Abstract - A space-time serial concatenated coding approach for spectrally efficient and reliable communications over fading channels is presented. The approach is based on bit-interleaved convolutionally coded modulation with multiple transmit antennas. The decoding problem of such a scheme is addressed in frequency selective fading channels, where an iterative procedure is proposed. At the first step a multiple-soft-in multiple-soft-out maximum a posteriori equalizer is deployed. Then, a soft-in soft-out maximum a posteriori convolutional decoder follows after deinterleaving. Soft information is exchanged between the decoder and the equalizer through multiple equalization and decoding iterations. A 2 bits/s/Hz system deploying two transmit antennas is presented. Simulation results show that, with just one receive antenna, few decoding iterations yield significant BER/FER gains in both frequency selective block fading and frequency selective fast fading over the uncoded system with dual receive diversity.

I. INTRODUCTION

It is well known that the reliability of wireless communications is significantly limited by the attenuation disturbances (fading) introduced by the propagation media. Diversity techniques are often necessary to counteract the detrimental effects of fading. They provide the receiver with replicas of the transmitted signal that may experience less attenuation through the use of temporal, frequency, polarization, and spatial resources [1].

Recently, schemes deploying multiple transmit and multiple receive antennas have gained a lot of attention since it has been shown that the capacity of a wireless link can be largely increased with such an architecture [2]. In order to enable reliable and spectrally efficient wireless communications, a systematic approach, known as space-time coding, has been presented in [3]. Space-time coding combines the design of channel coding, modulation, transmit and receive diversities.

In [3] performance criteria are derived to design space-time trellis codes for a frequency non-selective (i.e. flat) Rayleigh fading channel. Coding and modulation over multiple transmit antennas are jointly done, i.e. combined in a single entity, through a space-time extension of the trellis coded modulation concept [4]. This approach is referred to as STTCM, space-time trellis coded modulation. In STTCM a maximum likelihood decoder jointly decodes the signals that are simultaneously transmitted from different antennas. The structure of the space time code is such that destructive superposition is avoided.

Space-time block codes have been presented in [5] where it is shown that the orthogonal structure of these codes simplifies the maximum likelihood decoding algorithm, in flat fading channels, to simpler linear processing.

In this paper we deal with space-time coding over frequency selective fading channels that introduce inter-symbol interference. In particular we follow the alternative and novel space-time coding approach that has been presented in [6] for the flat fading case. The approach is based on the serial concatenation of a convolutional encoder, a bit-interleaver, and a space-time signal constellation mapper. We refer to it as space-time bit-interleaved coded modulation, STBICM. The information bits are encoded with a convolutional encoder. Then, they are appropriately interleaved and split into parallel bit streams that are mapped to signal constellation points using multi-level/phase modulators (e.g. $M$-PSK, $M$-QAM modulators). The parallel symbol streams are simultaneously transmitted from separate antennas. The resulting scheme is a space-time extension to multiple transmit antennas of the bit-interleaved coded modulation concept [7-8]. The bit interleaver deployed in STBICM keeps the channel encoder and the modulator separated, in contrast to STTCM [3]. Variable spectral efficiencies can be easily obtained by appropriate choice of the convolutional code rate, the number of transmit antennas, and the signal constellation mappers.

In [6] an iterative demapping/decoding procedure is applied over flat fading channels. Further, it is shown that the appropriate design of the convolutional code, the interleaver, and the bit-to-symbol mapping rule, allows optimizing the coding gain, and fully exploiting the spatial and temporal diversities. Code construction criteria are devised in [6] showing that differently from STTCM, the construction of STBICM with iterative decoding aims to maximize the distance properties (Hamming and Euclidean) at a bit level rather than at a symbol level. The diversity gain and the coding gain depend on the Hamming distance of certain coded bit sub-sequences. Further, appropriate bit mappings can enlarge the product Euclidean distance of the code and increase the coding gain.

In this paper we address the decoding problem of STBICM in a frequency selective fading channel that introduces time dispersion and consequently inter-symbol interference. The
proposed decoding strategy follows an iterative procedure that is applicable with one or more receive antennas, and that is constituted of two individually optimal steps. At the first step a multiple-soft-in multiple-soft-out MAP equalizer, i.e. based on the maximum a posteriori algorithm [9-10], is deployed. Then, a soft-in soft-out MAP convolutional decoder follows. Soft information is exchanged between the decoder and the equalizer through multiple equalization and decoding iterations.

