ANALYSIS OF THE ROBUSTNESS OF FILTERED MULTITONE MODULATION SCHEMES OVER SATELLITE CHANNELS

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ABSTRACT

In this paper we analyze the performance of Filtered Multitone (FMT) modulation systems in satellite channels. FMT is a generalized OFDM scheme that deploys sub-channel shaping filters. It is a spectral efficient scheme that can support orthogonal multiuser transmission. Herein, we carry out a detailed analysis to understand whether it yields increased robustness, compared to OFDM, in typical LEO satellite channels. We consider an asynchronous multiuser scenario that introduces carrier frequency offsets, time offsets, and time/frequency channel selectivity. The analysis that we present in this paper allows to benchmark the multitone system and understand how robust it is to frequency selective time-variant fading and carrier Doppler shifts. Quasi-closed form expressions in the case of rectangular frequency domain pulses and raised cosine pulses are derived for the signal-to-interference power ratio at the receiver. It is found that the sub-channel spectral containment of the FMT system can yield increased performance compared to OFDM.

1. INTRODUCTION

In this paper we analyze the performance of multicarrier modulation [1] based architectures in satellite time-frequency selective channels. We consider low earth orbit (LEO) channels where the satellite elevation angle seen by a given user changes continuously. This, together with the terminal movement, introduces channel time variations. The satellite movement also induces high carrier Doppler shifts [2], [3]. Furthermore, multipath propagation, and thus frequency selective fading, can be present in urban areas. Multicarrier modulation and in particular orthogonal frequency division multiplexing (OFDM) has been proposed as an attractive modulation technique for LEO satellite communications [2]. In this paper, we investigate the performance of a more general multicarrier scheme that is referred to as Filtered Multitone (FMT) modulation. FMT is a discrete time multicarrier system that deploys sub-channel shaping pulses (Fig.1) [4]. OFDM can be viewed as an FMT scheme that deploys rectangular time domain filters. OFDM is also referred to as discrete

Satellite Communications and Navigation Systems, E. Del Re, M. Ruggieri (ed.s), Springer, pp. 599-612, 2008. ISBN:978-0-387-47522-6

multitone modulation (DMT). FMT modulation has been proposed for spectral efficient transmission over broadband frequency selective channels both in wireline [4] and in wireless scenarios [5]. It has interesting properties in terms of spectral efficiency, efficient implementation, and capability to support orthogonal multiuser transmission [5]. The design of the sub-channel filters, and the choice of the sub-carrier spacing in an FMT system aims at subdividing the spectrum in a number of sub-channels that do not overlap in the frequency domain, such that we can avoid the ICI and get low ISI contributions. In an OFDM system the insertion of a cyclic prefix longer than the channel time dispersion is such that the ISI and ICI are eliminated, and the receiver simplifies to a simple one-tap equalizer per sub-channel.

Although multitone systems are robust to the channel frequency selectivity, they are sensitive to carrier frequency offsets, phase noise [6], and channel fast time variations [7], [8]. In [9] we have studied the performance limits of FMT modulation and we have shown that FMT can provide both frequency and time diversity gains when optimal multi-channel equalization is used. However, if complexity is an issue, it is likely that linear single channel equalizers are used [10]. In this paper we consider a multiuser FMT system where users are multiplexed via the assignment of a number of available sub-channels. In particular, we consider the uplink scenario where users are asynchronous. Our objective is to determine how robust the multiuser system is to users' time offsets, carrier frequency offsets, and channel time/frequency selectivity. The unified analysis that we carry out in this paper allows to evaluate the performance of both the FMT and the OFDM system.

This paper is organized as follows. In Section 2 we describe the multiuser FMT architecture. We particularize the description for the presence of time-frequency offsets only, and time-frequency channel selectivity only. In Section 3 we analyze the signal and interference power at the receiver outputs and we specialize the results to the FMT and OFDM cases. In Section 4 we report a performance comparison between FMT and OFDM. Finally, the conclusions follow.

2. MULTIUSER FMT SYSTEM MODEL

An asynchronous multiuser FMT architecture is depicted in Fig.1. The complex baseband transmitted signal $x^{(u)}(nT)$ of user u is obtained by a filter bank modulator with prototype pulse $g(nT)$ and sub-channel carrier frequency $f_k = k/(MT)$, $k = 0,...,M-1$, with T being the transmission period

$$x^{(u)}(nT) = \sum_{k=0}^{M-1} x^{(u,k)}(nT) \tag{1}$$

$$x^{(u,k)}(nT) = \sum_{m\in\mathbb{Z}} a^{(u,k)}(mT_0) g(nT - mT_0) e^{j2\pi f_k nT} , \tag{2}$$

