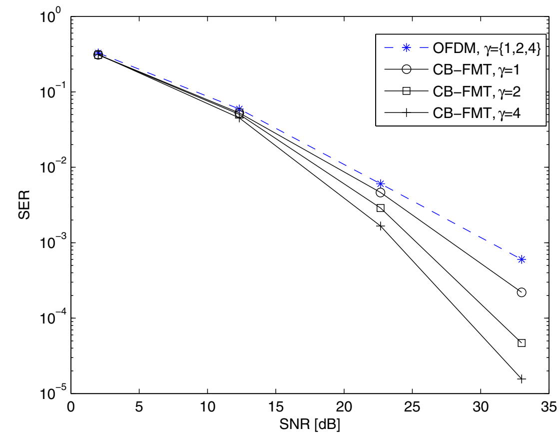

Fig. 7. SER vs. SNR varying the normalized delay spread γ.

Secondly, in Fig. 7 we report average symbol error rate (SER) versus average SNR for CB-FMT and OFDM for the values of delay spread $\gamma = \{1, 2, 4\}$. Both systems transmit QPSK data symbols and use a CP. The CB-FMT system has parameters $K = 8$, $N = 9$, $M_2 = 72$, and the outer CP has length $CP = 8$. The MMSE equalizer of Section VI is used. The baseline OFDM system has parameters similar to those of the WLAN IEEE 802.11 protocol, i.e., a number of sub-channels $K = 64$ and a CP with length 16 samples. Single tap equalization is performed at the receiver. With this choice of parameters both systems have the same transmission rate.

Fig. 7 shows that CB-FMT can significantly improve the SER, especially for high values of normalized delay spread γ. CB-FMT can transmit with a SER equal to 10^{-3} with SNR equal to $\{28, 26, 24\}$ [dB] respectively for delay spread values $\{1, 2, 4\}$, while OFDM achieves the same value of SER with SNR equal to 31 [dB] for all values of γ considered. This is due to the fact that CB-FMT in conjunction with the MMSE equalizer can exploit the sub-channel frequency diversity provided by the fading medium. Thus, the more dispersive the channel, the higher the gain is for CB-FMT. Since the CP is longer then the channel duration, in OFDM the sub-channels are flat faded and the single tap equalizer is sufficient and provides identical performance for all values of γ. Furthermore, the computational complexity for the considered systems is respectively equal to $\{11.6, 7.2\}$ $[cop/sample]$ for CB-FMT and OFDM (assuming the complexity of OFDM to be equal to $1.2\,log(K)$ $[cop/sample]$). Thus, with the chosen parameters there is a complexity increase which is however repaid with an increase in performance. Although not reported, the SER performance of conventional FMT with single tap equalization is worse than that of CB-FMT as it can also be deducted from the SIR results in Fig.6.

Other results about the application of CB-FMT in power line communication channels can be found in [10].

IX. Conclusions

We have presented a novel filter bank architecture referred to as Cyclic Block Filtered Multitone (CB-FMT) modulation. Differently from conventional filter bank modulation schemes, CB-FMT uses circular convolutions instead of linear convolutions. Furthermore, data symbols are transmitted in blocks.

We have derived an efficient frequency domain implementation for the CB-FMT scheme in the frequency domain and we have shown that it has lower complexity than conventional FMT even with longer pulses.

After having established the conditions under which the system is orthogonal, we have presented a simple procedure to design an orthogonal CB-FMT system. Furthermore, when an outer cyclic prefix is appended, simple frequency domain ZF or MMSE equalization can be used.

Numerical results have shown that CB-FMT has the potentiality to provide better SIR and SER performance than FMT and OFDM in frequency selective fading channels yet requiring low complexity.

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