Iterative decoding has originally found application in the context of turbo codes decoding [11]. Then, it has been applied to several detection/decoding problems such as iterative equalization of coded BPSK modulation [12], iterative demapping of coded QPSK [13], and iterative detection/equalization of coded M-DPSK [14] in the presence of co-channel interference. In the context of STBICM decoding, the proposed algorithm is capable of de-coupling with few iterations the signals that are simultaneously transmitted by the multiple transmit antenna array and that experience inter-symbol interference due to the frequency selective channel. With this decoding strategy both the spatial (multiple transmit and receive antennas), the temporal (coding and interleaving), and the multi-path (frequency selective fading) diversities can be exploited.

The paper is organized as follows. Section II revises the STBICM concept [6]. Section III describes the channel model. Section IV addresses the decoding algorithm in frequency selective fading. In Section V, a 2 bits/s/Hz system that deploys two transmit antennas is described, and performance results from simulations are reported. Finally, Section VI concludes the paper.

II. TRANSMITTER BASED ON STBICM

We consider a wireless communication system comprising $N_t \geq 2$ transmit antennas and $N_r \geq 2$ receive antennas [6]. At the transmitter (Fig. 1) the information bit stream $b_t$ is first convolutionally encoded and then bit-interleaved to produce the bit stream $d_t$. The interleaved bit stream is S/P converted into $N_r$ streams $d^i_t$ ($t=1,...,N_t$, $i=-\infty,...,\infty$). Each parallel stream is mapped into complex constellation points $x^i_k$ ($t=1,...,N_t$, $k=-\infty,...,\infty$) belonging to a multi-phase/level signal set (i.e. M-PSK or M-QAM signal sets). At time $kT$ (with $1/T$ symbol rate) the components of the vector $x^i_k = [x^i_k(1),...,x^i_k(N_r)]^{\top}$, after pulse shaping with identical filters and RF modulation, are simultaneously transmitted each from a different antenna.

The purpose of the bit interleaver is twofold. First, it is used to de-correlate the fading channel and maximize the diversity order of the system. Second, it removes the correlation in the sequence of convolutionally coded bits, and this is an essential condition in the iterative decoding algorithm that we propose in Section IV. We emphasize that no orthogonality constraint is imposed on the antenna constellations, and that with ideal interleaving independent bits are mapped into antenna constellation points.

Fig. 1. STBICM base band transmitter.

The spectral efficiency of such a scheme is $R=R_cN_t\log_2 M$ bits/s/Hz, with $R_c$ convolutional encoder rate, and $M$ modulation order. Different spectral efficiencies can be easily obtained by appropriate choice of the convolutional encoder rate, of the modulation order, and of the number of transmit antennas. In particular, a space-time bit-interleaved coded modem for communications at 2 bits/s/Hz is described in Section V. Other schemes with different spectral efficiencies and/or deploying a higher number of transmit antennas are presented in [6] where general STBICM design guidelines are derived from performance analysis.

III. CHANNEL MODEL

We consider propagation through a frequency selective fading channel with $N_p$ resolvable $T$-spaced rays. One or more receive antennas are deployed at the receiver. The received signal, at each receive antenna, is RF demodulated, filtered with a filter matched to the transmit pulse, and sampled at rate $1/T$. The sequence of samples at the $r$-th antenna filter output is then modeled as

$$y^r_k = \sum_{p=1}^{N_r} h_{p,k}^r x^r_{k-p+1} + n^r_k.$$  

In (1) $h_{p,k}^r$ represents the $p$-th tap equivalent channel impulse response of the link between the $r$-th transmit antenna and the $r$-th receive antenna, at time $kT$ (i.e. $n^r_k$ is a sequence of i.i.d. complex Gaussian variables with zero mean and variance $N_0/2$ per dimension). Furthermore, the channel taps are complex Gaussian distributed with zero mean (Rayleigh fading). They are independent over distinct antenna links and over distinct multi-paths (i.e. rays) of a given link. We consider both the case of fast fading, i.e. temporal uncorrelated fading coefficients, and the case of block fading, i.e. static fading coefficients over a block of transmitted symbols, but distinct blocks experience independent fading.