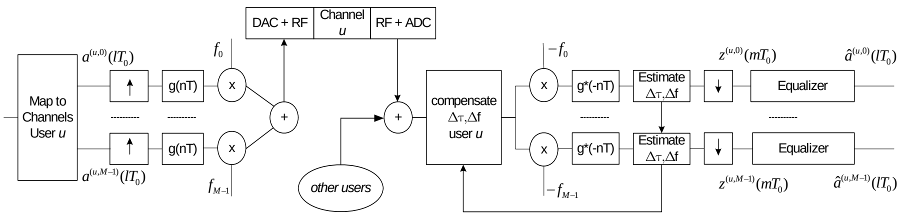

Fig. 1. Multiuser FMT system model with highlighted transmitter and receiver of user *u*.

where $a^{(u,k)}(mT_0)$ is the k -th sub-channel data stream of user u that we assume to belong to the M-PSK/M-QAM constellation set and that has rate $1/T_0$ with $T_0 = NT \geq MT$. If the sub-carrier spacing $f_k - f_{k-1}$ is larger than $1/T_0$ the scheme is referred to as non-critically sampled FMT, otherwise if $f_k - f_{k-1} = 1/T_0$ it is referred to as critically sampled FMT. In ideal FMT the prototype pulse has impulse response $g(nT) = \text{sinc}\left(nT / T_0\right)$. In this case a frequency guard equal to $f_G = 1/MT - 1/NT$ exists between sub-channels. A practical choice for the prototype pulse is to use a root-raised-cosine pulse. It is interesting to note that (2) allows to represent also a cyclically prefixed (CP) OFDM signal when the sub-carrier spacing is $1/MT$ and the prototype pulse is defined as $g(nT) = \text{rect}\left(nT / T_0\right)$. The interpolation factor N is chosen to increase the frequency separation between sub-channels, thus to minimize the amount of inter-carrier interference (ICI) and multiple access interference (MAI) at the receiver side. A possible efficient implementation of the transmitter that is based on polyphase filtering is described in [4]. The discrete time signal is digital-to-analog converted, RF modulated, and transmitted over the air. Distinct FMT sub-channels can be assigned to distinct users. In this case, the symbols are set to zero for the unassigned FMT sub-channels:

$$a^{(u,k)}(mT_0) = 0 \quad \text{for} \quad k \notin K_u\,, \tag{3}$$

where K_u denotes the set of M_u sub-channel indices assigned to user u .

At the receiver, after RF demodulation, and analog-to-digital conversion, the discrete time received signal can be written as

$$y(iT) = \sum_{u=1}^{N_U} \sum_{k=0}^{M-1} \sum_{n \in \mathbb{Z}} x^{(u,k)}(nT) g_{CH}^{(u)}(iT - nT; iT) + \eta(iT)\,, \tag{4}$$

where N_U is the number of users, $g_{CH}^{(u)}(nT; mT)$ is the channel impulse response and $\eta(iT)$ is the additive Gaussian noise with zero mean contribution. Then $y(iT)$ is passed through an analysis filter bank with prototype pulse

$h^{(u,k)}(nT)$. The sampled output at rate $1/T_0$ corresponding to user u and sub-channel k is

$$z^{(u,k)}(lT_0) = \sum_{i\in\mathbb{Z}} y(iT)h^{(u,k)}(lT_0 - iT) \,. \tag{5}$$

Defining in a convenient way the channel impulse response $g_{CH}^{(u)}(nT;mT)$ and the pulse $h^{(u,k)}(nT),$ we can analyze the effects of time and frequency offsets and of channel time-frequency selectivity as it is shown in the next sections.

2.1 Time and Frequency Offsets

In this section we consider only the effects of time and frequency offsets. Thus, we define

$$g_{CH}^{(u)}(iT - nT; iT) = e^{j2\pi\,\Delta_f^{(u)} iT}\delta(iT - nT - \Delta_\tau^{(u)}) \,, \tag{6}$$

where $\Delta_\tau^{(u)}$ is the time offset and $\Delta_f^{(u)}$ is the carrier frequency offset of user u . Note that we assume the time/frequency offsets to be identical for all sub-channels that are assigned to a given user.

Substituting (6) into (4) we can write the received signal as follows

$$y(iT) = \sum_{u=1}^{N_U}\sum_{k=0}^{M-1}\sum_{n\in\mathbb{Z}} x^{(u,k)}(iT - \Delta_\tau^{(u)})e^{j2\pi\,\Delta_f^{(u)} iT} + \eta(iT) \,. \tag{7}$$

We consider a single user based FMT receiver (Fig. 1) where we first acquire time/frequency synchronization with each active user. Then, for each user, we compensate the time/frequency offsets, we run FMT demodulation via a bank of filters that is matched to the transmitter bank, and we sample the outputs at rate $1/T_0$. In formulas we have

$$h^{(u,k)}(nT) = h(nT + \Delta_\tau^{(u)})e^{j2\pi(f_k + \Delta_f^{(u)})(nT + \Delta_\tau^{(u)} - lT_0)} \tag{8}$$

and substituting (8) into (6) we can write the sub-channel output as

$$\begin{aligned} z^{(u,k)}(lT_0) &= \sum_{i\in\mathbb{Z}} y(iT)h(lT_0 - iT + \Delta_\tau^{(u)})e^{j2\pi\left(f_k + \Delta_f^{(u)}\right)\left(-iT + \Delta_\tau^{(u)}\right)} \\ &= \sum_{u'=1}^{N_U}\sum_{k'=0}^{M-1}\sum_{l\in\mathbb{Z}} a^{(u',k')}(mT_0)g_{EQ}^{(u',k'),(u,k)}(lT_0;mT_0) + \eta^{(u,k)}(lT_0), \end{aligned} \tag{9}$$

where $\eta^{(u,k)}(lT_0)$ is the sequence of filtered noise samples, and where $g_{EQ}^{(u',k'),(u,k)}(lT_0;mT_0)$ is the multi-channel impulse response.

We can rewrite (9) as follows

$$z^{(u,k)}(lT_0) = \sum_{l \in \mathbb{Z}} a^{(u,k)}(mT_0) g_{EQ}^{(u,k),(u,k)}(lT_0; mT_0) + ICI^{(u,k)}(lT_0) + MAI^{(u,k)}(lT_0) + \eta^{(u,k)}(lT_0), \tag{10}$$

where we highlight the fact that the sub-channel filter output of index k may suffer from ISI, ICI (from the sub-channels of index $k' \neq k$ that are assigned to user u), and MAI (from all sub-channels that belong to the other users) as a consequence of frequency overlapping sub-channels, time/frequency offsets, and channel time/frequency selectivity. When the sub-channels do not overlap, e.g., with ideal root-raised-cosine pulses with appropriate sub-carrier spacing, and the carrier frequency offsets of all users do not exceed half the frequency guard the ICI and MAI components are zero. Some intersymbol interference over each sub-channel may be present and can be counteracted with sub-channel equalization.

In the uplink, multiuser OFDM is severely affected by time misalignments and carrier frequency offsets. This is due to the fact that in conventional OFDM, sub-channels exhibit *sinc* like frequency response, therefore their orthogonality can be easily lost in the absence of precise synchronization [2]. In an asynchronous multiuser environment, increased robustness and better performance can be obtained with filtered multitone (FMT) modulation architectures where the sub-channels are shaped with appropriate frequency concentrated pulses as proved above [5].

2.2 Channel Time-Frequency Selectivity

In this section we consider only the effects of time-frequency selectivity of the channel. We model the baseband channel with a discrete time-variant filter $g_{CH}^{(u)}(nT; mT)$ that comprises the effect of the DAC and ADC stages as follows

$$g_{CH}^{(u)}(iT - nT; iT) = \sum_{p \in \mathrm{P}} \alpha_p^{(u)}(iT) \delta(iT - nT - pT), \tag{11}$$

where the time-variant tap amplitudes $\alpha_p(nT)$ $p \in \mathrm{P} \subset \mathbb{Z}$ are stationary complex Gaussian. Under the WSSU isotropic scattering model [3] they have uncorrelated quadrature components, with zero mean, correlation

$$r_{p,p'}(nT) = E\left[\alpha_p(mT)^* \alpha_{p'}(mT + nT)\right] = \delta_{p,p'} \Omega_p J_0(2\pi f_D nT), \tag{12}$$

and power-spectral density $R_{p,p'}(f) = \delta_{p,p'} rep_{1/T}\left\{R_p(f)\right\}$ with

$$R_p(f) = \begin{cases} 0 & |f| > f_D \\ \Omega_p / (\pi f_D)\left(1 - (f/f_D)^2\right)^{-1/2} & |f| < f_D \end{cases} \tag{13}$$

Substituting (11) into (4) we obtain

$$y(iT) = \sum_{u=1}^{N_U} \sum_{k=0}^{M-1} \sum_{p \in \mathrm{P}} \alpha_p^{(u)}(iT) x^{(u,k)}(iT - pT) + \eta(iT) . \tag{14}$$

At the receiver we consider the following filter

$$h^{(u,k)}(nT) = h(nT) e^{j2\pi f_k (nT - lT_0)} \tag{15}$$

and substituting (15) into (5) we can write the $\hat{k}$-th sub-channel filter-bank output as

$$\begin{aligned} z^{(\hat{k})}(lT_0) &= \sum_{i \in \mathbb{Z}} y(iT) e^{-j2\pi f_{\hat{k}} iT} h(lT_0 - iT) \\ &= \sum_{k=0}^{M-1} \sum_{m=-\infty}^{\infty} a^{(k)}(mT_0) g_{EQ}^{(k,\hat{k})}(lT_0; mT_0) + \eta^{(\hat{k})}(lT_0), \end{aligned} \tag{16}$$

where $g_{EQ}^{(k,\hat{k})}(lT_0;mT_0)$ the equivalent impulse response between the input sub-channel k and output sub-channel $\hat{k}$.