We define the average bit-energy-to-noise-ratio as follows

$$\text{BER} = \frac{\text{Power of the transmitted signal}}{\text{Power of the noise}}.$$
where we assume a normalized symbol constellation and a normalized channel profile, i.e. 
\[ E[|x_t^r|^2] = E_s \] and 
\[ \sum_{r=1}^{N_r} E[|h_{p,k}^r|^2] = 1 \] for \( r = 1, \ldots, N_r \) and \( t = 1, \ldots, N_t \).

In vector notation (1) can be written as

\[
y_k = \begin{bmatrix} y_k^1 \\ \vdots \\ y_k^{N_t} \\ n_k^1 \\ \vdots \\ n_k^{N_t} \end{bmatrix} = \begin{bmatrix} h_k^1 \cdots h_k^{N_r} \\ \vdots \\ h_k^{N_r} \cdots h_k^{N_r} \end{bmatrix} \begin{bmatrix} x_k^1 \\ \vdots \\ x_k^{N_r} \\ x_k^{N_r} \\ \vdots \\ x_k^{N_r} \end{bmatrix} + \begin{bmatrix} n_k^1 \\ \vdots \\ n_k^{N_t} \end{bmatrix} \tag{3} \]

where \( h_k^r = [h_k^{r,1} \cdots h_k^{r,N_s}]^T \) is the equivalent impulse response of the channel at time \( kT \) of the \( r \)-th antenna link, and 
\[ [x_k^1 \cdots x_k^{N_s-1} \cdots x_k^{N_s}]^T = [x_k^1 \cdots x_k^{N_s-1} \cdots x_k^{N_s}]^T. \]

Assuming to process blocks of \( N_t \) received samples, it is possible to concisely represent the overall sequence in (3) as 
\[ y = Hy + e \] where \( y \) is the \( N_t \) by \( N_t \) matrix of received samples.

The time evolution of (3) can be represented with a Markov chain. In fact, if we define the state at time \( k \) as 
\[ S_{k-1} = [x_{k-1}^1 \cdots x_{k-N_s+1}^1 \cdots x_{k-N_s}^{N_t}]^T, \]
the noisy sample \( y_k \) depends on the state \( S_{k-1} \) and on the symbols transmitted at time \( k \) on each antenna, i.e. 
\[ X_k = [x_k^1 \cdots x_k^{N_t}]^T. \]
This Markov chain has \( |S_{k-1}| = M^{N_s(N_s-1)} \) states, and \( M^{N_t} \) branches leaving and entering each state.

IV. ITERATIVE DECODING OF STBICM IN FREQUENCY SELECTIVE FADING

Decoding of STBICM is here addressed for the frequency selective fading channel model in Section III. It is based on the turbo-processing concept [12-14] where two individually optimal steps can be iteratively repeated (Fig. 2). At the first step (referred to as equalization) from the block of noisy channel observations \( y \) we compute the a posteriori log-likelihood ratios of the coded and interleaved bits

\[
L(d_i^t) = \log[P(d_i^t = +1) / P(d_i^t = -1)] \tag{4}
\]
for each transmit branch \( t = 1, \ldots, N_t \). Assuming perfect knowledge of the channel state information (i.e. \( H \)), and noting that we are observing a Markovian source in a memoryless noisy channel, the values in (4) can be computed by application of the maximum a posteriori algorithm [9-10].

Since in general we have observations from an array of receive antennas, and we want to compute soft values for the bits that are transmitted on overlapping channels affected by inter-symbol interference, this module can be interpreted as a multiple-soft-in multiple-soft-out a posteriori probabilities calculator (MIMO-APP). However, when a single receive antenna is available the algorithm reduces to a single-soft-in multiple-soft out module (SIMO-APP).