It follows that the output in the absence of noise is

$$z^{(\hat{k})}(lT_0) = a^{(\hat{k})}(lT_0) g_{EQ}^{(\hat{k},\hat{k})}(lT_0; lT_0) + ISI^{(\hat{k})}(lT_0) + ICI^{(\hat{k})}(lT_0) , \tag{17}$$

where the first term represents the useful data contribution, the second additive term is the ISI contribution, the third term is the ICI contribution. In this case MAI is not present because for simplicity of the analytical evaluation we consider a single user scenario. An in-depth analysis of the signal over interference power ratio is treated in the next sections.

3. ANALYTICAL EVALUATION OF THE INTERFERENCE IN TIME-FREQUENCY SELECTIVE CHANNELS

Our objective is to determine the robustness of the system to the channel time and frequency selectivity as a function of the design parameters. To do so we evaluate the power of the interference components. The analysis is quite general and applies both to FMT and OFDM. The results have a practical relevance because allow to understand the sources of interference as a function of the design parameters. In the following we assume the data symbols to be i.i.d. with zero mean, and average power $M_a^{(k)} = E\left[| a^{(k)}(mT_0) |^2 \right]$.

First, it should be noted that

$$z^{(\hat{k})}(lT_0) = \sum_{k=0}^{M-1} z^{(k,\hat{k})}(lT_0) + \eta^{(\hat{k})}(lT_0) \tag{18}$$

where

$$z^{(k,\hat{k})}(lT_0) = \sum_{m=-\infty}^{\infty} a^{(k)}(mT_0) g_{EQ}^{(k,\hat{k})}(lT_0; mT_0) \quad (19)$$

is the contribution of the data stream transmitted on sub-channel k to the filter output of index $\hat{k}$. Further, the average power of (19) equals

$$M_z^{(k,\hat{k})} = E[|z^{(k,\hat{k})}(lT_0)|^2] = M_a^{(k)} \sum_m E\left[\left|g_{EQ}^{(k,\hat{k})}(lT_0; mT_0)\right|^2\right], \quad (20)$$

where the second equality holds with independent zero mean data symbols.

With the WSSU scattering tapped delay line channel model we can write

$$\begin{aligned} M_z^{(k,\hat{k})} = M_a \sum_m \sum_p \sum_{i,i'} r_{\alpha_p}(iT) g^{(k)}(iT + i'T - pT + lT_0 - mT_0) \\ \times h^{(\hat{k})}(-iT - i'T) g^{(k)*}(i'T - pT + lT_0 - mT_0) h^{(\hat{k})*}(-i'T). \end{aligned} \quad (21)$$

It should be noted that if we fix $k = \hat{k}$ in (21), and we isolate the term that corresponds to $m = 0$ we obtain the average signal power $S^{(\hat{k})} = M_a^{(\hat{k})} E\left[|g_{EQ}^{(\hat{k},\hat{k})}(lT_0; lT_0)|^2\right]$, while the sum of all other terms yields the ISI power $M_{ISI}^{(\hat{k})} = E\left[|ISI^{(\hat{k})}(lT_0)|^2\right]$. On the contrary, the total power of the ICI can be obtained as $M_{ICI}^{(\hat{k})} = \sum_{k \neq \hat{k}} M_z^{(k,\hat{k})}$.

To proceed we define the following function

$$gh^{(k,\hat{k})}(iT; sT) = g^{(k)}(iT - sT) h^{(\hat{k})}(-iT) \quad (22)$$

and we can rewrite (21) as

$$M_z^{(k,\hat{k})} = \frac{M_a^{(k)}}{T} \sum_m \sum_p \sum_i r_{\alpha_p}(iT) c_{gh}^{(k,\hat{k})}(iT; pT + mT_0 - lT_0), \quad (23)$$

where the autocorrelation of function (22) is defined as

$$c_{gh}^{(k,\hat{k})}(iT; sT) = T \sum_{i'} gh^{(k,\hat{k})}(iT + i'T; sT) gh^{(k,\hat{k})*}(i'T; sT). \quad (24)$$

The expression (23) is general and can be particularized for a certain choice of the sub-channel pulse as shown in the next sub-sections. In some cases it is convenient to calculate it in the frequency domain as

$$\begin{aligned} M_z^{(k,\hat{k})} &= \frac{M_a^{(k)}}{T} \sum_m \sum_p \sum_i r_{\alpha_p}(iT) \int_{-1/2T}^{1/2T} C_{gh}^{(k,\hat{k})}(f; pT + mT_0 - lT_0) e^{j2\pi f\, iT} df \\ &= \frac{M_a^{(k)}}{T^2} \sum_m \sum_p \int_{-1/2T}^{1/2T} R_{\alpha_p}(-f) C_{gh}^{(k,\hat{k})}(f; pT + mT_0 - lT_0) df \end{aligned} ,(25)$$

where we have used the discrete-time Fourier transforms $C_{gh}^{(k,\hat{k})}(f;sT) = T\sum_n c_{gh}^{(k,\hat{k})}(nT;sT)e^{-j2\pi fnT}$ and $R_{\alpha_p}(f) = T\sum_n r_{\alpha_p}(nT)e^{-j2\pi fnT}$. The first transform can be written as

$$C_{gh}^{(k,\hat{k})}(f;sT) = \left|GH^{(k,\hat{k})}(f;sT)\right|^2, \quad (26)$$

where $GH^{(k,\hat{k})}(f;sT)$ is the discrete-time Fourier transform of the function (22)

$$GH^{(k,\hat{k})}(f;sT) = rep_{1/T}\left[\left(G^{(k)}(f)e^{-j2\pi fsT}\right) * H^{(\hat{k})}(-f)\right] \quad (27)$$

and $G^{(k)}(f) = F\left[g^{(k)}(t)\right]$, $H^{(k)}(f) = F\left[h^{(k)}(t)\right]$.

3.1 Results for the FMT Case

In FMT the receiver filter-bank is matched to the transmitter filter-bank, i.e., $h^{(k)}(nT) = g^{(k)^*}(-nT)$. Thus, $H^{(k)}(f) = F\left[g^{(k)^*}(-nT)\right] = G^{(k)*}(f)$.

To proceed we need to define the prototype pulse. In the following sub-sections we obtain results when we deploy a sinc pulse, and a root-raised co-sine pulse.

3.1.1 FMT with Sinc Prototype Pulse

If we consider a sinc prototype pulse (rectangular frequency domain pulse)

$$g(nT) = \text{sinc}\left(\frac{nT}{T_0}\right) \quad (28) \qquad G(f) = T_0 rep_{1/T}\left\{rect\left(fT_0\right)\right\} \quad (29)$$

we obtain that

$$M_z^{(k,\hat{k})} = \frac{M_a^{(k)}T_0^{\,4}}{T^2\pi f_D}\sum_m\sum_p \Omega_p \int_{-f_D}^{f_D} \frac{(|\,f + f_{\hat{k}} - f_k\,| - 1/T_0)^2}{\sqrt{1-(f/f_D)^2}} \times \text{sinc}^2\left((|\,f + f_{\hat{k}} - f_k\,| - \frac{1}{T_0})(pT + mT_0)\right)df \quad (30)$$

assuming $f_D \le 1/MT$ (smaller than the sub-carrier spacing). It can be shown that (30) can be computed also in the time-domain as follows

$$M_z^{(k,\hat{k})} = \frac{M_a^{(k)}N^3}{\pi^2}\sum_m\sum_p\sum_i r_{\alpha_p}(iT)e^{j2\pi(f_k - f_{\hat{k}})iT} \times \frac{\text{sinc}\left(\frac{2(p+mN)}{N}\right) - \text{sinc}\left(\frac{2i}{N}\right)}{i^2 - (p+mN)^2}. \quad (31)$$

Now, the signal power can be obtained by isolating the term in (30) or (31) of index $m = 0$

$$S^{(\hat{k})} = \frac{2M_a^{(\hat{k})}T_0^{\,4}}{T^2\pi f_D}\sum_p \Omega_p \int_0^{f_D} \frac{(f - 1/T_0)^2}{\sqrt{1-(f/f_D)^2}} \text{sinc}^2\left((f - \frac{1}{T_0})pT\right)df \, . \quad (32)$$

The power of the sub-channel ISI is

$$M_{ISI}^{(\hat{k})} = \frac{2M_a^{(\hat{k})}T_0^{\,4}}{T^2\pi f_D}\sum_{m\neq 0}\sum_p \Omega_p \int_0^{f_D} \frac{(f - 1/T_0)^2}{\sqrt{1-(f/f_D)^2}} \times \text{sinc}^2\left((f - \frac{1}{T_0})(pT + mT_0)\right)df \, . \quad (33)$$

The total power of the ICI (assuming sub-channel data streams with identical power M_a) is

$$M_{ICI}^{(\hat{k})} = \sum_{k\neq\hat{k}} M_z^{(k,\hat{k})} = \frac{2M_a T_0^{\,4}}{T^2\pi f_D}\sum_m\sum_p \Omega_p \int_{f_G}^{f_D} \frac{(f - f_G)^2}{\sqrt{1-(f/f_D)^2}} \times \text{sinc}^2\left((f - f_G)(pT + mT_0)\right)df \, . \quad (34)$$

Now, it should be observed that if $f_D \leq f_G$, (34) is always zero for $k \neq \hat{k}$. Therefore, ICI is not present. Otherwise, if $f_G < f_D \leq 1/MT$ only two adjacent sub-channels can generate ICI. This is a very interesting aspect of FMT which exhibits no ICI when band limited pulses are used and a frequency guard larger than the maximum Doppler is used between sub-channels. Clearly, fast fading can introduce some ISI because it distorts the received sub-channel pulse as we will discuss in more detail in the following. If the channel is flat and static then there is neither ISI nor ICI, i.e., the system is orthogonal.