In the second step (referred to as decoding) the a posteriori log-likelihood ratios of the coded bits are P/S converted, deinterleaved, and fed to a soft-in soft-out convolutional decoder that is implemented according to the MAP algorithm [10]. The convolutional decoder provides both the log-likelihood ratios of the information bits \( L(b_i) \), and new/improved log-likelihood ratios of the coded bits \( L(c_i) \). Following the turbo decoding principle [11] extrinsic log-likelihood ratios of the coded bits are computed by subtracting the decoder inputs from the decoder outputs, \( L_s(c_i) = L(c_i) - L_a(c_i) \). The extrinsic values are interleaved, S/P converted, and fed back to the equalizer where they can be used in a new iteration as an estimate of the a priori log-likelihood ratios of the coded bits on each transmit branch, \( L_a(d_i^t) \). In order to minimize the correlation with previously computed soft information, extrinsic information is also computed at the equalizer output, \( L_e(d_i^t) = L(d_i^t) - L_a(d_i^t) \).

By repeating several times the above procedure, it is possible to greatly improve the performance of the system, where in the final iteration the decoded sequence of information bits is obtained from hard decisions on \( L(b_i) \).

Now, as we said the computation of the log-likelihood ratios in (4) can be carried out by application of the MAP algorithm. Here we summarize the fundamental steps. The details can be easily derived, and we refer the readers to [9-10], [14]. Let us assume to process blocks of \( N_t \) received samples. Denoting with \( d_i^t \) the bit that is mapped into the \( l \)-th bit position of the \( k \)-th complex symbol of the \( t \)-th transmit antenna \( (t = 1, \ldots, N_t; k = 1, \ldots, N_s; l = 1, \ldots, \log_2M = N) \), the
log-likelihood ratios in (4) can be computed as
\[ \ln \left( \frac{P(d_{k}^{ij}=+1|y,H)}{P(d_{k}^{ij}=-1|y,H)} \right) = \ln \left( \frac{\sum_{(S_{k},S_{k-1})} p(S_{k},S_{k-1}|y,H)}{\sum_{(S_{k},S_{k-1})} p(S_{k},S_{k-1}|y,H)} \right) \]

where \( D(d_{k}^{ij}) \) is the set of all possible state transitions \((S_{k-1},S_{k})\), from time instant \((k-1)T\) to time instant \(kT\), associated to the input bit \( d_{k}^{ij} = b, b = \pm 1 \). The product of a forward recursion, a backward recursion, and a transition probability yields the joint probabilities \( p(S_{k},S_{k-1},y,H) \) [10].

In turn, the transition probability is the product of the channel probability density function, and the a priori probability associated to the state transition \((S_{k-1},S_{k})\). Thus, under the AWGN and perfect CSI knowledge assumptions, it is given by
\[ \gamma_{k}(S_{k-1},S_{k}) = A_k e^{-\frac{1}{2} \sum_{l=0}^{N_c} \sum_{m=p}^{N_c} \sum_{p=1}^{N_c} d_{k}^{ij} L_a(d_{k}^{ij})} \]

where \( A_k \) is a constant. In (6) \( \hat{x}_{k-p}^{ij} \) and \( \hat{d}_{k}^{ij} \) are respectively the constellation symbols associated to the state transition \((S_{k-1},S_{k})\), and the bits that cause such a transition. At the first pass through the equalizer no a priori information on the coded bits is assumed, thus \( L_a(d_{k}^{ij}) \) is set to zero. In the following iterations, the a priori log-likelihood ratios for the bits of each transmit antenna branch associated to all state transitions are computed from the decoder outputs. This a priori knowledge helps improve the metric quality, and de-couple the signals that are simultaneously transmitted by the transmit array.

Finally, from a complexity standpoint, the implementation of both the equalizer and the decoder MAP algorithms can be simplified by the known max-log-MAP approximation [10].

V. A 2 BITS/S/Hz MODEM BASED ON STBICM

As an example we consider a STBICM based system that achieves a spectral efficiency of 2 bits/sHz and deploys \( N_t = 2 \) transmit antennas. We choose a tail terminated convolutional code with memory 2 (4 states) and rate \( R_a = 1/2 \). The code polynomials are in octal notation (7,5). We encode blocks (i.e. frames) of 260 bits. The stream of 260 parity bits from the first polynomial is sent to the first antenna while the stream of 260 parity bits from the second polynomial is sent to the second antenna. Each stream of bits is interleaved with a short random interleaver of length 260. In this way, as shown in [6], the code achieves full spatial transmit diversity. Finally, the interleaved bits are mapped to 4-PSK constellation points according to the Gray rule.