3.1.2 FMT with Root-Raised Cosine Prototype Pulse

Another possibility is to use root-raised cosine pulses

$$g(nT) = \text{rrcos}\left(\frac{nT}{T_0}\right) \quad (35) \qquad G(f) = T_0 rep_{1/T}\{\text{RRCOS}(fT_0)\} \quad (36)$$

where

$$\mathrm{rrcos}(t)=\mathrm{sinc}\left(\alpha t+\frac{1}{4}\right)\frac{\sin\left(\pi\left(t-\frac{1}{4}\right)\right)}{4t}+\mathrm{sinc}\left(\alpha t-\frac{1}{4}\right)\frac{\sin\left(\pi\left(t+\frac{1}{4}\right)\right)}{4t} \quad (37)$$

$$\mathrm{RRCOS}(f)=\begin{cases} 1 & 0\le|f|<f_1 \\ \cos\left(\frac{\pi}{2}\frac{|f|-f_1}{\alpha}\right) & f_1\le|f|<f_2 \\ 0 & |f|>f_2 \end{cases}. \quad (38)$$

with $f_1=0.5(1-\alpha)$ and $f_2=0.5(1+\alpha)$. Note that α is the roll-off factor of the filter. With this pulse the computation of (26) requires some cumbersome algebra. For space limitation we don't report herein the full analytical results but just the graphical comparison in Section 4.

3.2 Results for the OFDM Case

Now, we consider a CP-OFDM scheme for which the subchannel filters are $g^{(k)}(iT)=\mathrm{rect}(iT/T_0)e^{j2\pi f_k iT}$ and $h^{(k)}(iT)=\mathrm{rect}(-iT/T_1)e^{j2\pi f_k iT}$ where we have $T_0=NT$, $T_1=MT$, and $f_k=k/(MT)$. Using, the power of the interference seen by sub-channel $\hat{k}$ reads

$$\begin{aligned} M_z^{(k,\hat{k})}(lT_0) &= M_a^{(k)}\sum_m\sum_p\sum_{i=0}^{M-1}\sum_{i'=0}^{M-1} r_{\alpha_p}(iT-i'T)e^{j2\pi(f_k-f_{\hat{k}})(iT-i'T)} \\ &\times\mathrm{rect}\left(\frac{iT-lT_0+mT_0+pT}{T_0}\right)\mathrm{rect}\left(\frac{i'T-lT_0+mT_0+pT}{T_0}\right). \end{aligned} \quad (39)$$

Thus, the power of the useful term is

$$S^{(\hat{k})}=M_a^{(\hat{k})}\sum_p\sum_{i=0}^{M-1}\sum_{i'=0}^{M-1} r_{\alpha_p}(iT-i'T)\,\mathrm{rect}\left(\frac{iT+pT}{T_0}\right)\mathrm{rect}\left(\frac{i'T+pT}{T_0}\right). \quad (40)$$

The power of the sub-channel ISI is

$$\begin{aligned} M_{ISI}^{(\hat{k})} &= M_a^{(\hat{k})}\sum_{m\neq 0}\sum_p\sum_{i=0}^{M-1}\sum_{i'=0}^{M-1} r_{\alpha_p}(iT-i'T) \\ &\times\mathrm{rect}\left(\frac{iT+mT_0+pT}{T_0}\right)\mathrm{rect}\left(\frac{i'T+mT_0+pT}{T_0}\right). \end{aligned} \quad (41)$$

The power of the ICI is (assuming sub-channel data streams with identical power M_a)

$$M_{ICI}^{(\hat{k})} = M_{TOT}^{(\hat{k})} - (S^{(\hat{k})} + M_{ISI}^{(\hat{k})}) \,, \tag{42}$$

where the total power is

$$M_{TOT}^{(\hat{k})} = MM_a \sum_m \sum_p r_{\alpha_p}(0) \sum_{i'=0}^{M-1} \mathrm{rect}^2\left(\frac{i'T + mT_0 + pT}{T_0}\right). \tag{43}$$

4. FREQUENCY SELECTIVE FADING AND FLAT FADING

From the general expressions that we have obtained in the previous section we can evaluate the signal-to-interference power ratio:

$$SIR^{(k)} = \frac{S^{(k)}}{M_{ISI}^{(k)} + M_{ICI}^{(k)}} \,. \tag{44}$$

For ease of understanding we consider first a multipath channel with quasi-static fading, then a time-variant flat fading channel. The multipath channel has power delay profile $\Omega_p \sim e^{-pT/(\gamma T)}$. We truncate the channel at -20 dB obtaining N_P taps, and we normalize its power to one. The time-variant channel has the temporal correlation defined in (12).

4.1 FMT and OFDM Comparison in Frequency Selective Static Fading

Let us assume the channel to be quasi-static but frequency selective. Then, we can elaborate further the formulas and specialize the results as follows.

4.1.1 FMT with Sinc Prototype Pulse

$$S_{FMT}^{(\hat{k})} = M_a^{(\hat{k})} N^2 \sum_{p=0}^{N_P} \Omega_p \,\mathrm{sinc}^2\left(\frac{p}{N}\right), \tag{45}$$

$$M_{ISI-FMT}^{(\hat{k})} = M_a^{(\hat{k})} N^2 \sum_{m\neq 0} \sum_{p=0}^{N_P} \Omega_p \,\mathrm{sinc}^2\left(\frac{p+mN}{N}\right). \tag{46}$$

The total power of the ICI is always zero (assuming frequency confined pulses).

4.1.2 FMT with Root-Raised Cosine Prototype Pulse

$$S_{FMT}^{(\hat{k})} = M_a^{(\hat{k})} N^2 \sum_{p=0}^{N_P} \Omega_p \,\mathrm{rcos}^2\left(\frac{p}{N}\right) \tag{47}$$

$$M_{ISI-FMT}^{(\hat{k})} = M_a^{(\hat{k})} N^2 \sum_{m\neq 0} \sum_{p=0}^{N_P} \Omega_p \,\mathrm{rcos}^2\left(\frac{(p+mN)}{N}\right) \tag{48}$$

The total power of the ICI is always zero (assuming frequency confined pulses).

4.1.3 FMT with Rect Prototype Pulse (OFDM)

For the CP-OFDM system, the power of the useful term, the ISI and the ICI read as follows

$$S_{OFDM}^{(\hat{k})} = M_a^{(\hat{k})} \left(\sum_{p=0}^{\min(N-M,N_p)} \Omega_p M^2 + \sum_{p=N-M+1}^{\min(N-1,N_p)} \Omega_p (N-p)^2 \right) \tag{49}$$

$$M_{OFDM-ISI}^{(\hat{k})} = M_a^{(\hat{k})} \sum_{m\neq 0} \sum_{p=0}^{N_p} \Omega_p \sum_{i=0}^{M-1} \sum_{i'=0}^{M-1} \mathrm{rect}\left(\frac{iT + mT_0 + pT}{T_0} \right) \mathrm{rect}\left(\frac{i'T + mT_0 + pT}{T_0} \right) \tag{50}$$

$$M_{OFDM-ICI}^{(\hat{k})} = M_a \sum_{m} \sum_{p=0}^{N_p} \Omega_p \sum_{i'=0}^{M-1} \left[\mathrm{rect}\left(\frac{i'T + mT_0 + pT}{T_0} \right) \times \left(M - \sum_{i=0}^{M-1} \mathrm{rect}\left(\frac{iT + mT_0 + pT}{T_0} \right) \right) \right]. \tag{51}$$

In particular, as it is well known, when the channel is shorter than the CP ($N_p \le \mu = N - M$) the useful power is $S^{(\hat{k})} = M_a^{(\hat{k})} M^2 \sum_{p=0}^{N_P} \Omega_p$, while the ISI and ICI are zero.

In Fig.2.A we report the SIR as a function of the normalized delay spread for the FMT system while in Fig.2.B we consider the CP-OFDM system. The plot shows that the FMT architecture is robust to channel frequency selectivity. Indeed the CP-OFDM system maintains the orthogonality for channels shorter than the CP. But for channels longer than the CP it also suffers as a result of ISI and ICI.

4.2 FMT and OFDM Comparison in Flat Fast Fading

Now, let us assume the channel to be flat but time-variant. Elaborating further the formulas we obtain the following particular results.

4.2.1 FMT with Sinc Prototype Pulse

$$S_{FMT}^{(\hat{k})} = \frac{2M_a^{(\hat{k})} \Omega_0 N^4}{\pi} \left(\frac{(f_D T)^2 \pi}{4} - \frac{2 f_D T}{N} + \frac{\pi}{2N^2} \right), \tag{52}$$

$$M_{FMT-ISI}^{(\hat{k})} = \frac{M_a^{(\hat{k})} N^2 \Omega_0}{\pi^2} \sum_{m=1}^{\infty} \frac{1 - J_0(2\pi f_D m T_0)}{m^2}. \tag{53}$$

The total power of the ICI equals (assuming sub-channel data streams with identical power M_a)

$$M^{(\hat{k})}_{FMT-ICI} = \frac{2M_a T_0{}^4 \Omega_0}{T^2 \pi f_D} \sum_m \int_{f_G}^{f_D} \frac{(f - f_G)^2}{\sqrt{1-(f/f_D)^2}} \mathrm{sinc}^2\left((f - f_G) m T_0\right) df \; . \quad (54)$$

Note that (54) is zero when the sub-channels are separated more than the maximum Doppler.

4.2.2 FMT with Root-Raised Cosine Prototype Pulse

In the case of a root-raised cosine filter the useful and ISI term have complicated expressions that we don't report for space limitations. They are a combination of Bessel and StruveH functions [11]. For the ICI term instead we can't obtain a closed form, but as the sinc case we have that $M^{(\hat{k})}_{FMT-ICI}$ is zero if the sub-channels are separated more than the maximum Doppler.