We have evaluated through simulations (figures 3 and 4) the bit-error-rate BER and frame-error-rate FER performance of this system. We have assumed a two equal power rays channel. The rays are spaced by one symbol period, are statistically independent, and Rayleigh faded. Both a fast fading scenario and a block fading (static over 130 complex symbols) scenario are considered. It should be noted that while in the former case the fading channel is completely time-uncorrelated, in the latter case the inner interleaver herein considered leaves the block fading channel completely correlated. However, it is deployed to de-correlate the soft decisions exchanged between the equalizer and the decoder and to minimize the error propagation from feedback. In principle, better results are expected with longer length interleavers that, however, would increase the transmission delay. Furthermore, both the equalizer and the decoder stages are implemented using the max-log-MAP approximation, and up to three passes through the decoder are considered. The equalizer has 16 states with 16 transitions per state. Ideal channel state information is assumed.

As a baseline the performance of uncoded 4-PSK in both flat fading and 2-ray fading is also shown. In the latter case detection is performed through equalization with a MAP based equalizer.

It should be noted that according to (2) the curves do not show the antenna gain when two receive antennas are deployed.

A. STBICM with 2 Transmit and 1 Receive Antennas

Consider STBICM with two transmit antennas and one receive antenna. Then, Fig. 3 and Fig. 4 show that STBICM is capable of outperforming the uncoded system that deploys either one or two receive antennas. Running multiple iterations significantly improves the performance of STBICM. Most of the gain is achieved at the second pass (curves labeled with \( ir = 1 \)) through the convolutional decoder.

The gain in BER at \( BER = 10^{-5} \) in block fading with 3 iterations (Fig. 3a) is more than 10 dB over the uncoded system with a single receive antenna, and about 2 dB over the uncoded system with dual receive diversity (although we do not consider the receive antenna gain). In fast fading (Fig. 3b), the gain in BER is more than 16 dB over the uncoded system with one receive antenna, and 8.5 dB over the dual receive diversity uncoded system.

The gain in FER at \( FER = 10^{-3} \) in block fading with STBICM and 3 iterations (Fig. 4a) is more than 13 dB over the single receive diversity uncoded system, and about 4 dB over the dual diversity uncoded system. In fast fading (Fig. 4b) STBICM yields about 9.5 dB gain over the dual diversity uncoded system.

The diversity gain (slope of the curves) of STBICM over the uncoded system is clear. STBICM with iterative equalization/decoding is capable of exploiting the spatial, the multi-path, and the temporal diversities.

B. Performance of STBICM with 2 Transmit and 2 Receive Antennas

Performance can be further improved by deploying two receive antennas in the STBICM scheme. This improvement is more pronounced in block fading, about 6 dB improvement in BER and 4 dB improvement in FER over the single receive antenna STBICM system.
between the decoder and the equalizer through multiple equalization/decoding iterations.

A 2 bits/s/Hz system deploying 2 transmit antennas is described, and its performance evaluated by computer simulations, assuming perfect knowledge of the channel state information. By applying the iterative decoding procedure with just one receive antenna, such a system is capable of outperforming in BER/FER the uncoded system that deploys both single and dual receive diversity. Most of the gain is achieved with only two iterations, thus with a limited decoding complexity. The performance improvements are obtained in a frequency selective 2-ray channel with both block and fast fading. As expected higher gains are found in the fast fading scenario. However, even for the block fading case the gains are remarkable. The performance of STBICM in time-correlated fading channels with practical interleaving depths shall be between the bounds achieved with the block and fast fading channel models.

Based on the above performance results, STBICM is a promising alternative to space-time trellis codes [3] and space-time block codes [5] for designing spectrally efficient and reliable wireless systems. A performance comparison of space-time bit-interleaved coded modulation and space-time block and trellis codes is currently under investigation, including practical estimation of the channel state information.

VI. CONCLUSIONS

A space-time coding approach based on bit-interleaved coded modulation with multiple transmit antennas, referred to as space-time bit-interleaved coded modulation [6], has been presented. Decoding of such a scheme has been addressed in frequency selective fading channels. The decoding strategy follows two individually optimal steps in an iterative procedure. In the first step a MIMO soft-in soft-out MAP equalizer is deployed, followed by a soft-in soft-out MAP convolutional decoder. Soft information is exchanged between the decoder and the equalizer through multiple equalization/decoding iterations.

REFERENCES