4.2.3 FMT with Rect Prototype Pulse (OFDM)

For the CP-OFDM system the power of the useful term, the ISI and the ICI read as follows

$$S^{(\hat{k})}_{OFDM} = M_a \Omega_0 \sum_{i=0}^{M-1} \sum_{i'=0}^{M-1} J_0 (2\pi f_D T(i-i')) \,, \quad (55)$$

$$M^{(\hat{k})}_{OFDM-ICI} = M_a \Omega_0 \left(M^2 - \sum_{i'=0}^{M-1} \sum_{i=0}^{M-1} J_0 (2\pi f_D T(i-i')) \right). \quad (56)$$

The total power of the ISI is always zero. Note that (56) is identical to the one reported in [8]. In Fig.3.A we report the SIR as a function of the normalized delay spread for the FMT system, with sinc and root-raised cosine, while in Fig.3.B we consider the CP-OFDM system. The plot shows that the OFDM scheme is more robust than the ideal FMT scheme for large Doppler spreads. If we use root-raised cosine with roll-off $\alpha = 0.2$ we can improve the performance of the FMT. For moderate Doppler the performance of FMT can be improved also with sub-channel equalization.

Finally, we point out that when equalization is used in the FMT scheme we can exploit the sub-channel time-frequency diversity while the one tap equalizer in the OFDM scheme does not allow to pick any diversity gain [9]. As an example, we report in Fig.4 a comparison of bit-error rate performance in fast fading between FMT and OFDM that has been obtained via simulation. The OFDM scheme uses 128 tones while the FMT scheme uses 32 tones with a 11 taps sub-channel equalizer. The sub-channel filter in the simulation is a truncated root-raised-cosine pulse and 4-PSK modulation is used.

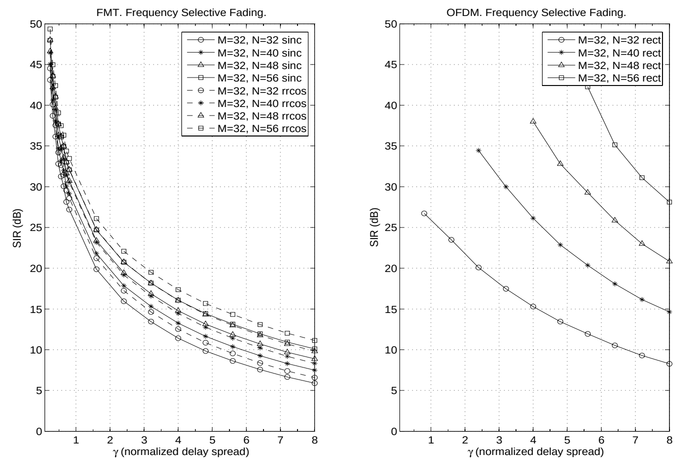

Fig. 2. SIR in frequency selective fading.

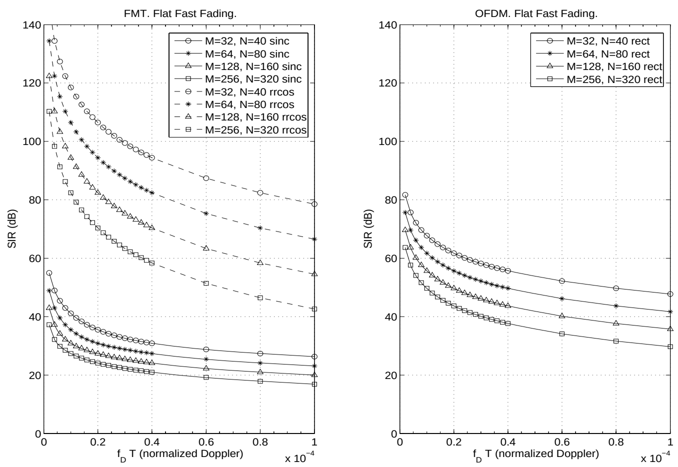

Fig. 3. SIR in flat time variant fading.

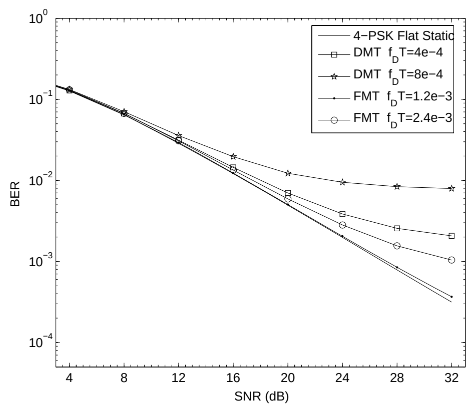

Fig. 4. BER performance in time variant fading for OFDM and FMT.

The figure shows that when equalization is used FMT has superior BER performance than OFDM.

5. CONCLUSIONS

We have presented an analysis of multiuser FMT in LEO satellite channels and in particular we have considered the effect of time-frequency offsets and time-frequency channel selectivity. We have obtained quasi-closed form expressions for the signal-to-interference power ratio in both FMT and OFDM systems. The sub-channel spectral containment of FMT yields increased robustness to the ICI and ISI compared to OFDM.

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